MEMOIRS

OF THE

AMERICAN ACADEMY

OF

ARTS AND SCIENCES.

NEW SERIES.
VOL. IX.—PART I.

CAMBRIDGE AND BOSTON:
WELCH & BIGELOW AND DAKIN & METCALF.
1867.

CONTENTS OF PART I.

OFFICERS OF THE AMERICAN ACADEMY.

FOR THE YEAR BEGINNING MAY, 1867.

President.

ASA GRAY.

Vice-President.

GEORGE T. BIGELOW.

STANDING COMMITTEES.

Rumford Committee.

JOSEPH LOVERING,
JOSIAH P. COOKE,
WOLCOTT GIBBS,
WILLIAM B. ROGERS,
FRANK H. STORER,
JOSEPH WINLOCK,
MORRILL WYMAN.

Committee of Publication.

JOSEPH LOVERING,
CHARLES W. ELIOT,
JEFFRIES WYMAN.

Committee on the Library.

JOHN BACON,
JOHN B. HENCK,
CHARLES PICKERING.

Committee of Finance.

ASA GRAY, *ex officio*,
JOHN C. LEE, "
THOMAS T. BOUVÉ.

Auditing Committee.

CHARLES E. WARE,
CHARLES J. SPRAGUE.

COUNCIL.

Class 1.

JOHN B. HENCK,
THOMAS HILL,
JOSEPH LOVERING.

Class 2.

LOUIS AGASSIZ,
CHARLES PICKERING,
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Class 3.

GEORGE E. ELLIS,
ANDREW P. PEABODY,
ROBERT C. WINTHROP.

CONTENTS OF PART II.

OFFICERS OF THE AMERICAN ACADEMY,

FOR THE YEAR BEGINNING MAY, 1872.

President.
ASA GRAY.

Vice-President.
CHARLES FRANCIS ADAMS.

JOSEPH LOVERING	*Corresponding Secretary.*
EDWARD C. PICKERING . . .	*Recording Secretary.*
EDMUND QUINCY	*Treasurer and Librarian.*

STANDING COMMITTEES.

Rumford Committee.

MORRILL WYMAN,
WOLCOTT GIBBS,
JOSIAH P. COOKE, JR.,
JAMES B. FRANCIS,
JOHN M. ORDWAY,
STEPHEN P. RUGGLES,
EDWARD C. PICKERING.

Committee of Publication.

W. W. GOODWIN,
JEFFRIES WYMAN,
JOHN TROWBRIDGE.

Committee on the Library.

H. G. DENNY,
J. D. RUNKLE,
JULES MARCOU.

Auditing Committee.

THEODORE LYMAN,
H. G. DENNY.

Committee on Finance.

ASA GRAY, EDMUND QUINCY, } *ex officio,*
THOMAS T. BOUVÉ.

COUNCIL.

Class 1.	*Class* 2.	*Class* 3.
THOMAS HILL,	ALEXANDER AGASSIZ,	ROBERT C. WINTHROP,
JOSIAH P. COOKE, JR.,	JEFFRIES WYMAN,	GEORGE E. ELLIS,
JOHN B. HENCK.	CHARLES PICKERING.	ANDREW P. PEABODY.

ERRATA.

Page 439, line 8 from the bottom, third word in the line; for *arc* read *time*.

Page 445, in last formula; insert $+$ between ω_0 and the fraction.

Page 455, in values of λ, Class II., Jan. 14: in set 4, *for* 42.991 *read* 41.991; in set 6, *for* 42.979 *read* 41.979.

Page 457, in value of λ, Class II., Jan. 28; *for* 42.152 *read* 43.152.

In the same table, on the right hand column, Division B, opposite Jan. 24, *for* 42.093 *read* 43.093; opposite Feb. 9, *for* 42.086 *read* 43.086.

Page 458, in values of $(\lambda - x)$, opposite Jan. 14; *for* 110.173 *read* 111.173, and *for* 110.181 *read* 111.181.

At the bottom of same page, in the column $(T' - T'')$, opposite Feb. 9, *for* -9.790 *read* $+9.790$; opposite Feb. 10, *for* -10.277 *read* $+10.277$.

Page 464, in line of means in the table: under Jan. 14, *for* 1.200 *read* 1.100; under Jan. 28, *for* 1.178 *read* 1.128.

The errors above named are simply misprints, which are made obvious by the context, and in no way affect the results.

On page 452, the first normal equation for obtaining the weight of $\Delta\theta$ should be $\Delta\theta + .616a - \frac{1}{24}$ (or .042) $= 0$. Hence the value of p (or the weight of $\Delta\theta$) is 19.86. Therefore, $r_0 = \frac{r}{\sqrt{p}} = 0^s.027$.

On page 453, according to the notation elsewhere used in the memoir, all the values of ΔT_0 for Cambridge and the *Hourly Rate* should have the sign *minus* prefixed to them. It should be understood that the *Hourly Rates* on pages 453 and 454 are the approximate values used in computing (ω_0), and not the corrected rates used in calculating the longitude.

The values found for the differences of longitude are not affected in any way by these corrections.

STATUTES

AND

STANDING VOTES

OF THE

AMERICAN ACADEMY OF ARTS AND SCIENCES.

(*Adopted May* 30, 1854: *amended September* 8, 1857, *November* 12, 1862, *May* 24, 1864, *November* 9, 1869, *and March* 14, 1870.)

CHAPTER I.

Of Fellows and Foreign Honorary Members.

1. The Academy consists of *Fellows* and *Foreign Honorary Members*. They are arranged in three classes, according to the Arts and Sciences in which they are severally proficient, viz.: Class I. The Mathematical and Physical Sciences; Class II. The Natural and Physiological Sciences; Class III. The Moral and Political Sciences. Each Class is divided into four Sections, viz.: Class I. Section 1. Mathematics; Section 2. Practical Astronomy and Geodesy; Section 3. Physics and Chemistry; Section 4. Technology and Engineering. Class II. Section 1. Geology, Mineralogy, and Physics of the Globe; Section 2. Botany; Section 3. Zoölogy and Physiology; Section 4. Medicine and Surgery. Class III. Section 1. Philosophy and Jurisprudence; Section 2. Philology and Archæology; Section 3. Political Economy and History; Section 4. Literature and the Fine Arts.

2. Fellows resident in the State of Massachusetts can alone vote at the meetings of the Academy.* They shall each pay to the Treasurer the sum of five dollars on admission, and an annual assessment of five dollars, with such additional sum, not exceeding five dollars, as the Academy shall, by a standing vote, from time to time determine.

3. Fellows residing out of the State of Massachusetts shall be known and distinguished as Associate Fellows. They shall not be liable to the payment of any fees or annual dues, but,

* The number of Resident Fellows is limited by the Charter to 200.

on removing within the State, shall be admitted to the privileges, and be subject to the obligations, of Resident Fellows. The number of Associate Fellows shall not exceed *one hundred*, of whom there shall not be more than *forty* in either of the three classes of the Academy.

4. The number of Foreign Honorary Members shall not exceed *seventy-five;* and they shall be chosen from among persons most eminent in foreign countries for their discoveries and attainments in either of the three departments of knowledge above enumerated. And there shall not be more than thirty Foreign Members in either of these departments.

CHAPTER II.

Of Officers.

1. There shall be a President, a Vice-President, a Corresponding Secretary, a Recording Secretary, a Treasurer, and a Librarian, which officers shall be annually elected, by written votes, at the Annual Meeting, on the day next preceding the last Wednesday in May.

2. At the same time and in the same manner, nine Councillors shall be elected, three from each Class of the Academy, who, with the President, Vice-President, and the two Secretaries, shall constitute a Council for Nomination. It shall also be the duty of this Council to exercise a discreet supervision over all nominations and elections, and to exert their influence to obtain and preserve a due proportion in the number of Fellows and Members in each of the Sections.

3. If any office shall become vacant during the year, the vacancy shall be filled by a new election, and at the next stated meeting.

CHAPTER III.

Of the President.

1. It shall be the duty of the President, and, in his absence, of the Vice-President or next officer in order, as above enumerated, to preside at the meetings of the Academy; to summon extraordinary meetings, upon any urgent occasion; and to execute or see to the execution of the Statutes of the Academy.

2. The President, or, in his absence, the next officer as above enumerated, is empowered to draw upon the Treasurer for such sums of money as the Academy shall direct. Bills presented on account of the Library, or the publications of the Academy, must be previously approved by the respective committees on these departments.

3. The President, or, in his absence, the next officer as above enumerated, shall nominate members to serve on the different committees of the Academy which are not chosen by ballot.

4. Any deed or writing, to which the common seal is to be affixed, shall be signed and sealed by the President, when thereto authorized by the Academy.

CHAPTER IV

Of Standing Committees.

1. At the Annual Meeting there shall be chosen the following Standing Committees, to serve for the year ensuing, viz.: —

2. The Committee of Finance, to consist of the President, Treasurer, and one Fellow chosen by ballot, who shall have charge of the investment and management of the funds and trusts of the Academy. The general appropriations for the expenditures of the Academy shall be moved by this Committee at the Annual Meeting, and all special appropriations from the general and publication funds shall be referred to or proposed by this Committee.

3. The Rumford Committee, of seven Fellows, to be chosen by ballot, who shall consider and report on all applications and claims for the Rumford Premium, also on all appropriations from the income of the Rumford Fund, and generally see to the due and proper execution of this trust.

4. The Committee of Publication, of three Fellows, to whom all memoirs submitted to the Academy shall be referred, and to whom the printing of memoirs accepted for publication shall be intrusted.

5. The Committee on the Library, of three Fellows, who shall examine the Library, and make an annual report on its condition and management.

6. An Auditing Committee, of two Fellows, for auditing the accounts of the Treasurer.

CHAPTER V.

Of the Secretaries.

1. The Corresponding Secretary shall conduct the correspondence of the Academy, recording or making an entry of all letters written in its name, and preserving on file all letters which are

received; and at each meeting he shall present the letters which have been addressed to the Academy since the last meeting. With the advice and consent of the President, he may effect exchanges with other scientific associations, and also distribute copies of the publications of the Academy among the Associate Fellows and Foreign Honorary Members, as shall be deemed expedient; making a report of his proceedings at the Annual Meeting. Under the direction of the Council for Nomination, he shall keep a list of the Fellows, Associate Fellows, and Foreign Honorary Members, arranged in their Classes and in Sections in respect to the special sciences in which they are severally proficient; and he shall act as secretary to the Council.

2. The Recording Secretary shall have charge of the Charter and Statute-book, journals, and all literary papers belonging to the Academy. He shall record the proceedings of the Academy at its meetings; and after each meeting is duly opened, he shall read the record of the preceding meeting. He shall notify the meetings of the Academy, and apprise committees of their appointment. He shall post up in the Hall a list of the persons nominated for election into the Academy; and when any individual is chosen, he shall insert in the record the names of the Fellows by whom he was nominated.

3. The two Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the proceedings of the Academy as may seem to them calculated to promote the interests of science.

CHAPTER VI.

Of the Treasurer.

1. The Treasurer shall give such security for the trust reposed in him as the Academy shall require.

2. He shall receive officially all moneys due or payable, and all bequests or donations made to the Academy, and by order of the President or presiding officer shall pay such sums as the Academy may direct. He shall keep an account of all receipts and expenditures; shall submit his accounts to the Auditing Committee; and shall report the same at the expiration of his term of office.

3. The Treasurer shall keep a separate account of the income and appropriation of the Rumford Fund, and report the same annually.

4. All moneys which there shall not be present occasion to expend shall be invested by the Treasurer, under the direction of the Finance Committee, on such securities as the Academy shall direct.

5. The Treasurer shall have the power, with the consent of the Committee of Finance, to remit the admission fee of five dollars, and likewise any annual assessment above the sum of two dollars, in all such cases as he shall deem reasonable and proper.

CHAPTER VII.

Of the Librarian and Library.

1. It shall be the duty of the Librarian to take charge of the books, to keep a correct catalogue of the same, and to provide for the delivery of books from the Library. He shall also have the custody of the publications of the Academy.

2. The Librarian, in conjunction with the Committee on the Library, shall have authority to expend, as they may deem expedient, such sums as may be appropriated, either from the Rumford or the General Fund of the Academy, for the purchase of books and for defraying other necessary expenses connected with the Library. They shall have authority to propose rules and regulations concerning the circulation, return, and safe-keeping of books; and to appoint such agents for these purposes as they may think necessary.

3. To all books in the Library procured from the income of the Rumford Fund, the Librarian shall cause a stamp or label to be affixed, expressing the fact that they were so procured.

4. Every person who takes a book from the Library shall give a receipt for the same to the Librarian or his assistant.

5. Every book shall be returned in good order, regard being had to the necessary wear of the book with good usage. And if any book shall be lost or injured, the person to whom it stands charged shall replace it by a new volume or set, if it belong to a set, or pay the current price of the volume or set to the Librarian; and thereupon the remainder of the set, if the volume belonged to a set, shall be delivered to the person so paying for the same.

6. All books shall be returned to the Library for examination, at least one week before the Annual Meeting.

CHAPTER VIII.

Of Meetings.

1. There shall be annually four stated meetings of the Academy; namely, on the day next preceding the last Wednesday in May (the Annual Meeting), on the second Wednesday in August, on the second Wednesday in November, and on the last Wednesday in January; to

be held in the Hall of the Academy, in Boston. At these meetings only, or at meetings adjourned from these and regularly notified, shall appropriations of money be made, or alterations of the statutes or standing votes of the Academy be effected.

2. Fifteen Fellows shall constitute a quorum for the transaction of business at a stated meeting. Seven Fellows shall be sufficient to constitute a meeting for scientific communications and discussions.

3. The Recording Secretary shall notify the meetings of the Academy to each Fellow residing in Boston and the vicinity; and he may cause the meetings to be advertised, whenever he deems such further notice to be needful.

CHAPTER IX.

Of the Election of Fellows and Honorary Members.

1. Elections shall be made by ballot, and only at the stated meetings in May, November, and January.

2. Candidates for election as Resident Fellows must be proposed by two or more Resident Fellows, in a recommendation signed by them, specifying the section to which the nomination is made; which recommendation shall be read at a stated meeting, and then stand on the nomination list during the interval between two stated meetings, and until the balloting. No person shall be elected a Resident Fellow, unless he shall have been resident in this Commonwealth one year next preceding his election; and any Resident Fellow, hereafter elected, who shall reside out of the Commonwealth for the term of five years, and shall discontinue the payment of his assessments during that time, shall be deemed to have abandoned his fellowship; provided, nevertheless, that this abandonment of fellowship for non-residence shall not apply to persons engaged in the service of the State, or of the United States.

3. The nomination of Associate Fellows shall take place in the manner prescribed in reference to Resident Fellows; and after such nomination shall have been publicly read at a stated meeting previous to that when the balloting takes place, it shall be referred to a Council for Nomination; and a written approval, authorized and signed at a meeting of said Council by at least seven of its members, shall be requisite to entitle the candidate to be balloted for. The Council may in like manner originate nominations of Associate Fellows; which must be read at a stated meeting previous to the election, and be exposed on the nomination list during the interval.

4. Foreign Honorary Members shall be chosen only after a nomination made at a meeting of the Council, signed at the time by at least seven of its members, and read at a stated meeting previous to that on which the balloting takes place.

5. Three fourths of the ballots cast must be affirmative, and the number of affirmative ballots must amount to eleven, to effect an election of Fellows or Foreign Honorary Members.

6. Each section of the Academy is empowered to present lists of persons deemed best qualified to fill vacancies occurring in the number of Foreign Honorary Members or Associate Fellows allotted to it; and such lists, after being read at a stated meeting, shall be referred to the Council for Nomination.

CHAPTER X.

Of Amendments of the Statutes.

1. All proposed alterations of the Statutes, or additions to them, shall be referred to a committee, and, on their report at a subsequent meeting, shall require for enactment a majority of two thirds of the members present, and at least eighteen affirmative votes.

2. Standing Votes may be passed, amended, or rescinded, at any stated meeting, by a majority of two thirds of the members present. They may be suspended by a unanimous vote.

CHAPTER XI.

Of Literary Performances.

1. The Academy will not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its publications.

RUMFORD PREMIUM.

In conformity with the last will of Benjamin Count Rumford, granting a certain fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his said will, the Academy is empowered to make from the income of said fund, as it now exists, at any annual meeting, an award of a gold and silver medal, being together of the intrinsic value of three hundred dollars, as a premium, to the author of any important discovery or useful improvement in light or in heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference being always given to such discoveries as shall, in the opinion of the Academy, tend most to promote the good of mankind; and to add to such medals, as a further premium for such discovery and improvement, if the Academy see fit so to do, a sum of money not exceeding three hundred dollars.

STANDING VOTES.

1. Communications of which notice has been given to the Secretary shall take precedence of those not so notified.

2. Resident Fellows who have paid all fees and dues chargeable to them are entitled to receive one copy of each volume or article printed by the Academy, on application to the Librarian personally or by written order, within two years from the date of publication. And the current issues of the Proceedings shall be supplied, when ready for publication, free of charge to all the Fellows and Members of the Academy who desire to receive them.

3. The Committee of Publication shall fix from time to time the price at which the publications of the Academy may be sold. But members may be supplied at half this price with volumes which they are not entitled to receive free, and which are needed to complete their sets.

4. One hundred extra copies of each paper accepted for the Memoirs of the Academy shall be separately printed, of which fifty shall be placed at the disposal of the author, free of charge.

5. Resident Fellows may borrow and have out from the Library six volumes at any one time, and may retain the same for three months, and no longer.

6. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library, for a limited period.

7. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy, except by special leave of the Librarian.

8. Books, publications, or apparatus shall be procured from the income of the Rumford Fund only on the certificate of the Rumford Committee, that they, in their opinion, will best facilitate and encourage the making of discoveries and improvements which may merit the Rumford Premium.

9. The annual assessment upon Resident Fellows shall be eight dollars, until otherwise ordered.

10. The annual meeting shall be holden at half past three o'clock, P. M. The other stated meetings at half past seven o'clock, P. M.

11. A meeting for receiving and discussing scientific communications shall be held on the second Tuesday of each month, excepting the three summer months.

LIST

OF THE

FELLOWS,

ASSOCIATE FELLOWS,

AND

FOREIGN HONORARY MEMBERS

OF THE

AMERICAN ACADEMY OF ARTS AND SCIENCES.

MAY, 1873.

FELLOWS.

CLASS I.

Mathematical and Physical Sciences.

Section I.

Mathematics.

Ezekiel B. Elliott,	Washington, D. C.
William Ferrel,	Cambridge.
Benjamin A. Gould,	Cambridge.
Gustavus Hay,	Boston.
John B. Henck,	Boston.
Thomas Hill,	Waltham.
Edward Pearce,	Providence, R. I.
Benjamin Peirce,	Cambridge.
James M. Peirce,	Cambridge.
John D. Runkle,	Boston.
Edwin P. Seaver,	Cambridge.
Chauncey Wright,	Cambridge.
Joseph Winlock,	Cambridge.

Section II.

Practical Astronomy and Geodesy.

J. Ingersoll Bowditch,	Boston.
Alvan Clark,	Cambridgeport.
Henry Mitchell,	Needham.
Robert Treat Paine,	Boston.
William A. Rogers,	Cambridge.
George M. Searle,	New York.
Henry L. Whiting,	Boston.

Section III.

Physics and Chemistry.

Joseph Hale Abbot,	Beverly.
John Bacon,	Boston.
John H. Blake,	Boston.
Thomas Edwards Clark,	Williamstown.
W. J. Clark,	Amherst.
Josiah P. Cooke, Jr.,	Cambridge.
James M. Crafts,	Boston.
William P. Dexter,	Roxbury.
Charles W. Eliot,	Cambridge.
Moses G. Farmer,	Salem.
Wolcott Gibbs,	Cambridge.
Augustus A. Hayes,	Roxbury.
Eben N. Horsford,	Cambridge.
T. Sterry Hunt,	Boston.
Charles L. Jackson,	Cambridge.
Joseph Lovering,	Cambridge.
John M. Merrick,	Boston.
William R. Nichols,	Boston.
John M. Ordway,	Boston.
Edward C. Pickering,	Boston.
Edward S. Ritchie,	Boston.
S. P. Sharples,	Cambridge.
Frank H. Storer,	Boston.
John Trowbridge,	Cambridge.
Cyrus M. Warren,	Boston.

Section IV.

Technology and Engineering.

Henry L. Abbot,	Cambridge.
G. R. Baldwin,	Quebec, Canada.
John M. Batchelder,	Cambridge.
C. O. Boutelle,	Washington, D. C.
Edward C. Cabot,	Boston.
Henry L. Eustis,	Cambridge.
James B. Francis,	Lowell.
John C. Lee,	Salem.
William R. Lee,	Roxbury.
Alfred P. Rockwell,	Boston.
John Rodgers,	Washington, D. C.
Stephen P. Ruggles,	Boston.
Charles S. Storrow,	Boston.
William H. Swift,	Boston.
John H. Temple,	Boston.
William R. Ware,	Boston.
William Watson,	Boston.
Morrill Wyman	Cambridge.

CLASS II.

Natural and Physiological Sciences.

SECTION I.

Geology, Mineralogy, and Physics of the Globe.

Thomas T. Bouvé,	Boston.
William T. Brigham,	Boston.
Algernon Coolidge,	Boston.
John L. Hayes,	Cambridge.
Charles T. Jackson,	Boston.
Jules Marcou,	Cambridge.
William H. Pettee,	Cambridge.
Raphael Pumpelly,	Cambridge.
William B. Rogers,	Boston.
Nathaniel S. Shaler,	Cambridge.
Charles U. Shepard,	Amherst.
Josiah D. Whitney,	Cambridge.

SECTION II.

Botany.

Jacob Bigelow,	Boston.
George B. Emerson,	Boston.
Asa Gray,	Cambridge.
H. H. Hunnewell,	Wellesley.
John A. Lowell,	Boston.
John L. Russell,	Salem.
Charles James Sprague,	Boston.
Edward Tuckerman,	Amherst.

SECTION III.

Zoölogy and Physiology.

Alexander E. R. Agassiz,	Cambridge.
Louis Agassiz,	Cambridge.
J. A. Allen,	Cambridge.
Robert Amory,	Boston.
Nathaniel E. Atwood,	Provincetown.
James M. Barnard,	Boston.
Thomas M. Brewer,	Boston.
Samuel Cabot,	Boston.
George Derby,	Boston
John Dean,	Boston.
Silas Durkee,	Boston.
Herrmann A. Hagen,	Cambridge.
Alpheus S. Hyatt,	Salem.
Samuel Kneeland,	Boston.
Theodore Lyman,	Boston.
Edward S. Morse,	Salem.
Alpheus S. Packard, Jr.,	Salem.
Charles Pickering,	Boston.
Francis L. Pourtales,	Cambridge.
Frederic W. Putnam,	Salem.
Samuel H. Scudder,	Cambridge.
D. Humphreys Storer,	Boston.
Henry Wheatland,	Salem.
James C. White,	Boston.
Jeffries Wyman,	Cambridge.

SECTION IV.

Medicine and Surgery.

Samuel L. Abbot,	Boston.
Henry J. Bigelow,	Boston.
Henry I. Bowditch,	Boston.
Henry P. Bowditch,	Boston.
Edward H. Clarke,	Boston.
Benjamin E. Cotting,	Roxbury.
Calvin Ellis,	Boston.
Richard M. Hodges,	Boston.
Oliver W. Holmes,	Boston.
R. W. Hooper,	Boston.
John B. S. Jackson,	Boston.
Edward Jarvis,	Dorchester.
Charles G. Putnam,	Boston.
Edward Reynolds,	Boston.
Horatio R. Storer,	Boston.
John E. Tyler,	Somerville.
J. Baxter Upham,	Boston.
Charles E. Ware,	Boston.
Henry W. Williams,	Boston.

CLASS III.

Moral and Political Sciences.

SECTION I.

Philosophy and Jurisprudence.

George Bemis,	Boston.
George T. Bigelow,	Boston.
Francis Bowen,	Cambridge.
John Henry Clifford,	New Bedford.
Benjamin R. Curtis,	Boston.
Richard H. Dana, Jr.,	Boston.
C. C. Everett,	Cambridge.
Horace Gray, Jr.,	Boston.
Nicholas St. John Green,	Cambridge.
Frederic H. Hedge,	Cambridge.
L. P. Hickok,	Northampton.
Nathaniel Holmes,	Cambridge.
Mark Hopkins,	Williamstown.
C. C. Langdell,	Cambridge.
Henry W. Paine,	Cambridge.
Joel Parker,	Cambridge.
Theophilus Parsons,	Cambridge.
Charles S. Peirce,	Washington, D.C.
William A. Stearns,	Amherst.
Benjamin F. Thomas,	Boston.
James Walker,	Cambridge.
Emory Washburn,	Cambridge.

SECTION II.

Philology and Archæology.

Ezra Abbot,	Cambridge.
Albert N. Arnold,	Hamilton, N. Y.
William P. Atkinson,	Boston.
H. G. Denny,	Boston.
Epes S. Dixwell,	Cambridge.
William Everett,	Cambridge.
William W. Goodwin,	Cambridge.
Ephraim W. Gurney,	Cambridge.
Horatio B. Hackett,	Newton Centre.
George M. Lane,	Cambridge.
Chandler Robbins,	Boston.
Nathaniel B. Shurtleff,	Boston.
E. A. Sophocles,	Cambridge,
Edward J. Young,	Cambridge.

SECTION III.

Political Economy and History.

Charles F. Adams, Jr.,	Cambridge.
Erastus B. Bigelow,	Boston.
Caleb Cushing,	Newburyport.
Charles Deane,	Cambridge.
Charles F. Dunbar,	Cambridge.
Samuel Eliot,	Boston.
George E. Ellis,	Boston.
William Gray,	Boston.
Edward Everett Hale,	Boston.
J. L. Motley,	Boston.
Francis Parkman,	Brookline.
A. P. Peabody,	Cambridge.
Willard Phillips,	Cambridge.
Edmund Quincy,	Dedham.
Charles Sumner,	Boston.
Nathaniel Thayer,	Boston.
Henry W. Torrey,	Cambridge.
Robert C. Winthrop,	Brookline.

SECTION IV.

Literature and the Fine Arts.

Charles F. Adams,	Boston.
William T. Andrews,	Boston.
George S. Boutwell,	Groton.
J. Elliot Cabot,	Brookline.
Francis J. Child,	Cambridge.
Ralph Waldo Emerson,	Concord.
John C. Gray,	Boston.
Richard Greenough,	Boston.
Henry W. Longfellow,	Cambridge.
Francis C. Lowell,	Boston.
James Russell Lowell,	Cambridge.
Charles Eliot Norton,	Cambridge.
John K. Paine,	Cambridge.
Thomas W. Parsons,	Boston.
Charles C. Perkins,	Boston.
John G. Whittier,	Amesbury.
Edward Wigglesworth,	Boston.

ASSOCIATE FELLOWS.

CLASS I.

Mathematical and Physical Sciences.

Section I.

Mathematics.

Charles Avery,	Clinton, N. Y.
Alexis Caswell,	Providence, R. I.
Charles Davies,	New York.
Simon Newcomb,	Washington, D. C.
H. A. Newton,	New Haven, Conn.
James E. Oliver,	Ithaca, N. Y.
Truman H. Safford,	Chicago, Ill.

Section II.

Practical Astronomy and Geodesy.

Stephen Alexander,	Princeton, N. J.
W. H. C. Bartlett,	West Point, N. Y.
J. H. C. Coffin,	Washington, D. C.
Charles Henry Davis,	Washington, D. C.
William H. Emory,	Washington, D. C.
George W. Hill,	Nyack, N. Y.
Elias Loomis,	New Haven, Conn.
Maria Mitchell,	Poughkeepsie, N. Y.
C. H. F. Peters,	Clinton, N. Y.
Charles Wilkes,	Washington, D. C.
Charles A. Young,	Hanover, N. H.

Section III.

Physics and Chemistry.

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I.

On the Embryology of Echinoderms.

BY ALEXANDER AGASSIZ.

Communicated February 2d, 1864.

THE following account of the embryology of a few of our Echinoderms, though by no means complete, will fill several of the gaps hitherto existing in the knowledge of the development of Echinoderms; and in the hope that something may be added to our understanding of the general plan of development of Echinoderms, it is here given in its present condition, in order to show how far the plan of development is identical in the different orders of Echinoderms. I shall take up in turn the Echinoids, Ophiurans, and Holothurians, and then compare these different larvæ with what we know already of the development of our common Starfish, to see how close the agreement of the mode of formation of the young Echinoderm is in these four orders, and to satisfy ourselves how true are the suggestions made at that time, on a very superficial acquaintance with any other Echinoderm larva except that of Asteracanthion, concerning the function of many of the organs of these wonderful larvæ.

TOXOPNEUSTES DROBACHIENSIS AG.

The larvæ of our common Echinus resemble most closely, in some of their earlier stages, those of *Toxopneustes lividus* Ag., figured by Müller on Plates VI. and VII. of his fourth Memoir.* The figures which Müller gives correspond with what I have

* MÜLLER, J. Ueber die Larven und die Metamorphose der Echinodermen. Vierte Abhandlung. Berlin, 1852.

observed in larvæ obtained by artificial fecundation. He succeeded in tracing them for about three weeks, which is not quite as long as I have kept them alive. Müller has unfortunately not given us any figures of the very first stages of this species, nor has he found the adult larvæ swimming about immediately before the absorption of the Pluteus. The series of figures found in this paper will give us a more complete idea of the different phases of growth of one species of Echinus, than can be gathered from a comparison even of the different species which Müller has investigated. It will enable us to trace the order of appearance of the arms of the Pluteus, and the last changes which the larva undergoes immediately before the young Sea-urchin has resorbed the whole framework of the Pluteus.

The process of segmentation of the yolk is entirely similar to what we observe in the Starfish; the main differences in the eggs are simply of proportion between the relative size of the yolk-mass and the outer envelope. My observations agree with the account of the segmentation given by Derbès.* The formation of the "Richtung's Bläschen" is very easily followed in the Sea-urchin.

The yolk contracts somewhat immediately after the fecundation takes place, and we might repeat here, word for word, the description of the changes which the yolk undergoes in the Starfishes, and have the history of the changes during the segmentation of the yolk of an Echinus. For an account of this I would refer the reader to the fifth volume of the Contributions to the Natural History of the United States, by Professor Agassiz. The embryo, on escaping from the egg, resembles a Starfish embryo, and it would greatly puzzle any one to perceive any difference between them. The formation of the stomach, of the œsophagus, the intestine, and the water-tubes takes place in exactly the same manner as in the Starfish, the time only at which these different organs are differentiated not being the same. We have thus very early in the history of these two orders differences which to a practised eye tell at once to which of them the young larva belongs. What is a particularly important difference is the formation in Ophiurans and in Echinoids of calcareous rods at an early period of the larval condition.

The embryo after its escape from the egg.

In the spherical embryo soon after its escape from the egg we perceive a thickening of the walls at one of the poles; a depression is then formed at this extremity, which becomes more and more marked;† the wall then turns in, and a small cavity

* Derbès. Ann. d. Scien. Nat. 3e Ser. VIII. p. 80. 1847.

† See figs. 4 and 5 of my paper on Asteracanthion in Proc. Am. Acad., Vol. VI., April 14, 1863, which represent the corresponding stages of the embryo of the Starfish.

is formed, which goes on increasing in length until we have a hollow cylinder (*d*) extending half the length of the larva, as in figs. 1 and 2, which correspond to fig. 6 of my paper referred to above. In the profile, fig. 1, we notice the same tendency in the digestive cavity (*d*) to incline towards the lower side, after the dorsal portion has increased more rapidly, giving the anal part of the larva a bevelled appearance. In a somewhat older stage, fig. 3, the digestive cavity is still longer, and almost touches the lower side. We notice a difference between the Starfish and the Sea-urchin in the time of formation of the alimentary canal, the stomach, and the œsophagus. In the Starfish the mouth is formed before the differentiation of these organs takes place; while in the Sea-urchin the mouth is not formed until the alimentary canal and the œsophagus, as well as the water-tubes, are quite distinctly defined. (See fig. 8.) What is also peculiar to Echini is the presence of large masses of yolk-cells along the sides of the digestive cavity, indicative of the great changes which take place at the points where these masses of yolk-cells are most numerous. We have observed that the yolk-cells are always present wherever any new organ is developed; in these larvæ the appearance of the water-tubes is preceded by an accumulation of yolk-masses at the extremity of the digestive cavity (see fig. 3); and the place of the limestone rods, fig. 4, *r′*, is first seen filled by clusters of these yolk-masses, in the midst of which the rods are deposited. Rods extending into the arms are characteristic of Echinoids and Ophiurans; we find nothing of the kind in Starfish or Holothurian larvæ.

In the next stage, fig. 4, the original cylindrical digestive cavity has already a decided tendency to differentiation, the walls of the stomach (*d*) and of the œsophagus (*o*) being of very different thickness; from *o*, the pouches which are to become the water-tubes (*w w′*) project far beyond the outline of the digestive cavity. The limestone rods (*r′ r′*) can faintly be distinguished from the mass of yolk-cells which surround them. The calcareous cells which take such a great prominence in older larvæ (see fig. 19) make their appearance as early as the stage of fig. 3; they are quite large in the condition represented in fig. 4. The strong contrast which already exists between the different parts of the digestive cavity is still more apparent in a stage but slightly more advanced, fig. 5. The dorsal portion of the larva has up to this time been growing most rapidly, changing the outline of the larva, particularly when seen in profile; in the subsequent figures, the outline, when seen from above, is also undergoing great changes. The larva assumes a more rectangular shape (fig. 6) when seen either from above or from below. The water-tubes are almost separated from the digestive cavity, which has been divided into three very distinct regions (*c*, *d*, *o*, fig. 8), the limestone rods, simply T-shaped before, fig. 5, are sending off small processes, and the chords

of vibratile cilia (*v*, *v'*), which were a simple button (*v*, fig. 5), are quite stout, projecting beyond the general outline when seen from above or below (fig. 6, *v*). It is at this advanced condition only that the œsophagus touches the lower surface, previous to the formation of the mouth, which takes place only when the larva has reached the condition of fig. 9. A view of fig. 6 seen from the anal extremity, fig. 7, shows how far it has lost its cylindrical shape and become wedge-shaped. In fig. 8, which is the same larva seen in profile, the indentation which indicates the position of the mouth (*m*) has changed somewhat the even outline of the lower surface; there is a marked bending of the alimentary canal, bringing the anal opening (*a*) still nearer the lower surface. In fact, since the first formation of an opening in fig. 1 (*a m*), which is at once strictly mouth and anus, there is a continued tendency to bend the anal part of the digestive cavity towards the oral surface, even while this single opening performs the functions of mouth and anus, during the period which precedes the formation of the mouth, after the alimentary canal, the true stomach, and the œsophagus have been differentiated. This is somewhat different from what we notice in the Starfish embryo, where the mouth is formed before the anus has been much bent from its original position.*

The large accumulations of yolk-masses round the rods *r' r'* cannot fail to be noticed in the stages just described, figs. 6, 7, 8. This contrasts strikingly with the Starfish larvæ, in which we find nothing of the kind; the body of the young embryo is quite remarkable for its great transparency, which only increases with age, while in the Echinus larvæ the great accumulations of yolk-masses renders them somewhat opaque even in their early stages, and the increase of calcareous cells in somewhat more advanced forms, as figs. 17, 19, makes it more difficult than in the Starfish to trace accurately the minute changes which the rods and water-tubes undergo. We now come to conditions, figs. 9 and 10, which are sufficiently advanced to enable us by comparing them with still older forms, such as figs. 16, 17, 19, to form a correct idea of the mode of transformation from the shape of figs. 9, 10, to the complicated larva represented in fig. 20. I merely refer to this comparison in a general way, as in the explanation of the different stages it will be carried out more fully, to call attention to the periods which first give us a clew to the development of the different parts, by showing us plainly which portions of the embryo must assume a great prominence, and obtain a more rapid development than others, to pass gradually through the stages which are hereafter figured. In fig. 9 the difference in the rapidity of the development

* See fig. 8, Proc. Am. Acad., Vol. VI., quoted above.

of the two water-tubes w, w', is quite striking. The left water-tube (w') — left when seen from the aboral side, the anal extremity being turned down as in these figures — is fast pushing through the mass of the larva, and finds its way to the surface at about the condition represented in fig. 15, where the water-pore, the madreporic body, allows the water to enter freely into this water-tube. In a somewhat more advanced larva, seen obliquely from the oral side, fig. 10, we can trace the mode of development of the chord of vibratile cilia; it is formed of a single continuous line, extending round the mouth; it forms but a single shield, and not two as in the Starfish, where the first trace of this chord is the appearance of two separate arcs forming eventually two distinct plastrons. The little larva looks in this condition like a quadrangular pyramid with a rounded apex and rounded angles at the base. The corners of the base e', e''' are the origin of the first arms of the larvæ (figs. 9, 10). Owing to the great increase of the dorsal and oral parts of the larvæ, they change their general appearance very rapidly. (See figs. 11, 12.) As the intestine becomes more distinct from the stomach, the angle which their axes make grows more acute (fig. 11 c, d); the mouth (m) is removed further from the anus. The walls of the œsophagus (o) are now capable of considerable expansion and contraction; they are much thinner than those of the intestine or stomach. Fig. 12, which is fig. 11 seen from the oral side, shows the course of the vibratile chord, the position of the arms e', e''', the great size of the rods (r'), with their branches, and the difference of level between the opening of the mouth and anus. From an examination of figs. 11 and 12 the position of the rods can be determined, one main part extending from the anal extremity to the arms e', another extending in a curved line (fig. 11) from e' to e''', and sending off a small branch which runs between the anus and the digestive cavity (fig. 12). This will, perhaps, be more clear on examining the larva in such a way (fig. 13) as to bring the vibratile chord into the field; this stage does not differ materially from that represented in fig. 10, the changes which have taken place defining the arms more sharply by the indentations of the vibratile chord. The intestine, the stomach, and the œsophagus are clearly distinguished by the different character of their walls. The water-tubes are not united, and have not increased in size. This condition presents a material difference in the degree of development when compared with the corresponding stage of a Starfish (fig. 11, Proc. Am. Acad.). Here the water-tubes occupy the most important portion of the larva, while in the Sea-urchin larva the most striking characteristic is the amount of room taken up by the stomach and œsophagus compared with that occupied by the water-tubes.

In the subsequent stages (figs. 14 and 15), the Echinus larvæ have reached forms

which are already more familiar to us from the drawings of Müller; they resemble closely some of the figures given by him of *Toxopneustes lividus* in his fourth Memoir. A good deal of allowance must be made for the differences of outline between the figures given here and the drawings of Müller. From the evidence of the drawings themselves, it is plain that nearly all the specimens drawn by him are compressed. I have endeavored to represent these larvæ as they appear swimming about; it is by no means an easy task to follow them in their almost unceasing movements with the magnifying power required to introduce the necessary details, but I trust I have succeeded in giving a tolerably accurate idea of their appearance in these outline drawings. In a larva during the tenth day after fecundation (fig. 14), the most important changes are the increase of the arms e', e''', and the formation of rudiments of another pair of arms, e^{iv}; the vibratile epaulettes, v'', as Müller has called the peculiar accumulation of vibratile cilia situated between the base of the adjoining arms e', e''', make their appearance at this stage. It is easy to follow them from their origin, when they are simply a thickening of the vibratile chord v'', figs. 14, 15, until they have passed through the successive stages represented in figs. 16, 17, 19, to attain the great size observed in fig. 20, when they appear in certain positions as having no connection whatever with the vibratile chord, and to have originated independently of the main chord. Müller had not traced their development, and laid great stress on their presence in distinguishing the different species of Sea-urchin larvæ.

With the development of the arms, the intestine loses its former shape; it has now assumed the appearance of a large elliptical receiver with thin walls. The stomach is somewhat dumb-bell-shaped, and the left water-tube connects with the surrounding water, through the water-pore *b*, having pushed its way to the surface. The rods keep pace with the growth of the arms, fig. 15; the water-tubes have not increased in size; they are still two distinct bodies. The anal part of the outline of the larva is quite pointed; the aboral side is regularly arched, with a slight depression at the point where the water-pore opens, *b*, fig. 14. The opacity of the larva has increased to such an extent that it becomes impossible to define clearly the outline of the water-tubes in the stages which come between figs. 14 and 16. I am unable to state positively whether the two water-tubes are united in this and older larvæ. All I could distinctly see was the great increase in size of the water-tubes; but at the same time it becomes a puzzling matter to trace the limits of these tubes, owing to the delicate walls which bound them. Their presence can only be traced by the fine line which runs across the œsophagus from each side, and by the water-pore and the tube leading to it; *b*, fig. 16. In a profile view of a larva considerably older than that represented in fig. 15, the

epaulettes, v'', fig. 16, have assumed a more independent position, forming a curve somewhat similar to the arc from which the median anal arms of the Brachiolaria are developed; the third pair of arms bulges out quite prominently, e^{IV}, when seen in profile; the fourth pair of arms is visible, e''; the rod which eventually extends in the interior of it is a straight rod (r'') with a slight point in the middle, at present disconnected from the remaining part of the calcareous framework. This set of rods and the fork r''', which extends into the arms e^{IV}, take their origin independently from the main rod, extending from the anal part round by the mouth, from which branches are sent into the arms e' and e'''. The rod r'' ultimately combines with the main system, but the rod r''' always remains separate from the others. The position of these rods is better understood from fig. 17 when seen from the aboral side.

The stage represented in fig. 17 is particularly important, as it is at this time that we notice the first trace of what I suppose becomes the tentacular pentagon of the young Sea-urchin. On the left water-tube we notice a very prominent loop, *t*, which, from its resemblance to the tentacular loops of Brachiolaria and from its position on the water-tube connecting with the water-pore, I have no hesitation in considering to be the first tentacular loop formed. Compare figs. 1, 2, of Plate VII. of Müller's seventh Memoir,* where he figures a similar tentacular loop in two different stages of development; unfortunately, there is nothing in the text to explain what Müller considered it to be. The relation of this loop to the madreporic body is perfectly plain in this same larva seen from above as it is floating in the water, fig. 18, the epaulettes appearing like great flaps extending between the base of the arms e' and e'', in continuation of the chord of cilia extending along these arms.

Figs. 17 and 18 are larvæ twenty-three days old; during the next four days no change of any importance could be perceived; the tentacular loop remained the same, the arms alone increasing in size, and a few dark pigment spots appeared in the arms. Unfortunately, at the end of these four weeks the young Sea-urchins all died. I have only once succeeded in keeping them such a long time, and that was during the coldest winter weather. In the attempts made in the spring, whenever a warm day came, it was sure to kill everything; while in the summer, though the facilities I had were infinitely greater, I never could keep these larvæ alive more than three or four days. The Sea-urchins spawn during the whole year. Successful artificial fecundations have been made in December, January, and during every month from that time till the middle of October.

* MÜLLER, J. Ueber die Gattungen der Seeigellarven. Berlin, 1855.

The remaining observations of this paper were all made from specimens caught swimming on the surface of the water; this applies to the fully-formed Sea-urchins as well as the larvæ, only the most advanced specimens (fig. 28) were found thrown up on the beach after a storm, attached to Laminaria. The specimens obtained in this way, the various stages of which were traced until there could be no doubt to what species they belong, connect so nearly with specimens obtained from artificial fecundation as to leave but few gaps to fill, in the larval condition, to give us all their transformations. The Sea-urchins raised from larvæ caught swimming freely about were kept in confinement until they had attained the size of some of the more advanced nomadic Sea-urchins, figs. 26, 27. This can leave no doubt to which of our two species of Echinoids these larvæ should be referred.

The next oldest larva, which is fig. 19, shows that since the last stage represented the principal changes have taken place in the oral part of the larva, fig. 19; the arms e'', e^{iv} especially have increased greatly in length, the outline of the anal extremity is somewhat rounded, the rod which runs along its edges is made up of short, stout pieces with strong pointed projections, and the rods of the arms are composed of three rods connected together by transverse spokes; it requires close examination to distinguish this. On the aboral side two very prominent spurs project over the stomach, somewhat below the point of junction of the rods of the arms e', e'', e'''. Additional tentacular loops have been formed; we can distinctly trace three on the surface of the left water-tube w'; the outline of a part of the right water-tube (w) shows great increase in the volume of the tube. In an adult Sea-urchin Pluteus, fig. 20, the Sea-urchin has encroached so much on the anal extremity as to conceal the shape of the digestive cavity. The spines are so large, that we are unable to trace the position of the tentacular system; the anal opening is very conspicuous. The vibratile epaulettes, v'', are remarkably powerful. The arms have attained nearly the same length. The vibratile chord has been twisted in such a manner as to assume the appearance of binding an anal and an oral plastron, of which e', e'', e''', and e^{v} are respectively the arms; the mode of formation of the chord and of the arms shows that all these arms in reality belong to one plastron (see figs. 12, 13), notwithstanding the great resemblance to the two distinct plastrons of a Brachiolaria. Two very prominent black spots are seen in the arms e', e'', similar to those observed by Müller in his *Pluteus quadrimaculatus;* a few small spots are scattered over the other arms. The Pluteus figured here in its natural attitude does not undergo any further changes of form; it now enters a stage when the Sea-urchin goes through its greatest transformations; these unfortunately cannot be followed, owing to the opacity of the larva.

The presence of rods in the plutean forms of Ophiurans and Echinoids of course restricts considerably the play of the arms in assisting the motion of the larva. The arms cannot be bent and twisted in the graceful manner so peculiar to Brachiolaria. They are only capable of opening and shutting like the rods of an umbrella. Fig. 21, which is fig. 20 seen from above, when left in its natural attitude, shows the extent to which the arms can be spread. This does not prevent the larva from moving quite rapidly by means of a kind of gliding motion, in which the vibratile epaulettes perform an important part in propelling the Pluteus. While moving, the anal extremity is usually kept below, as in the position which has been given to all the figures in this Memoir, which is their *natural position.* Previous to the time when the anal extremity is loaded down by the presence of the Sea-urchin (figs. 1 – 19), it is quite common to see them moving in every possible direction, so that it would be difficult, from a knowledge of the earlier stages alone, to ascertain with precision what the natural attitude is; although we notice even in the early periods a very strong tendency to assume the natural position of the adult larva. The larvæ also assume during their movements the oblique position described in Brachiolaria; this seems characteristic of all the Echinoderm larvæ I have had occasion to examine, whether Ophiuran, Holothurian, Echinoid, or Asterian. A natural profile view of an adult larva (fig. 22) cannot be made with great accuracy, and the outline here given is added simply to show the position of the arms; rotating as they do almost continually on their vertical axis, we catch only passing glimpses of the exact profile outline. The only adult larva figured in profile by Müller is on Plate V. of his seventh Memoir.

An adult Pluteus, in the condition of fig. 20, requires several weeks for the completion of the Echinus and the absorption of the plutean framework. The Echinus encroaches gradually on the anal extremity; the base of the arms, e' and e'', is soon lost in the midst of the spines of the young Sea-urchin, which are arranged in a conical open spiral wreath surrounding the mouth, fig. 23. While this encroachment of the anal extremity is going on, the œsophagus has contracted to such an extent that the base of the oral arms, e^{IV}, e''', is brought directly in contact with the anal vibratile chord. During the process of resorption the arms have lost their mobility; they appear like helpless rods, stretching at every conceivable angle from the Pluteus, which has lost entirely its former symmetrical appearance, fig. 23.

The young Echinus after the Resorption of the Pluteus.

The figures given by Müller on Plate III. of his first Memoir* represent several

* Müller, J. Ueber die Larven und die Metamorphose der Ophiuren und Seeigel. Berlin, 1848.

Echinoid larvæ in which the young Echinus has absorbed more or less of the plutean frame. From what I have observed on several of these larvæ, the Pluteus is as completely resorbed as is the case in the Brachiolaria observed by me. Not a single part of the framework is thrown off; this process of resorption begins at the base of the arms; they are thus gradually shortened, the rods apparently melt away before our eyes, the extremity of the arms is the last to disappear, and immediately before the time when the young Echinus is freed from the plutean appendages, the extremities of all the arms are still there, as perfect as when these appendages stretched symmetrically on both sides of the longitudinal axis. From many of the figures of Müller himself it is evident that, in the larvæ he has observed, the young Echinus resorbs the whole of the framework, and does not separate from it by losing the arms, as he has stated. See Plates III., IV., V., VI. of the first, and Plate VIII. of his seventh Memoir. The larva represented in fig. 20 of this paper was kept in confinement from the 1st of October to the 20th of November before every trace of the arms had disappeared, and the young Sea-urchin had assumed the appearance of fig. 24. Fig. 24 was drawn from a specimen found floating on the surface in the middle of June. This young Sea-urchin bears a striking resemblance to a young Echinocidaris figured by Müller on Plate IV. of his seventh Memoir. The development of the separate parts is very different in the two. The number of spines is much greater in our Toxopneustes, and they are of an entirely different shape. Pedicellaria are likewise present in Echinocidaris;* these do not make their appearance till a much later period in our young Sea-urchins. (See fig. 28, *p.*) What is particularly characteristic of these earlier stages of the young Sea-urchins is the great size and small number of the spines. Their position is also peculiar; they are all placed on the edge of the test, which is exceedingly flat. (Compare this with Podophora.) Five of the tentacles are strikingly prominent, equalling in length the diameter of the shell; they are also remarkable for their great thickness, and the presence of a calcareous ring in the sucker, which is entirely wanting in young Starfishes. A similar calcareous ring is figured by Müller,

* The function of pedicellaria has long been a puzzle to naturalists. While watching a Sea-urchin which was discharging its excrements, I noticed that the pellets always moved in definite paths, down the interambulacral spaces, till they were pushed off from the test. On examining this with a magnifying-glass, I could distinctly see the innumerable pedicellaria hard at work seizing in their tiny prongs the pellets which they pushed down the interambulacral areas. If by chance a pellet found its way into the ambulacral zone and came in contact with the suckers, it was immediately seized by the ambulacral pedicellaria and thrown back into the interambulacral zone to move on in its accustomed path. The pedicellaria are more numerous in the interambulacral zone than in the ambulacral. This will explain the function of these pedicellaria, at least for Sea-urchins, as that of scavengers.

Plate IV. fig. 13, seventh Memoir, and Plate VII. fig. 2, first Memoir. The whole abactinal surface is thickly covered with dark crimson pigment-cells. The younger spines resemble those of the young Starfish (see fig. 16, Proc. Am. Ac., l. c.), while the more advanced spines are not fan-shaped, but slightly pointed, reminding us of spines of Cidaris. On turning this Sea-urchin on its actinal side, as in fig. 26, we find near the base of each of the five large tentacles four others, which are not as advanced, and are incapable of expanding beyond the edge of the test.

Additional spines are formed on the abactinal side of the test of older specimens (fig. 25), so that they cover the whole of that surface, and are no longer limited to its edge, as in fig. 24; the large spines become more pointed, the tentacles grow slender, and they can all expand beyond the edge of the test. The odd tentacle expands and contracts to a remarkable extent, sometimes as much as three times the radius. The four other tentacles are somewhat more stout, and not capable of such extensive expansion and contraction; the pair of tentacles placed nearest the mouth is the stoutest. The position of the tentacles is best seen from the actinal side (fig. 26). The whole actinal surface is covered with a plating of limestone cells, which leave but a small circular opening, the mouth, in which the points of the teeth project. This actinal system is circular; there are no notches for the passage of the gills, as in adult Echini; the ambulacral tentacles are placed one above the other. The long spines move in every direction, as they are already provided with the peculiar ball-and-socket joint of Echinoids. On removing the spines of one of these young Sea-urchins, the great size of the tubercles (*n*, *n'*, fig. 27) and the large circular actinal system give them an aspect totally different from what we are accustomed to associate with the genus Toxopneustes. The teeth (*k*, fig. 27) fill but a small part of the actinal system; they are five narrow triangular wedges, extending from the centre to the edge of the shell, covered partially by the network of limestone plates. (See fig. 27.) The test thus denuded of its spines resembles in all the general features that of a Cidaris. With the exception of the formation of the abactinal system, which is not yet developed, the striking features of the young Sea-urchin — such as the circular actinal system, its large size, the great prominence of the tubercles, the position of the pores one above the other — are all characters belonging to a different family from that to which the adult Sea-urchin belongs. The little Sea-urchin does not long retain these anomalous features; with every day of increasing age the changes which it undergoes bring it closer and closer to the condition of the adult. In a young Sea-urchin of a diameter of one fifteenth of an inch (fig. 28) the spines have lost almost entirely their embryonic character, the tentacles are much more numerous, and pedicellaria have made

their appearance; in the interambulacral space they are more thickly scattered than in the ambulacral, where there are merely three or four. The abactinal system consists of a single large plate covering the opening of the anus (a) which leads out on one side of it. The additional spines and plates which have been formed are all developed from the abactinal region. The new plates are added in a spiral manner round the anal plate by additions to the limestone mass, pushing further away from the abactinal pole the first formed plates. The outline of the new plates is at first indicated on the lower edge, which becomes somewhat undulated; then the transverse divisions are made, and a spine is formed on the plate soon after that. There are no spines on the last formed plates; the spines when they first appear have the same fan-shaped character as the earliest formed spines of the abactinal surface. (Figs. 24, 25.) This shape they lose soon, and pass at once into spines resembling the older ones in every respect except size.

The mode of formation of the new plates was discovered by Professor Agassiz as early as 1834, when he gave a short account of it in the Edinburgh Philosophical Journal. The spiral arrangement of the plates is still very plainly visible in adult specimens. Although the Sea-urchins are circular, we have in their mode of growth something which reminds us of the earlier embryonic stages of the Starfish. I have not been able to trace all the various stages of growth of the young Sea-urchin, how it passes from fig. 28 to the condition when the pores, instead of being arranged in single rows one above the other, are placed in arcs on both sides of a median ambulacral row covered with spines. We may, however, form a tolerably accurate idea of the changes which must be gone through by examining the abactinal part of the ambulacral area of an adult Sea-urchin. The mode of formation of the ovarian and ocular plates remains still to be traced. The oldest of the young Sea-urchins (fig. 28) has advanced sufficiently to enable us to see that the subsequent changes which are required to make it agree with its adult condition are by no means as great as the changes which the young Sea-urchin has undergone up to the present time. It has reached a condition which assures us that we deal with a young Toxopneustes, and nothing else. The pigment spots, so marked in the younger stages, are smaller and scattered more uniformly, the muscular band around the mouth is well developed, the plate covering the actinal area has separated from the edge of the test, and is moved by the muscular membrane which covers the actinal system. There are no notches as yet in the actinal part of the test. The teeth have not changed their form from that found in earlier stages (fig. 27); there are from seven to eight tubercles in each vertical row of the ambulacral and interambulacral zones. I was unable to distinguish

among the many tentacles the original odd tentacle which was so prominent in the younger stages. Neither have I succeeded in determining the position of the eye in any of the stages of these young Sea-urchins, owing to the early presence of the spines and of the large pigment-cells, which prevent us from obtaining a favorable view of the odd terminal tentacle in the young forms; neither have I been able to satisfy myself whether this odd terminal tentacle retains its original position during the whole life of the Sea-urchin, as is the case in the Starfish. I can likewise say nothing concerning the development of additional ambulacral tentacles.

The figures of young Sea-urchins which Müller has given belong unfortunately nearly all to different suborders from our Toxopneustes, so that we cannot make the comparison with our young Sea-urchins as close as we might wish; and besides this, the figures of Müller are not drawn in such a way as to discriminate between the parts which belong to the ambulacral and interambulacral spaces. This is particularly evident in his figures from the mouth side, where we frequently find tentacles in such numbers as must make the development of the different ambulacra unequal. The same is the case with the spines. Any one who will take the trouble to compare the figures of young Sea-urchins of Plates IV. and VII. of his first Memoir, Plate VII. fourth Memoir, and Plate IV. seventh Memoir, with the figures given here (figs. 24–28), will see that, although they agree in their general characters, yet it is impossible to place the different spines or tentacles in such positions that they will be divided into ambulacral and interambulacral regions, which is easily done with the figures I have given. We must remember, however, that most of Müller's figures are Clypeastroids and Spatangoids, which may make it difficult, if not impossible, to divide the young Sea-urchin into ambulacral and interambulacral areas, where we have nothing like regular vertical rows to guide us, as in our Toxopneustes. One great difference, however, will strike us at once; it is, that what Müller has called anus I have in my figures called mouth. The view he has taken is probably due to the fact that the young Sea-urchins from which he made his drawings were compressed. Having followed the mouth in the different stages which have been represented here, I think there can be but little doubt that Müller was mistaken. Compare fig. 26 of this paper with his fig. 3, Plate VII. first Memoir, and we cannot fail to come to the conclusion that it is the mouth which is turned towards us in both cases. If Müller's statement were correct, we should have the anomaly in young Sea-urchins of finding all the tentacles between the spines and the anus on the abactinal side of the test, while on the actinal area we should have nothing but a closed membrane. This is so contrary to the plan of development of Echinoderms, whether Echinoids, Starfishes, or Ophiurans,

from the observations of Müller himself, that I give the accompanying explanation, which seems to bring the figures of Müller in accordance with what I have observed. The appearance of the teeth, in Müller's figures, on what seems the abactinal side, is due to compression also. The spines of the young Sea-urchins observed by Müller have a very uniform appearance; they are nearly all hexagonal prisms in their earliest stages. The same is the case with our young Sea-urchins, though they lose their embryonic character at an earlier period than is the case in any species observed by Müller.

Embryological classification of Echinoids.

From a careful examination of the different stages through which our young Sea-urchin passes during its development, and after comparing these stages to forms resembling them which are now found living in our seas, we obtain valuable hints as to the relative standing of the different families of some Echinoids. We can extend this somewhat to other orders by availing ourselves of the additional observations of Müller, and of a few facts concerning the development of Clypeastroids noticed by Professor Agassiz, which will be given here. Professor Agassiz's observations were made upon young specimens of *Mellita testudinata* Kl., which is so common along the whole of our Southern coast, from Beaufort, N. C., to Texas; upon the young stages of *Echinarachnius parma* Gray, which are found in great quantities in the stomach of our Cods; and on the young of an Encope, probably *Encope Valenciennesii* Ag. The observations on Mellita I had occasion to repeat on an allied species, *Mellita longifissa* Mich., which is exceedingly numerous on the beaches in the neighborhood of Acapulco, Mexico.

The smallest specimens of Mellita observed, measuring about one seventh of an inch in diameter, are nearly circular; they have but a single lunule placed in the posterior interambulacral space; this is no larger than the prick of a needle. With advancing age, measuring one fifth of an inch, its outline has not varied much; the lunule has grown larger. When it has attained a diameter of one third of an inch, the outline becomes slightly indented at the place where the two posterior ambulacral lunules will eventually be developed. As the posterior interambulacral lunule is not homologous to the others, but is simply an opening for the anus, we have in the first stages described a condition which reminds us at once of Echinarachnius. It next loses its circular outline, becoming somewhat eccentrically elliptical, as in Dendraster; while after the formation of the two posterior lunules it has all the aspect of a Lobophora. As it increases in size two other indentations are formed, the indication of the position of the two anterior ambulacral lunules. These lunules remain open for a considerable period of the growth of the urchin; we cannot fail to recognize in this state of the

young Mellita a strong resemblance to *Encope Michelini* Ag. and Echinoglycus (*Lobophora Stokesii* Ag.). The anterior ambulacral lunule is last formed. The posterior lunules then close, and had we not the anterior notches we should have an Amphiope. The last step, which is the closing of the three anterior lunules, gives to the young Mellita the general aspect of the adult; it has then attained a diameter of three quarters of an inch. In the young of Encope the same succession of changes is observed; at first they are circular, with simple notches in place of the posterior lunules; the anterior lunules are then formed, the posterior ones close, and, last of all, the anterior ones. In the early stages it is a Lobophora, then an Echinodiscus (*Lobofora bifora* Ag.).

In the young of Echinarachnius the changes of form are equally important; small specimens measuring not more than one fourteenth of an inch are very elliptical in outline, the longer axis being twice as great as the shorter; the test is quite convex, resembling strikingly Echinocyamus and Fibularia; the anus is placed on the abactinal side, and not on the edge of the test, as in the adult. This shape the young soon loses to attain a more circular outline, becoming flattened, while the anus is pushed nearer the edge. In specimens in which the longer axis measured one fifth of an inch, the test is quite flat, and the anus is placed on the edge of the disk. The outline is still different from that of the adult, the longer axis being the anterior axis in these young specimens, while in the adult it is the axis at right angles to the odd ambulacrum which is the longest.

From these data important results have been drawn for the classification of Echinoids.* All Echinoids pass, in their early stages, through a condition which recalls to us the first Echinoids which made their appearance in geological ages. We should, then, on embryological grounds, place true Echini lowest, then the Clypeastroids, next the Echinolamps, and finally the Spatangoids. The true Echini are an embryological suborder; the Clypeastroids recall to us, in their young stages only, this lowest suborder. The Echinolamps have lost the teeth which characterize the first suborders, and they show a tendency to develop an upper and a lower side, a front and a behind, which is carried to its extreme among Echinoids in the Spatangoids, the highest suborder, the young of which cannot be distinguished from those of the lower suborders at the time when they are provided with few stout spines. There are not a sufficient number of living representatives of Spatangoids and Echinolamps of which we know the development to go into more detail with respect to the standing of minor groups. In Clypeastroids what has preceded seems to show that we must place Fibularia and

* L. Agassiz. Essay on Classification.

Echinocyamus lowest; then the rounded Clypeasters, Clypeaster and Rhaphydoclypus, in which we have the floor connected by but few supports; then forms such as Laganum; then the circular Scutellæ, such as Arachnoides, in which the anus is supramarginal; then Echinarachnius, Dendraster, Scaphechinus; then the genera Lobophora, Echinodiscus, Echinoglycus, Encope, Rotula, and Mellita.

Adapting in a similar way the observations of the different stages of our young Echinus given in this Memoir to the true Echinoids, we should for similar reasons place lowest the family of Cidaridæ; next, the Diadematidæ; then that peculiarly embryonic family, the Echinometradæ, in which the unwinding of the pentagons leaves the Sea-urchin with oblique axes; then Sea-urchins with few larger spines, such as the Echinocidaridæ and Heliocidaridæ. In all these families, with the exception of the Heliocidaridæ and Echinometradæ, the ambulacral system is particularly simple. We next have those Sea-urchins with a more complicated ambulacral system, in which the tubercles become numerous and are not arranged in such regular vertical rows as in the true Echinidæ, Toxopneustes, and the like; then the Hipponoidæ, and the like, in which the development of the ambulacral system reaches its greatest complication, in which the spines are exceedingly fine, and in many genera (such as Salmacis and Mespilia) resemble more what we find in the Clypeastroids; passing gradually through forms such as Boletia, Tripneustes, Hipponoë, Salmacis, Mespilia, to take their greatest degree of complication, both in the ambulacral and interambulacral regions, in Holopneustes.

The correspondence between the embryological development and the order of succession of Echinoids in geological times is so striking, that it may not be out of place to show some of the principal points of agreement.* The number of fossil Echinoids known is so great, that, when we have as large a number of embryonic forms for each species to compare with them as are here given for a couple of species, we shall not fail to draw most important conclusions for our knowledge of the classification of these animals.

The earliest Echinoids,† which make their appearance with the Trias, are without exception Sea-urchins, belonging to families which have eminently embryonic charac-

* See AGASSIZ, L. Catalogue Raisonné . . . des Echinodermes . . . An. des Sc. Nat., 1846 – 47.

† I omit intentionally the presence of such forms as Palæchinus in the Silurian, Eocidaris in the Devonian, and the many Archæocidaris and Melonites of the Carboniferous, until a comparative study of these forms with the younger stages of Comatula has been made. I am convinced from what I know of the embryology of Echinoderms, that they are only synthetic and prophetic Crinoids, and have therefore nothing to do with our present subject.

ters. The Cidaridæ make their appearance first, and the other genera, of which a few representatives are found, belong to a family closely allied to the Cidaridæ, the Hemicidaridæ, a prophetic and synthetic family combining features of the Cidaridæ, which have preceded them, and of the Pseudo-Diadematidæ, which are to follow them. We have not a single Clypeastroid, Echinolamp, or Spatangoid. During the Lias the Clypeastroids make their appearance, but the type which represents the suborder, the genus Collyrites, is not a Clypeastroid, as we understand them in the present epoch. It is a type which is the forerunner of the true Clypeastroids, and which apes the characters of the Spatangoids. The Echinoids of the Lias have not the strong, embryonic character which belonged to the Trias; we find such genera as Hemipedina, Diademopsis, and the like, which remind us forcibly of the Diadematidæ of our present time. In the lower stages of the Jurassic period the Echinoids are of still more varied genera, — such as Holectypus and Pygaster, — being forms which recall to us the embryonic stages of our Clypeastroids. The Echinolamps are likewise introduced with Pygurus and Echinobrissus; while it is only in the lowest Cretaceous deposits that the first Spatangoids appear as Holaster and Toxaster. If we compare the appearance of the Clypeastroids in geological ages with the embryonic stages of which I have given a short account, we shall find that the first true Clypeastroids (Clypeastroids such as we know them in our own time) are such forms as Scutellina and Lenita, in the Eocene; it is only in the Myocene that the genus Clypeaster appears and takes its greatest development, accompanied with a large number of true Scutellæ; and it is not till the Pliocene that genera resembling Amphiope, Encope, and finally Mellita, make their appearance, showing a closeness of agreement between the order of development and the geological succession carried out to the fullest possible extent.

OPHIURANS.

To the extensive investigations of Müller about the embryology of Ophiurans and Holothurians I have but little to add, and I consider myself very fortunate to have been able to increase our knowledge of two of the orders of Echinoderms after what has been done by him.

Ophiopholis bellis Lym.

Two Ophiurans are quite common at Nahant; one, *Amphiura squamata* Sars; the other, *Ophiopholis bellis* Lym. The latter, unfortunately, does not lay eggs from which the plutean stage is developed, as the young Ophiurans are never nomadic. The eggs are laid in bunches, from which, according to the observations of Professor Agassiz, made as far back as 1849, the young Ophiopholis is developed, very much

after the manner of *Asteracanthion Mülleri* of Sars, without passing through the plutean stage. So many points of investigation have arisen since those observations were made that they must necessarily be very incomplete without a renewed comparative study of the whole subject. I here copy from the drawings of Professor Agassiz two stages of the young Ophiopholis which will enable us to compare the two modes of development, and show us as clearly for Ophiurans as I have shown for Asteracanthion in the fifth volume of Professor Agassiz's Contributions, that these two modes of development are but longer or shorter ways of arriving at the same point. Fig. 29 is one of the more advanced young of Ophiopholis seen from the abactinal side. At this stage it shows prominently the arrangement of the abactinal plates and the position of the arm-joints. As in young Asteracanthion, we have a central plate, and five radial and five interradial plates. No hooks or granules are as yet developed; only two of the tentacles have reached any great development, and project beyond the edge of the arms. A careful examination of the young Ophiura in this stage would enable us to determine the exact place where new arm-joints are added. There seems some doubt, from the observations of Müller * and of Lütken,† as to whether the new joints are formed at the base of the arms or at the extremity. From what I have observed in Starfishes, it is evident that the new parts of the actinal and abactinal portion of the arms are not added at the same place. The new suckers in the Starfishes are formed nearest to the odd terminal ocular tentacle, while the new spines of the abactinal side of the arm are formed at the base of the arm. (See Vol. V. of the Contributions of Professor Agassiz, where a full account of the increase of the arms of the young Starfish will be found.) Something similar may occur in Ophiuridæ, and would account for the difference of opinion entertained by Müller at different times as to where the new arm-joints were added. In fig. 30 we have the actinal side of a young Ophiopholis somewhat less advanced than fig. 29; there are as yet no arm-joints, and the whole outline is pentagonal; two of the tentacles alone make their way through the actinal limestone floor.

Amphiura squamata Sars.

The Pluteus of which a figure is here given (fig. 31) is probably the larva of *Amphiura squamata*. I have found these larvæ only three times during two summers, — once in June, once in July, and then the young Ophiuran (figs. 32 and 33) during the first week of October. It probably requires as long a time as this for the development of the Echinoderm, as I have kept the larvæ which were caught in June

* Müller, J. Memoir I., 1848, and Memoir V., 1852.

† Lütken, C. F. Additamenta ad Historiam Ophiuridarum. Kjobenhavn, 1858.

and July for weeks without noticing any striking changes. These Ophiuran larvæ resemble very closely the Pluteus of *Ophiothrix fragilis* and the *Pluteus bimaculatus* of Trieste, figured by Müller in his Memoir on the Ophiuran Larvæ of the Adriatic.* Fig. 31 resembles more Müller's fig. 1, Plate VII. of the Memoir just quoted, than any other. The order of appearance of the arms of this Pluteus agrees with his observations on the younger stages of the Pluteus of *Ophiothrix fragilis.* (Memoir V., Plate VI. figs. 8–12.) This has been traced in a specimen considerably younger than the one figured here, resembling Müller's figure of *Pluteus bimaculatus* (Plate I. fig. 1, Memoir V.), with the exception of the different degree of development of the arms C and D of that figure. In this younger Pluteus, the arms corresponding to e' of fig. 31 were the longest; next the arms e^{iv}, then e''', the arms e'' scarcely projecting from the curve joining e''' and e'. The arms e' in the adult larvæ are twice as long as the others (fig. 31), while the other arms, e'', e''', e^{iv}, are all nearly equally developed. With the exception of this difference in the proportion of the arms, the younger larvæ did not differ in the essential parts from the one which is figured here. The most complete histories which Müller has given us of the development of any Echinoderms are those of *Pluteus bimaculatus* and of *Ophiothrix fragilis.* He gives us not only the complete history of the changes of the larva, but follows the young Ophiuran after it has absorbed its calcareous framework. Here, again, the figures of Müller seem to contradict his text; he says the framework is dropped, but his figures show that, on the contrary, nothing is lost, that every part of the Pluteus is absorbed by the Echinoderm. See Müller's figures of Memoir V., Plates IV., V., VI., VII., VIII. Fig. 5, Plate VII. agrees exactly with the stage represented by me in figs. 32, 33; the long arms e' are cut off for want of room; every other arm can be traced, with its extremity perfect, disappearing gradually into the substance of the Ophiuran. This Pluteus is quite transparent, and did it occur in sufficient numbers it would be as favorable a species to follow the development of the Echinoderm as the Brachiolaria of our Starfishes.

The greater similarity of the Ophiuran larva to Echinoids than to Starfishes is something very peculiar. The same thing we find again repeated for two of the other orders; the larvæ of Holothurians and of Starfishes resemble each other to a remarkable extent, in the character of the arms, the absence of rods, and the aspect of the water-tubes. In Ophiurans and Echinoids this resemblance is still closer. The arms are supported by rods in both, the water-tubes are small, and the general outline of

* Müller, J. Ueber die Ophiurenlarven des Adriatischen Meeres. Fünfte Abhandlung. Berlin, 1852.

the plutean forms is so similar that they might easily, on first examination, be mistaken for larvæ of the same orders.

All I wish to show from the figure of this Pluteus (fig. 31) is that in Ophiurans, as well as in Echinoids and in Starfishes, the Echinoderm is developed on the water-tubes, — one (the left) developing the actinal, the other (the right) developing the abactinal region, — that the tentacular pentagon is open in the larval condition, and is only closed by a process of unwinding, some time after the actinal and abactinal surfaces have been formed, though this closing takes place in Ophiurans and Echinoids while the Echinoderm still retains parts of the framework. In Starfishes, on the contrary, the closing of the pentagons takes place after the resorption of the Brachiolaria. From the position of the water-tubes it is self-evident that the loops of the pentagon of tentacles (fig. 31, *t*) are not in one plane; they connect with the water-tube *b*, and we find all the essential features of the tentacular water-tube (w') of the Brachiolaria. (See Proc. Am. Acad., l. c., fig. 12.) The tube *w* is not connected with the tentacular water-tube w'. It would be interesting to observe whether such a connection takes place, also the mode of formation of the first limestone particles on the water-tube *w*, as this would facilitate the study of the development of the same parts in Echinoids where the opacity of the larva prevents accurate observations. Müller has, in nearly all his figures of the Ophiuran plutean forms, represented the water-tubes as made up of two distinct parts, having no connection. This probably arose from the strong contraction of the water-tubes in certain parts. I have noticed in the larvæ which I observed a marked tendency in the anal portion of the water-tubes to contract, and thus apparently to divide off from the remainder of the water-tubes; this was, however, but temporary, and the moment afterwards the water-tubes had assumed again their fully expanded shape, as in fig. 31, *w*, w'. The young Ophiurans (figs. 32, 33) are remarkable for the total absence of the plates of the disk; they are not even indicated by the presence of Y-shaped limestone rods. The only calcareous deposits we have (y, y', fig. 32) are evidently parts of the first arm-joints, the dorsal (y, fig. 32) and side arm-shields (y', fig. 32) of that joint, which consist at present of but a few rods, indicating their future position. The arms e' are the last to be resorbed; in the stage of fig. 32 they retain their full length; the other arms, e'', e''', e^{v}, are in a very contracted condition, as they have been almost entirely resorbed by the young Ophiuran, leaving nothing but the extremities; the rods of two of the arms are not entirely resorbed (fig. 32).

This young Ophiuran, seen from the lower side, fig. 33, shows that the tentacular pentagon has entirely closed. The ambulacral system is similar to the tentacles of

the young Starfish, and must have been formed in a similar way from the original simple loops of fig. 31. (See Proc. Am. Acad., fig. 15.) Additional tentacles are therefore formed at the base of the odd loop *t'*, probably in the same manner as in Asteracanthion. The mouth is not limited by the formation of the actinal floor; this, as well as the abactinal area, is in the present stage almost entirely made up of the remaining portions of the Pluteus, which have not been resorbed, and through which the parts of the young Ophiuran are seen.

The number of young Ophiuridæ observed is not sufficient to enable us to make much use of their earlier condition for a classification. The figures of the young given by Müller, and the few species of which Lütken has studied some of the earlier forms, together with the observations of Kröyer,* are all we have. The figures of young *Astrophyton eucnemis* M. T., given by Lütken in his *Additamenta ad Historiam Ophiuridarum*, throw some light on the classification of Ophiuridæ. Having had the opportunity to examine very young specimens of *Astrophyton Agassizii* Stimp., collected at Eastport, Me. by Mr. Verrill for the Museum at Cambridge, I was enabled to repeat his observations, and find the same remarkable differences between young and adult which had already been pointed out by Lütken. A young Astrophyton would seem at first glance to belong rather to Asteronyx or Asteroporpa than to Astrophyton. The disk is circular; there are no ribs; the arms have but a single fork. The ribs on the disk make their appearance when each arm has divided three times; that is, when there are twenty terminal points. Up to that period the rounded plates of the disk were quite prominent, somewhat resembling in their arrangement those of the disk of Ophiopholis. This is sufficient to show that the Astrophytidæ stand highest among Ophiurans; that Ophiurans with smooth arms, as Ophiura, are lowest; next come such genera as Ophioglypha and Amphiura; Ophiopholis next; while Ophiocoma, Ophiothrix, and the like, in which the spines take their greatest development, stand highest among Ophiuridæ proper.

The stages represented in figs. 32 and 33 are somewhat different from any given by Müller; the nearest conditions are those of fig. 2, Plate VII. and fig. 4, Plate IV. fifth Memoir, in which the outline of the young Ophiuran is far less well defined, but in which the abactinal plates, as well as the arm plates, are further advanced; the condition of the tentacles is nearly identical. In its general outline fig. 33 differs but little from fig. 30, where the abactinal and actinal floors are more advanced, showing otherwise no differences which would lead us to suppose that the mode of develop-

* KRÖYER. Nat. Tidskrift, III., 1840.

ment of these two forms was apparently so contrary. Let us take the Pluteus at the time of the appearance of the young Ophiuran, and deprive it of its arms, we should soon find our Pluteus in a condition in which the nomadic and sedentary mode of development could not be distinguished; showing us that for Ophiurans, as well as for Starfishes, these two modes of growth, at first so different, reach, early in the development of the young, stages which are identical.*

HOLOTHURIANS.

Cuvieria Fabricii Dub. et Kor.

In Holothurians the two modes of development seem to combine in a very remarkable manner. The larva has short arms only in the early stages; it remains nomadic after they have disappeared, when it would be very difficult to tell by which of the two modes the young Holothurians have been developed. See Müller's figures, in which the Auricularia approaches the "Walzenförmige" form, fourth Memoir, Plate I. From the young Holothurians which I have myself observed I am not able to say anything concerning the water-tubes, and must therefore make a comparison of the Memoirs of Müller with what has been suggested here. Although we have a Synapta and a Chirodota (*Caudina arenata* Stimp.) which are very common here, all my attempts at artificial fecundation, or at finding their plutean stages with the other Echinoderm larvæ, have completely failed. The only Holothurian of which I have seen the young is Cuvieria, which is tolerably abundant in deep water off Nahant, and of which the young, in the more advanced stages only, however, are found from June to October. The study of the development of an Auricularia with reference to the part the water-tubes play in this would be particularly interesting.

Holothurians are the only Echinoderms studied by Müller in which he distinctly says that the young Holothurian resorbs the whole of the pupa, as he calls the envelope of the Holothurian, and which lose nothing during the development. On account of this resorption, he considers the plan of the Holothurian development to be something special. I trust I have made it sufficiently clear that resorption takes

* The fact that Ophiopholis lays its eggs in bunches seems to lessen the difference between the Echinoderms which lay eggs and those which are viviparous, or retain the eggs in bunches in a sort of pouch at the base of the anus, as in fig. 34, which figure shows the manner in which a small Asteracanthion, allied to *A. Mülleri* Sars, retains its eggs till they have reached a very advanced stage of development. This figure is borrowed from drawings made for Professor Agassiz in 1848. In the Ophiurans, also, the young of some species are retained in the body of the parent till they reach a very advanced condition. Quatrefages has observed this. Professor Agassiz and Dr. Simpson have also noticed it in a species from Charleston, and Mr. Theodore Lyman observed it on the coast of France. See also Schultze and Krohn in Müll. Arch. for 1852 and 1857.

place in Starfishes, Echinoids, and Ophiurans, as well as Holothurians, and thus the distinction drawn by Müller falls to the ground. The young of Cuvieria figured here resemble the figures of Müller on Plate IV. third Memoir,* and Plate VII. fourth Memoir.† The young Holothurians (figs. 35 – 38) can without doubt be referred to Cuvieria on account of their color; they are of a brilliant vermilion. This unfortunately renders them so opaque that we cannot trace the position of the different organs without strong compression. In the youngest Cuvieria observed (fig. 35), the pupa (*l*) is very large; the Holothurian has not resorbed it to any extent; the tentacles (*g*) cannot be protruded. In a somewhat more advanced specimen (fig. 36), the Holothurian seems to have resorbed a considerable portion of the pupa (*l*); the tentacles (*g*) protrude; they are simple, do not branch, and terminate with knobs. The only tentacles present are those round the mouth. In a still more advanced stage (fig. 37), the tentacles show the first tendency to forking (*g*); other tentacles, ambulacral tentacles (*g'*), are also developed. The madreporic body stands out at the extremity of a small tentacle, *b*. That this is truly the madreporic body can easily be seen by compressing specimens of the age of fig. 36, when the madreporic body will be seen at the extremity of a tube, connecting thus the outer medium with the water-tubes in exactly the same manner as in Starfishes, Echinoids, and Ophiurans. The presence, also, of a calcareous network, is the best proof of its future function. During resorption this canal becomes liberated, but shrinks soon so as to bring the madreporic body on a level with the general outline.

On compressing a young Holothurian, about in the state of fig. 37, we shall obtain an idea of the state of development of the different parts. Fig. 38 (a Holothurian thus compressed) shows the separation of the main cavity into a kind of œsophagus, a stomach, *d*; and an intestine, *c*, with the anus, *a*. In younger specimens, when compressed, there is no anus, and the stomach is a simple sac without the peculiar bent addition of the intestine, *c*, of fig. 38. There are likewise no Y-shaped rods deposited on the surface of the body, and the ring of limestone particles at the base of the tentacles is not as fully developed as in this figure. (Fig. 38.) The madreporic body is seen more clearly in younger stages to connect with a large sac, *w'*, which is probably the remnant of one of the water-tubes, from its connection with the tube leading to *b*. Although externally (fig. 37) we see but two ambulacral tentacles, yet on compressing this specimen we should find that there were others already formed, but in such a rudimentary state that they do not as yet force their

* Müller, J. Ueber die Larven und die Metamorphose der Holothurien und Asterien. Berlin, 1850.

† Müller, J. Ueber die Larven und die Metamorphose der Echinodermen. Berlin, 1852.

way through the envelope, *l*. When compressed, the mode of branching of the young tentacles is clearly seen.

Incomplete as these observations are, they are of considerable value in a zoölogical point of view. It is evident that those Holothurians which want the ambulacral tentacles,* and have only a limited number of tentacles round the mouth, such as Caudina, Synapta, Fistularia (the Apodes), in which there are but very few deposits of limestone particles, stand lower than those Holothurians in which, as in Cuvieria and Psolus, some of the ambulacra are provided with suckers, forming a kind of sole, and in which the limestone particles perform an important part in covering the body. We should therefore place higher still those Holothurians in which, as in the preceding group, the tentacles round the mouth have a highly ramified character, and which have besides ambulacral suckers equally developed on the different ambulacra. It is evident from this that several of the forms observed by Forbes,† such as Psolinus, etc., will prove to be only the young of Pentacta, since he has distinguished genera principally from the degree of development of the gills and of the ambulacral system.

General Plan of Development.

The figures of Müller show without doubt that in Holothurians as well as Echinoids the young Echinoderm is in its earliest stages an open spiral star. An examination of the figures of Plates III. figs. 6–9, V. figs. 5–8, and Plate VI. first Memoir, and of Plates VI. fig. 14, VII. figs. 4, 6, 7, 9, and IX. figs. 3, 4, fourth Memoir, will satisfy any one that his figures cannot be made to agree with one another on any other supposition than that of an open pentagonal spiral surface which develops the actinal system of Echinoids on the surface of one water-tube, and of a similar spiral surface on the other water-tube which develops the abactinal area. In the same way Müller's figures of Auricularia, particularly those of Plate I. fourth Memoir, and Plate III. sixth Memoir, show that the young Holothurian also commences by having an open spiral actinal tentacular system. The figures of young Auricularians which are given on Plate I. fourth Memoir also show without doubt that the tube which leads to *b*, in fig. 36 of this Memoir, is the tube leading from the water-tubes to the dorsal pore, and that *b* is really the madreporic body. Müller also considers it as such in his figures, and it is certainly very remarkable that, with the admirable figures he has given of the young Ophiuran and Auricularian, he should not have noticed the intimate connection of the water-tubes and of the young Echinoderms. It is natural that the

* L. Agassiz. Methods of Study. 1863. † Forbes, E. A History of the British Starfishes.

idea which he entertained, that we had in Echinoderms a passage from the bilateral to the radiated form, should have made such a strong impression as to prevent his noticing the radiated character of the young embryo, hidden as it is by all this external appearance of bilateral symmetry. And had it not been for the clear idea we have now of the character of the parts of radiated animals,* I doubt not that Müller's view would have gained general acceptance among investigators, and the whole framework of classification, based upon the idea that a plan pervades the different types of the animal kingdom, would have fallen to the ground, if it could have been clearly proven that in Echinoderms we had a transition from one of these plans to another.

As embryology gives us the means of distinguishing on broad principles the class of Batrachians from that of the true Reptiles, since it has been shown conclusively by Professor Agassiz that the Batrachians are an eminently embryonic class, while the Reptiles proper are a synthetic type, so the embryology of Echinoderms throws a new light on the character of the orders which compose that class. Particularly important is this knowledge when applied to those early forms which have been considered by some geologists as Starfishes, Echinoids, or Ophiurans, thus placing the first appearance of these orders far back in geological times. A comparison of these types with the embryonic forms of our Starfishes, Sea-urchins, and Ophiurans, will show us plainly that they have nothing in common with them. The few features which have misled investigators, and have prevented their recognition as true Crinoids, are either synthetic or prophetic characters. Crinoids are an eminently synthetic and prophetic type. From the time of the earliest appearance of Crinoids the characters which they combined foreshadowed the advent of the true Starfishes, the Ophiurans, and the Echinoids. The synthetic characters were so prominently developed that many of them are readily mistaken for Starfishes or Sea-urchins, for the same reasons which have made it so difficult to recognize as true Reptiles those synthetic forms in which Fish or Batrachian features concealed the true Reptilian character, until we had obtained a reliable guide in the distinctions pointed out by embryology. If these views are correct, the Crinoids are the only Echinoderms which are found in the Palæozoic period, and it is not until the Secondary that the other orders appear.

The Starfishes as an order are characterized by the absence of prophetic features. They are rather a parembryonic order; that is, certain features which are character-

* L. Agassiz. Contributions to the Nat. Hist. of the U. S., Vols. III., IV.

istic of the embryos of Echinoderms are carried to a great prominence in the different families. These are, to give a few examples, the great development of the marginal plates of the arms, the character of the spines on the abactinal area, and the presence of pointed tentacles in the adult Starfishes. Ophiurans, on the contrary, are a peculiarly embryonic order. They never develop interambulacral plates, which, as we shall see in Asteracanthion,* develop quite late in the life of the young Starfish. What is very remarkable in all the young Echinoderms is their Crinoidal character. A stem added to a very young Starfish, Ophiuran, or Echinoid recalls to us many of the forms with which we are familiar in the palæontological history of our earth, and I have no doubt that a comparative study of the innumerable Crinoids known, and of the living and fossil Echinoids, Starfishes, and Ophiurans, will bring out many more points of interest than have been here alluded to, and give us a correct idea, not only of the nature of the orders, but also of the families which compose them. I have here pointed out a few of the characters which distinguish the different orders of Echinoderms; I shall endeavor to adopt the same method, to show how far what we know of the embryology of Echinoderms will assist us in forming a true conception of the classification of Radiates, reserving closer comparisons between the development of Acalephs and Echinoderms for another occasion, when I shall treat more fully than I have room for here of the development of the Ctenophoræ, which gives us the connecting link between the Polypoidal and Echinodermal mode of development. The division of Cœlenterata, proposed by Leuckart in contradistinction to Echinoderms, does not correspond to any natural distinction we can draw between the mode of development of Echinoderms on the one side, and that of Polyps and Acalephs on the other. The mode of development of Polyps differs more from that of the higher Acalephs — the Ctenophoræ, for instance — than that of the Ctenophoræ differs from the Echinoderms. But what is a fatal objection to the division proposed by Leuckart is the appearance at the earliest stages of development of definite numbers of spheromeres, whether we deal with a Polyp, an Acaleph, or an Echinoderm; and these spheromeres are not simply analogous parts, as the tentacles of the embryos of some Annelids and the arms of the plutean state of Echinoderms, but are strictly homologous, showing plainly that the same plan underlies the mode of development of these three classes, though it is carried out in such different ways in Polyps, Acalephs, and Echinoderms, and that the separation of the great type of Radiates into two branches, as proposed by Leuckart, is an artificial division which has no true foundation in nature.

* See Vol. V. of Contrib. Nat. Hist. U. S., by L. Agassiz.

EXPLANATION OF THE FIGURES.

WITH the exception of figs. 29, 30, and 34, which were copied from drawings made by Mr. Tappan, under the direction of Professor Agassiz, the figures were drawn on wood by myself from the drawings I made at the time of this investigation.

TOXOPNEUSTES DROBACHIENSIS AG.

FIGS. 1 – 28.

The specimens from figs. 1 – 18 inclusive were all obtained from artificial fecundation; figs. 19 – 27 were caught with the dip-net; fig. 28 was thrown up on the beach after a storm.

EXPLANATION OF LETTERING.

a, anus.
b, madreporic body.
c, alimentary canal (intestine).
d, digestive cavity (stomach).
e′, *e″*, *e‴*, *e*ᴵⱽ, arms of the plutean form.
f, brachiolar arms?
g, tentacles round actinostome in Cuvieria.
g′, ambulacral tentacles of Cuvieria.
k, teeth of young Echinus.
l, body of larva of Cuvieria (pupa of Müller).
m, mouth.
n, interambulacral tubercles of Echinus.
n′, ambulacral tubercles.
o, œsophagus.
p, pedicellaria.
r, abactinal region.
r′, main rod, formed first in the Echinus Pluteus.
r″, rod of arm *e″*.
r‴, independent rod above œsophagus.
*r*ᴵⱽ, anal point of junction of rods in the Ophiuran Pluteus.
s, actinal region.
s′, spots of arms *e′*, *e″* of the Pluteus of Echinus.
s″, interambulacral spines of Echinus.
s‴, younger spines of Echinus.
t, tentacles.
t′, odd terminal tentacle.
v, anal part of vibratile chord.
v′, oral portion of vibratile chord.
v″, vibratile epaulettes.
w, water-tube.
w′, water-tube leading to madreporic body.
y, terminal arm-plate of Ophiuran.
y′, side arm-plates of Ophiuran.

In fig. 31 *v* should be *v′*, and *vice versa*.

For the earlier stages compare figs. 1 – 5 of Proc. Am. Acad., from which the earlier stages of the Echinus Pluteus do not differ greatly.

Fig. 1. Profile view of an Echinus Pluteus eighty-eight hours after artificial fecundation.

Fig. 2. The same as fig. 1, seen from above.

Fig. 3. Profile view of a plutean form somewhat more advanced than in fig. 1; the digestive cavity, *d*, is bent very considerably towards the actinal side.

Fig. 4. View from above of a Pluteus in which the digestive cavity, *d*, is somewhat differentiated by con-

strictions; the pouch, *o*, formed at the blind extremity, has sent off two diverticula, *w*, *w'*; the first appearance of the water-tubes. End of fourth day.

Fig. 5. Profile view of an embryo somewhat more advanced than fig. 4, at the beginning of the fifth day; the intestine, *c*, stomach, *d*, and œsophagus, *o*, are well separated; the water-tube, *w'*, also stands out prominently from the extremity of the œsophagus, *o*; the anal part of vibratile chord, *v*, bulges out considerably.

Fig. 6 is a view, from the mouth-side, of the stage the embryo has attained at the end of the fifth day. The water-tubes, *w*, *w'*, have different degrees of development, and are less closely connected with the œsophagus; the rods have sent out processes at points where the new arms are to be developed, at *v*.

Fig. 7 is a view of fig. 6 from the anal extremity, to show the changes of form since fig. 2, and the position of the vibratile chord.

Fig. 8 is a profile view of fig. 6; the mouth, *m*, is not yet opened.

Fig. 9. A profile view of an embryo taken at the beginning of the seventh day; the mouth, *m*, is opened; the water-tube, *w'*, reaches nearly the dorsal surface. The currents which previously to this stage had carried the food through the only opening, *a*, as far as *o*, and then were reversed to eject the digested matter, now come in through the mouth, *m*, pass through the œsophagus, rotate about in the stomach, *d*, and pass out through the first-formed opening, the anus *a*, which is hereafter used only to eject the food.

Fig. 10 represents a Pluteus at the beginning of the eighth day, seen in such a manner as to show the whole of the vibratile chord, when looking on the side where the mouth and anus open.

Fig. 11 is a profile view of an embryo slightly more advanced than the one represented in fig. 10.

Fig. 12. The same as fig. 11, seen from the mouth side.

Fig. 13. Somewhat more advanced than fig. 12, seen so as to show the changes the vibratile chord has undergone since the stage represented in fig. 10.

Fig. 14. An embryo during the tenth day, seen in profile; shows the position of the arms e''' and e^{IV}.

Fig. 15. The same as fig. 14, seen from the mouth side; the arms, *e'*, have been greatly developed; the differentiation of the intestine, *c*, the stomach, *d*, and œsophagus, *o*, is quite complete. First appearance of the vibratile epaulettes, *v''*. The water-tubes have not united; they have not greatly increased in size.

Fig. 16. Profile view during the twenty-third day; the arm, *e''*, has made its appearance, and is already quite prominent; a still greater development of the vibratile epaulettes is perceived.

Fig. 17. The same as fig. 16, seen from above, to show the relation of the different rods to each other, as well as the first appearance of the tentacular loop, *t*.

Fig. 18. View of fig. 17, corresponding to the less advanced stages of figs. 10 and 13, showing the connection of the different parts of the vibratile chord.

Fig. 19. A much more advanced stage than fig. 17, seen from above; found swimming freely on the surface of the water. The rods extending into the arms are made up of three sets of rods united by short transverse bars; additional tentacular loops have been formed. The water-tubes have greatly increased in size, and appear to have united.

Fig. 20. An adult Pluteus of *Toxopneustes drobachiensis*, in which the young Sea-urchin has already en-

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 8.

Fig. 9.

Fig. 7

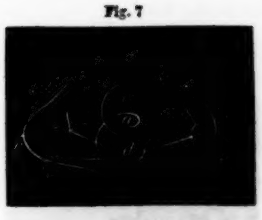

Fig. 11.

Fig. 12.

Fig. 10.

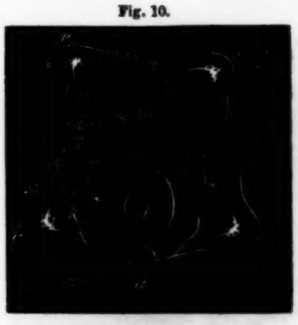

Fig. 14.

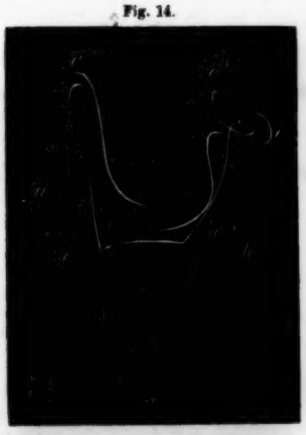

Fig. 15.

Fig. 13.

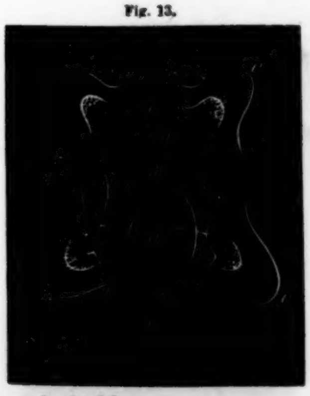

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Fig. 16.

Fig. 17.

Fig. 18.

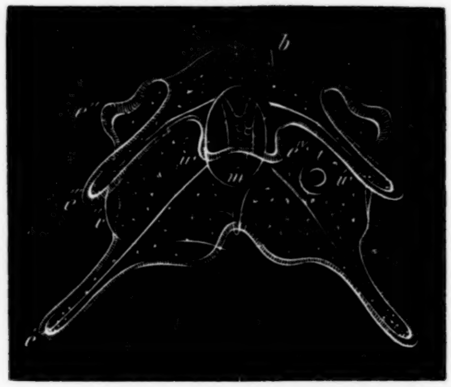

Fig. 19.

Fig. 20.

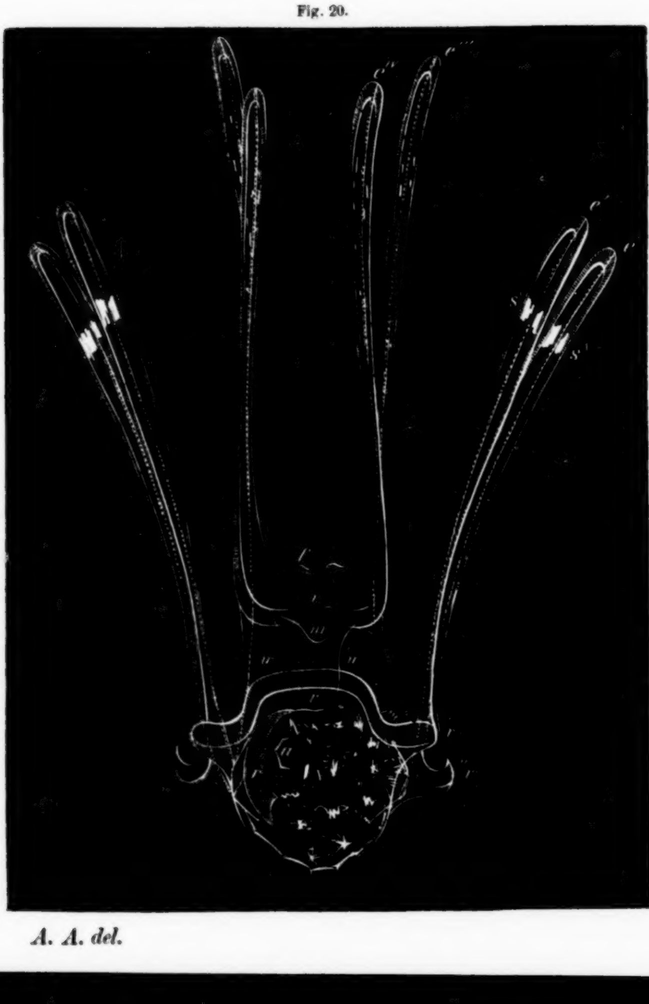

Fig. 23.

A. A. del.

Fig. 22.

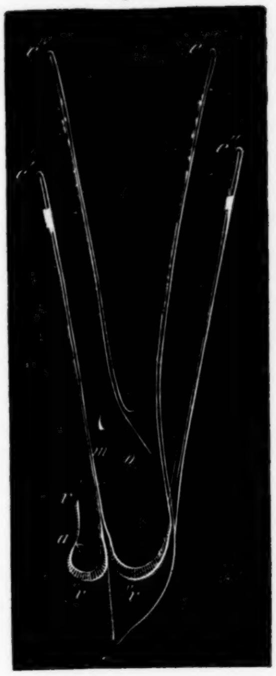

Fig. 21.

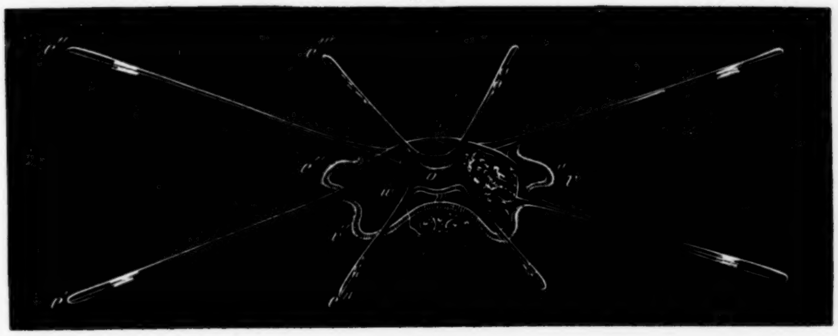

Fig. 34.

Fig. 28.

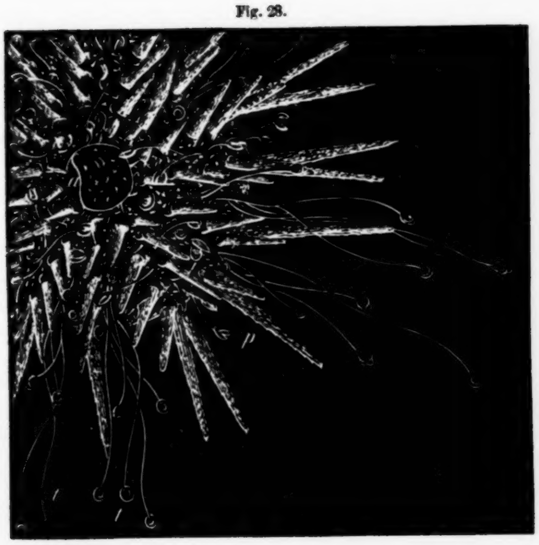

Fig. 24.

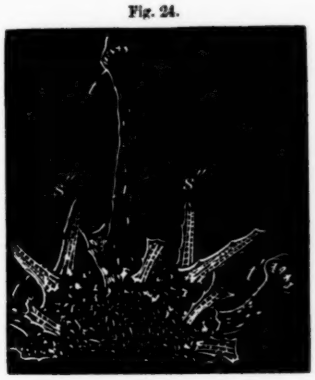

Fig. 27.

Fig. 26.

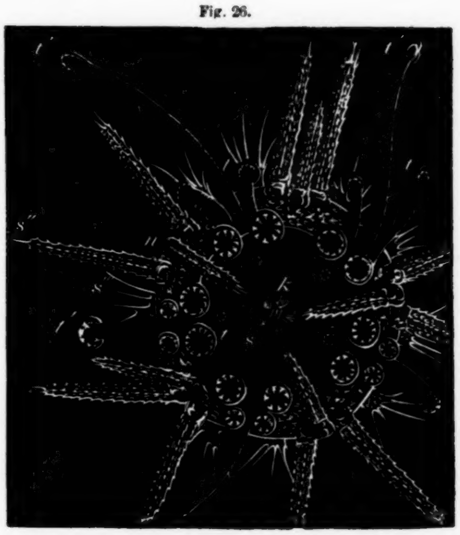

Fig. 25.

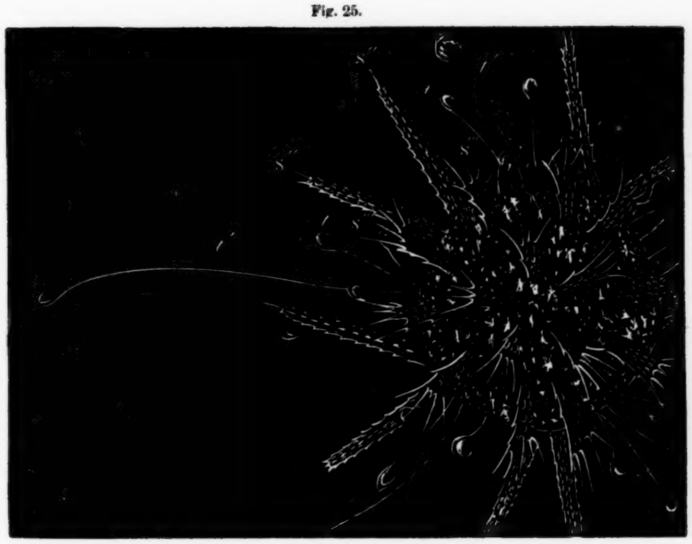

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Fig. 32.

Fig. 29.

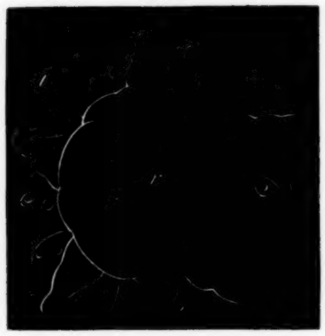


Fig. 30.

Fig. 35.

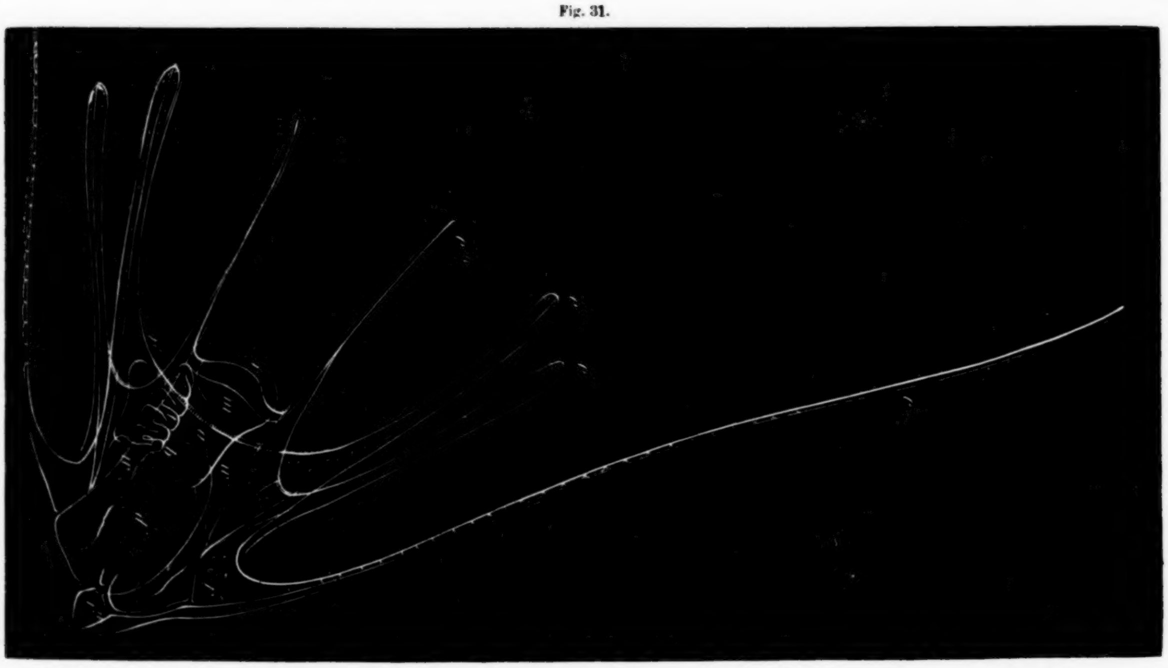
Fig. 37.

Fig. 33.

Fig. 36.

Fig. 38.

Fig. 31.



A. A. del.

croached somewhat on the anal extremity. The spines are quite prominent; the vibratile epaulettes have acquired a very large size; two very prominent spots, s', s', in the arms, e', e''; f is perhaps something homologous to the brachiolarian arms of Brachiolaria.

Fig. 21. View of fig. 20, corresponding to figs. 10, 13, and 18.

Fig. 22. Profile view of fig. 20.

Fig. 23. The young Echinus has materially encroached on the Pluteus, and resorption has been carried on to such an extent that there is very little left of the Pluteus except the arms, which have lost their mobility.

Fig. 24. A young Echinus immediately after the resorption of the Pluteus, seen from the abactinal side.

Fig. 25. Young Sea-urchin somewhat more advanced than that of fig. 24, seen from the abactinal side. The opening of the anus (a, fig. 28) cannot be traced in these younger specimens, though it is very apparent in somewhat more advanced ones.

Fig. 26. The same as fig. 25, seen from the actinal side; the tentacles have become more slender; the spines more numerous than in younger stages. The actinal membrane is well developed.

Fig. 27. The same as figs. 25 and 26, stripped of its spines, seen from the actinal side.

Fig. 28 is a young Toxopneustes measuring one fifteenth of an inch in diameter, including the length of the spines, in which pedicellaria have developed, and the spines have assumed the general appearance of those of the adult.

OPHIOPHOLIS BELLIS Lyman.

Figs. 29, 30.

Fig. 29. Abactinal view of a young Ophiopholis, to show the arrangement of the plates of the disk.

Fig. 30. A somewhat younger Ophiopholis than fig. 29, seen from the actinal side.

AMPHIURA SQUAMATA Sars.

Figs. 31 – 33.

Fig. 31. An adult Pluteus of Amphiura, in which the water-tubes are plainly seen. The figure is taken on the side of the madreporic body; one of the long arms is cut off. In this figure v' is the anal, and v the oral vibratile chord.

Fig. 32. The arms of the Pluteus are almost entirely resorbed, except the two long arms, e', which are still intact, as in fig. 31; they are here cut off for want of space. The Ophiuran is seen from the abactinal side, to show the arrangement of the arm-plates, y, y'.

Fig. 33. The same as fig. 32, seen from the actinal side. The mouth is still very large; the tentacles, t, t', mere loops.

ASTERACANTHION FLACCIDA Ag.

Fig. 34. Mode of carrying the embryos in *Asteracanthion flaccida*.

CUVIERIA FABRICII DÜB. ET KOR.

FIGS. 35–38.

Fig. 35. Youngest Cuvieria observed, in which the tentacles of the mouth do not yet protrude beyond the general outline of the pupa.

Fig. 36. A much more advanced Cuvieria than fig. 35; the tentacles, *g*, are very prominent.

Fig. 37. A still more advanced Cuvieria, in which the position of the madreporic body, *b*, can be seen. Ambulacral tentacles, *g′*, have also been formed.

Fig. 38. A young Cuvieria nearly in the state of fig. 37, compressed to show the relation of the different parts to each other.

II.

Observations on the Development of Raia batis.

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The following observations relate mostly to the changes which a single species of skate undergoes in its outward form and structure during development. They are based upon the examination of a series of eggs, collected in the spring of 1851 and of the three subsequent years. The publication of them has been delayed in the hope that other specimens might be obtained which would make it practicable to give an account of the evolution of the internal organs. Having been disappointed in this, it was thought desirable to publish such results as have already been reached, believing that some of them, at least, are additions to the previous knowledge of the subject.

Egg case. — This singular structure has the general form of such parts in egg-laying Selachians. The whole case, in the species here described, is between six and seven inches in length, of a deep greenish-brown color, and composed of minute parallel filaments, which give it a striated appearance and a silky lustre. The central pouch (fig. 1, *a*), for the protection of the yelk and the embryo, is about two inches long, an inch and a half wide, bulges in the middle, and has a hollow, slender, curved horn projecting from each corner. The fore end of the pouch is deeply concave, and thickest, while the hinder is thin, nearly square, and ragged; it is from this part that the embryo escapes, after the separation of the upper and under walls from each other. The hinder horns project backwards as they lie in the oviduct, and are of about twice the length of those at the other end. The outer edge of each horn is the more rounded, and near the free end has an oblong slit (*b b′*) for

the inward and outward flow of the water which passes through the egg during incubation.

At the base of each fore horn is a slender projection or spur (*c*), about half an inch in length, the whole outer border of which breaks up into a series of silky filaments, and these are especially abundant near the free end. Similar filaments are given off from the whole border of the capsule, and all become tangled and woven together in such a manner as to form a broad and somewhat thick membrane on each side (*d*). This membrane was found entire only on cases taken from the oviduct, and on those newly laid. In all such, however, as have embryos somewhat advanced, it is more or less destroyed, and for the most part only tufts of it remain at the base of each horn. The object of it is not apparent, unless it be to assist in securing an anchorage, by the entanglement of its filaments with submarine plants or rough surfaces.

In a single instance, in the dissection of skates, an imperfect egg-case was found in each oviduct, the development of it having just begun.* The hinder horns and the hinder edge of the capsule were the only parts completed. They were contained in the glandular portion of the oviduct, which is quite thick during the reproductive season, and is mostly made up of very minute and slender follicles, of great length. From some of them fibrils protruded, identical in structure with those out of which the cases are made, and which, after being liberated, are doubtless moulded into the shape of these cases, and cemented together by some secretion from the oviduct. The horns are formed in grooves on either side of the duct, and the pouch for the yelk in the intervening space.† A careful examination of the ovary and oviduct in the above instance showed the singular fact, that, although some of the yelks were mature, none had as yet been detached from the ovisacs. This circumstance renders it probable, that, after the horny pouch is partially formed, the yelk descends and enters it, and that then the other portions are completed. If this supposition, based upon a single observed instance, were to be confirmed by further examinations, it would prove the existence of an interesting deviation from a rule among animals generally supposed to be without exception, viz. that the presence of the yelk in the oviduct is necessary before the formation of the egg-coverings can begin.

* For the specimen here referred to, as well as for several others which have given me important aid, I am indebted to my friend Dr. John Green of Boston.

† Aristotle was familiar with the eggs of Plagiostomes. See Hist. of Animals, Book VI. Section 10. Ruysch figures them for the first time in the *Thesaurus Animalium*, Pl. III. figs. 3 and 4, showing the embryo *in situ*. Cuvier ascribes the materials of the case to the follicles, and the form to the glandular surface. *Leçons*, Tome VIII. p. 90.

In none of the cases which we have examined have we found the fœtus surrounded either by a membrane or by albuminous matter, but in every instance the yelk and the embryo were fully exposed to contact with the water, which entered by the openings already described. An albuminous covering may have existed at an earlier period, and have been absorbed.

Yelk.—After the body of the embryo has become well defined, it is attached to the yelk by a slender umbilical cord about half an inch in length (fig. 2). The yelk has not the pyriform shape so common in other Selachians, but is nearly spherical, though somewhat flattened above and below. The cord has the length just mentioned only temporarily, and soon begins to shorten, and contracts until the fœtus rests once more upon the surface of the yelk (figs. 6 and 7). The two omphalo-mesenteric vessels, common to all vertebrates, carry the blood from the embryo to the yelk and back. The artery, a branch of the mesenteric (figs. 2 and 7, *a*), passes out beneath the head, over the front of the yelk, and descends to the under surface, giving off minute twigs to the right and left; but the trunk itself does not branch. Dr. John Davy,* in his observations on the development of the torpedo, although he figures a vein surrounding the vascular area in the younger specimens, yet makes no reference to it in the text. Agassiz has observed a similar vessel in the yelk of a dog-fish, and has for the first time pointed out its resemblance to the *sinus terminalis* of birds. Dr. Davy's figures, taken in connection with those here given, form a complete series. In the youngest of the specimens described by him the sinus is found on the upper surface of the yelk, and quite near the embryo; in the second, it has receded toward the sides, and the vascular area enlarged to a corresponding degree. In our specimens it is found on the under surface, is of a triangular form (fig. 3), and encloses only a small area. Eventually it contracts still further, and at last wholly disappears, and thus the entire surface of the yelk becomes vascular (figs. 6 and 7).

As development advances the yelk is gradually withdrawn into the cavity of the abdomen, as in birds; but the retraction does not appear to be quite complete in the skates until a short time after hatching. In one instance a fully formed skate taken from the egg-case had the yelk reduced to a small flattened mass about two lines in diameter. Very nearly the same condition existed in another, which was already hatched. In a third instance, where the young had been hatched for a longer time, the yelk had been wholly introduced into the cavity of the abdomen; but a con-

* Researches Anatomical and Physiological, Vol. I. p. 61.

siderable mass of it, still within the abdominal cavity, remained to be absorbed (fig. 11, *a*), where, as in the newly hatched chick, it serves as a reservoir of nourishment. Dr. Davy states that, in the torpedo, the young fish is nourished by the yelk for six weeks after birth. In all cases we have found the vitelline duct entering the intestine just above the spiral valve.

Form of the Fœtus. — The general form of the youngest specimens is long, slender, and gradually tapers to a point backward, as in fig. 2, and may be described in one word as eel-shaped. The head presents two rounded projections, one of them forward (figs. 4^a and 5, *d*), forming the foremost part of the embryo; this is made by the protrusion of the optic lobes, and closely resembles the same part in the embryos of birds; the second (fig. 5, *e*) is directed downward, and contains the cerebral and olfactory lobes, behind which are the eyes. These last, which in the earlier stages, as in figs. 2, 4, 10, are on the same level with the surrounding parts, soon become remarkably prominent, as in fig. 8, where they remind us of the eyes of the young of Malleus. In the fully formed fish they are again reduced to nearly the same level with the adjoining integuments. As development advances, the optic lobes cease to form the most prominent part of the head, in consequence of a change of position of the cerebral hemispheres, which rise to the same level with the organs just mentioned, as the facial disk (figs. 7, 9, 10, *b*) advances beneath them. In fig. 6 the embryo has many of the features of a shark; and in fig. 8, with the expansion of the pectoral and ventral fins, it begins to take on the form of the skate. For further details the reader is referred to the different sections of this article.

Fins. — In the youngest specimen examined (fig. 2), a vertical fold of skin stretches along the middle line, from near the head almost to the end of the tail above, and from near the umbilical cord to the same point below. These folds do not pass beyond or become connected around the end of the tail. The dorsals (figs. 2 and 7, *c*) are formed by two vertical extensions of the upper fold, and in this early stage of their existence are placed midway between the base of the tail and its tip, which last tapers to a slender point. The anals (figs. 2, 7, and 9, *d*) are formed from a similar extension of the under fold, and are situated somewhat farther forward than are the dorsals. The first (fig. 6, *d*) grows very rapidly, and soon acquires a disproportionately large size; but the second (fig. 6, *d'*) is quite diminutive. Both upper and lower folds and fins have their edges bordered with follicles.

Both dorsal and anal fins undergo a very remarkable change as development advances. The first in the adult are found quite at the end of the tail, instead of the middle, as in the early stage. This change of relative position seems to be

effected in part by the more rapid growth of that portion of the tail which is in front of them, while that which is behind scarcely increases in size, and thus the fins are soon nearer the end than the middle. At the time of hatching, the terminal portion is still present (fig. 11, *c*); but subsequently it is either absorbed, or, what is not improbable, is covered by the extension of the dorsals backward.

The anal fins, the first of which, as already stated, attains to a remarkably large size, are gradually absorbed, and are wholly removed before the end of gestation. From the fact of these fins having a temporary existence in the skate, and a permanent one in many sharks, it is not improbable that they may be present in the embryos of all Plagiostomes. In the torpedo the dorsal fins seem to retain their primary or embryonic position, as they remain permanently in the middle of the tail; and Uraptera has the slender terminal portion behind the dorsals persistent, just as in the newly hatched *Raia batis*.

This development, temporary existence, and early removal of the anal fins, gives us another interesting example of the formation of parts which have no obvious use in the economy, and which must be regarded as having merely a morphological value. It falls into the same category with the caudal fin of the embryo of Pipa, which is never used, the teeth of certain Cetaceans, the inferior incisors of the female mastodon, which are all removed without being used, and the milk incisors of the Guinea-pig, which are shed *in utero*.

There is still another point of interest in the morphology of the tail of the species we are here considering; for although symmetrical, it does not at any period assume the heterocereal form, but retains permanently its primary embryonic or protocereal condition.

In this respect the skates hold a lower position than the sharks, nearly all of whom pass through the protocereal into the heterocereal stage.

The pectoral and ventral fins begin as slight ridges on either side, but each soon takes on the form of a half-oval disk (fig. 4, *a*, *b*). At first the two are nearly continuous in the same plane (fig. 4), but the pectorals (figs. 8 and 9, *a*) grow the most rapidly, gradually assume a somewhat oblique position, and in a short time partially cover the ventrals. None of the specimens were of a proper age to show whether or not the pectorals were formed first, as is the case with the fore limbs of all vertebrates whose development has been thus far studied. As they grow, they advance on either side of the head in the form of horns (figs. 8 and 9, *a′*), but by degrees the space between these horns and the side of the head is filled up, and thus the eyes and the persistent portion of the first branchial fissure are pushed to the upper surface, and

eventually the pectorals gain the foremost part of the side of the head, at the same time becoming united with the extended facial disk.

The tail, as the whole animal increases in size, becomes relatively very much shorter. In the earlier stages the body is only three sixteenths of the entire length of the embryo, but subsequently it is about one half that length, as will be seen by a comparison of figs. 7, 8, 9, and 11.

Mouth. — In the youngest embryos this has an elongated shape, is broadest at the fore part, and, contrary to what is usually the case in the embryos of other orders, has its longest diameter directed from before backwards, which circumstance gives it a very singular appearance (fig. 4^a). Its borders are formed by the first branchial arch (figs. 4^a and 5, *b*), and, as development advances, its longest diameter begins to shorten, and the arch bends on either side, its upper and lower, or its fore and hinder portions, forming an angle with each other; and thus begin the future angles of the mouth. That portion of the arch below these is without a doubt transformed into the lower jaw, and the "upper jaw" is formed of what remains in front of them.

The homology of this so-called upper jaw has led to much discussion. One thing is certain, there is at no time to be seen in the circumference of the mouth anything which corresponds with the "intermaxillary bud" or "fronto-nasal protuberance"; nor is there anything corresponding with an upper "maxillary bud." The only other parts to which the "upper jaw" could be said to correspond would be the palatines, as asserted by Cuvier; or more probably, as maintained by Mr. Huxley, the bones just mentioned, the pterygoids and quadrate bones together, all of which are believed to be developed from one and the same primary cartilage.

If the maxillary and intermaxillary bones exist at all in the head of the skate, their homologues must be found in the parts farther forward than those just mentioned, and in some way connected with the nostrils. We shall, therefore, speak of them when these last-named parts are described.

There is a very important change, which, though not directly connected with the mouth, yet involves the region of it, and may properly be mentioned here; namely, the formation, just in front of the nostrils, across the whole breadth of the under side of the head from one pectoral fin to the other, of a ridge of thickened integument, which gradually extends forward in a horizontal plane, forming what may be called the facial disk, and is most prominent in the middle (figs. 7, 9, and 10, *b*). It is by the extension of this forward till it passes beyond the foremost part of the cranium, and its fusion with the pectorals, that the pointed rostrum of the adult skate is formed (fig. 11, *e*). Even after this is nearly completed, the cranium remains prominent above it, but eventually both come to the same level.

In advanced embryos there is formed behind the mouth a semicircular fold of skin, which extends from one angle to the other, and very closely resembles the lower lip of mammals in its first stage of development. We did not find this in an adult specimen which was especially examined for comparison.

Nostrils. — In the adult, each nostril is lodged in an inverted cartilaginous cup connected with the base of the cranium, and lined with a folded sensitive mucous membrane. From this there extends to the angle of the mouth a deep groove, which ends just in front of the "upper jaw." The integument between the right and left grooves projects from the general surface, forming a sort of upper lip (fig. 11, *d*), and the angles are developed into fringed lobes which cover the corners of the mouth. Each lobe contains a cartilage, which Müller compares with the cartilages of the wings of the nose, as also the outer border of each groove opposite to them. The peculiar configuration of these parts in the adult skate, and the resemblance of the whole to an embryonic condition of the higher vertebrates, renders the study of them quite important. As Professor Agassiz states, "no one can fail to be impressed with this resemblance who compares the head of an embryo quadruped, looking at it in front face, with the adult skate." * To give demonstrative evidence that the parts thus compared are homologous, can only be done by an examination of a larger number of specimens, in different stages of development, than have as yet been studied. The series here described contributes something to this end, and enables us to determine some points which have not thus far been noticed.

The first traces of the olfactory fossæ which we have seen consisted of two small, but elongated and well-defined pits (fig. 5, *c*), somewhat enlarged at each end, and converging towards each other backward. They are at a distance from the mouth, and have no connection whatever with it, nor is there as yet the beginning of the groove which is found at a later period. In these respects they resemble the primary form of the nostrils of vertebrates in general. The position of them is such that they might be easily overlooked, for they are confined mostly to the hinder face of that portion of the head which is formed by the projection of the cerebral lobes downward, and can only be wholly seen by standing the embryo on its head. As the mouth takes its permanent shape, the nostrils lengthen, and a process forms on the inner border of each of them (fig. 10, *a*), which is the first stage of the lobe already described as existing in the adult. By a gradual thickening of the integuments, these processes become connected with each other across the middle line, when the whole skin between the nos-

* Methods of Study in Natural History, (Boston, 1863,) p. 317.

trils projects above the surface, and forms the upper lip, or the portion already spoken of as having so strong a resemblance to the intermaxillary bud or protuberance of the higher vertebrates (fig. 11, *d*). The edges of this form the inner borders of the nasal grooves, the outer ones being the result of a corresponding thickening of the integuments on the other side of this groove. Both inner and outer borders rise to the same level.

If now we compare the phases which these parts pass through in skates with the permanent conditions of them in other Selachians, it will be found that, in one species or another, these permanent conditions are arrests of development in various stages. In *Oxyrhina gomphodon* the nostrils retain permanently the primitive form of the olfactory fossæ, like that of the youngest skates noticed above; in *Pristiurus melanostoma*, and in *Mustellus*, *Carcharias laticaudus*, and many other species, the nostrils have superadded the lobe on their inner border, but no further thickening of the integument between them; in *Scyllium Bergeri* this lobe is extended by the thickening of the integument towards the middle line, as in Fig. 10; and in *Raia torpedo*, *Scyllium maculatum*, *Tæniurus Meyeri*, and others, it extends across the whole space between the nostrils, and forms above the mouth a continuous upper lip, as already described.

If the part the development of which has just been described is to be compared with the intermaxillary bud, the grooves on either side must be compared with the unclosed nostrils of the embryos of air-breathing vertebrates, or in other words to the "hare-lip." In air-breathing animals the nostrils open into the mouth, either by a canal between the maxillary and intermaxillary bones, as in many reptiles, or by a canal extending farther back, and separated from the mouth by the bones just mentioned and the palatines in addition, as in mammals; or by these bones and the pterygoids, as in the crocodiles. In the Proteus, Axolotl, and Menobranchus, however, the nostrils cannot be said to enter the mouth at all, but pass through the upper lip at a point corresponding with the union of the maxillaries and intermaxillaries, but still exteriorly to the dental arch. Bearing this in mind, we are led to look for the homologues of these bones in the immediate neighborhood of the nostrils in the skate. The only parts which occupy the position indicated are the cartilages, already referred to, contained in the nasal lobes, and in the parts just outside of the nasal groove. Is not the cartilage which extends from the olfactory fossæ towards the pectoral fin the homologue of a maxillary bone, and that in the lobe, of an intermaxillary? If so, the skates and Proteiform reptiles agree in having the nostrils open in front of the dental arch, but at a point corresponding with the union of the maxillaries and intermaxillaries; they differ in this, that while in all Batrachians the nasal groove becomes closed, in the skates it

remains permanently open. Should this prove to be a correct determination of the parts, it will add another feature which justifies Owen, Agassiz, and others, who have so far dissented from Cuvier as to give the Selachians a place in the zoölogical series higher than that of the bony fishes. At the same time, it will give corroborative proof of the correctness of Cuvier's view, that "the rudiments of the maxillaries, intermaxillaries, . . . are evident in the skeleton." * Furthermore, we may also assert that among Selachians we have numerous instances of a double hare-lip being a normal adult condition.

Branchial Fissures and Gills. — In nearly all adult Selachians there are five gill-openings in each side; Hexanchus and Heptanchus have respectively six and seven such openings. In addition to these, all of the skates and some of the sharks have a peculiar opening just behind the eyes, or at some point between these and the first branchial fissure, which makes a direct communication, for the most part of a large size, between the top of the head and the pharynx, and to which the terms "spiracle," "event," "Spritz-locher," "foramina temporalia," etc., have been applied.

In the youngest embryos of skates here described, we have found the number of gill-openings or branchial fissures seven on each side, all well defined except the last, which is the smallest of the series (figs. 4 and 4[a]). These are all in the same range from before backward, and at this stage the spiracle, as such, is not distinguished from the others. It is characteristic of the early embryos of all Selachians, to have developed, in connection with branchial apparatus, temporary gills, which are seen in the form of long and slender filaments projecting from the sides of the neck. They are generally described as coming out through the gill-openings, and as prolongations of the internal gills. Cornalia,† who has made a special study of these organs, so describes and figures them. We believe that, in consequence of not having seen embryos sufficiently young, he has been led into an error.

We have found them, when first formed, growing from the outer edge of the branchial arch (figs. 4[a] and 5), and at that time in no way connected with the branchial fissures. In the skate, the first and seventh arch had no fringes at any period, and of the five which had them, the fringes of the foremost ones were the longest, the hindmost being merely short, conical projections. As development advances, the bases of the fringes are gradually covered up, as it seems, by the growth of the portion of each arch in front of them, which is thus projected outward as the body

* Animal Kingdom, McMurtrie's translation, (New York, 1831,) Vol. II. p. 283.

† Sulle Branchie Transitorie dei Feti Plagiostomi. Memoria del Dottore Emilio Cornalia. 1856.

becomes thicker from side to side; the line of attachment of the fringe, which retains its original position, being thus buried between two adjoining arches.

From the fact that the temporary gills are formed before the permanent ones, and from the outer surface of the arch, it is obvious that they cannot be — as commonly described — prolongations of these last-mentioned breathing organs.

The fringes do not cover the whole border of the arch, but are confined to its central portion, and consist of from six to eight filaments each.

We have made no observations on the formation of the internal gills, and cannot therefore explain the connection which eventually exists between these and the fringes, and which at a later period correspond exactly with the descriptions usually given.

The existence of temporary branchial fringes, and their subsequent absorption, is one of the most remarkable characteristics of Selachians, and one in which they differ from all osseous fishes, unless it be the Lepidosiren.* All vertebrates, as embryos, agree in this, that they are in their early stages provided with "branchial fissures" and "arches," or, as they have sometimes been called, "visceral arches." Gills or gill fringes, either as temporary or permanent structures, are never formed in any scaly reptile, bird, or mammal.† Much confusion and misapprehension have arisen from the constant reiteration of the opinion put forth in the early days of embryology, that all vertebrates at one time have a branchial respiration, an error which is repeated by naturalists even at the present day. Among Batrachians some genera, as Menobranchus, Siren, Axolotl, etc., have external fringes permanently attached to their branchial arches, which are not known to be replaced by, or to coexist with, internal gills. They are their sole organs of respiration, for their lungs are too imperfect and rudimentary to have much physiological importance. In frogs, toads, and salamanders, the external gills are replaced by internal ones, and these in turn by lungs. Thus it will be seen that no Batrachian is permanently provided with internal gills.

Selachians and Batrachians agree in this, that their embryos have in their first stages external fringes growing from the outer surface of the gill arches, and these

* "In the *Lepidosiren annectens*, three small external branchial filaments project from the single opercular aperture on each side, and are long retained, if they be not permanent, in that remarkable osculant form between the osseous and the cartilaginous fishes." Owen, Lects. on Comp. Anat., Vol. II. p. 301. See also Jardine, Ann. Nat. Hist., Vol. VII., (1843,) and Peters, Müller's Archives (1845).

† The recent investigations of Lereboullet on the development of the lizard are, as regards this animal, to the same effect. "The branchial arches do not, in the evolution, pass through the same phases as those of fishes. They never become provided with fringes, and never perform the functions of respiratory organs." Annales des Sciences Naturelles, Tom. XVII., (1862,) p. 127.

fringes have the same structure in both.* The Selachians still further agree with frogs, toads, and salamanders, in the fact that the outer fringes are absorbed, and are replaced by internal gills. They differ from them, however, in the following particular. Selachians retain their internal gills permanently through life, while, if such exist at all in the Batrachians just mentioned, it is only during the larval stage, and they are soon replaced by lungs. Selachians may therefore be said to pass through stages analogous to the first and second stages of Anourous Batrachians and salamanders.

The other changes which the fissures pass through before the skate acquires its permanent form are as follows. The seventh fissure is closed up at a very early period, about the time that the dorsals are beginning to be formed. While the first arch bends and is drawn forward as already described in connection with the formation of the jaws, it at the same time becomes broader, so as to widen the distance between the mouth and the first fissure, or the second, after the first is partially closed. The inner part of the first closes up, while the outer remains open (fig. 5, *a*), is somewhat enlarged, and retains its relative position to the eye. It is very soon widely separated from the other fissures by the rapid growth of the intervening parts, and still further by the extension of the pectoral fins forward between this remnant of the first fissure and those behind it, the former being thus thrown to the upper, and the latter to the under surface. The unclosed portion of the first branchial fissure is thus converted into the spiracle.

The transformation thus described is of very great interest when compared with the changes which occur in the corresponding fissures of the air-breathing vertebrates, and enables us to establish an unexpected homology. Reichert,† in his most important investigations of the development of the gill arches ("visceral Bogen") of the pig, has shown that in this animal the first fissure is gradually separated from the others by the widening of the second arch, and for a time, even after all the others are closed up, forms a direct opening from the side of the neck into the pharynx. Afterwards it is

* Cornalia states that these respiratory fringes are not found in the "cotylophorus" sharks, as in Centrina, for in such the fœtus forms a direct communication with the oviduct of the parent, and the fringes are therefore unnecessary. This statement may be questioned with propriety, on the ground of analogy, unless it were based upon observations made upon very young embryos. This, however, does not appear to have been the case, for the figure of the fœtus referred to by him as evidence (see Carus, Entwickelung der Thiere im Allgemeine, Tab. VI. fig. 9) shows that the specimen was quite advanced, and has reached a period when the fringes might have been absorbed. It is quite probable that, as the young of such sharks have the advantage of a vitelline placenta, the fringes would disappear at a very early stage. See Cornalia, Sulle Branchie Transitorie dei Feti Plagiostomi, (Nizza, 1856,) p. 22.

† Meckel's Archives, (1837,) p. 120.

divided into an outer and inner portion by a membranous septum; the former being the external auditory canal, and the latter the Eustachian tube and the cavity of the tympanum. It will thus be seen that the spiracle is not only a true branchial fissure in the first place, but that in the end it is homologous with the Eustachian tube and the outer auditory passage before these are separated from each other by the membrane of the tympanum.

Professor Huxley, in a series of lectures * on the Vertebrate Skeleton, in which the homologies and development of it are discussed with great ability, sets forth a somewhat different view with regard to the formation of the external ear, and maintains that the first step is similar to that in the case of the eyes and nose, viz. an "involution" or a "pushing in" of the integument. Professor Huxley's observations were made on the chick, and he arrives at the same conclusions as Remak, leaving us to infer that the auditory passage and Eustachian tube have no connection with the branchial fissures. We have gone over the same ground in the pig, and have found Reichert's observations, as mentioned above, fully confirmed.

The relation of the spiracle to the branchial fissures is still further shown by the fact that in some species, as in Scyllium and Læmargus, it, like the others, is provided with respiratory fringes. In the skate this is not the case, but in the adult a comb-like fold, resembling, and probably having the functions of, a gill, is found just within the spiracular opening.

The following is a general summary of the results contained in the preceding pages.

1. The yelk case is formed in the glandular portion of the oviduct, and is begun previously to the detachment from the ovary of the yelk which is to occupy it.

2. The embryo, before assuming its adult form, is at first eel-shaped, and then shark-shaped.

3. The embryo is for a short time connected with the yelk by means of a slender umbilical cord; the cord afterwards shortens, and the young skate remains in contact with the yelk until the end of incubation.

4. There are seven branchial fissures at first; the foremost of these is converted into the spiracle, which is the homologue of the Eustachian tube and the outer ear canal; the seventh is wholly closed up, and no trace remains; the others remain permanently open.

* Structure and Development of the Vertebrate Skeleton, London Lancet, July, 1863, p. 427, American reprint.

5. There are no temporary branchial fringes or filaments on the first and seventh arches; on the others the fringes are developed from the outer and convex portion of the arch, and are not at first prolongations of the internal gills.

6. The nostrils, as in all vertebrates, consist at first of pits or indentations in the integuments; secondly, a lobe is developed on the inner border of each; and finally, the two lobes become connected, and thus form the homologue of the fronto-nasal protuberance. The transitional stages of these correspond with the adult conditions of them in other species of Selachians.

7. The nasal grooves are compared with the nasal passages of air-breathing animals, and the cartilages on either side of these to the maxillary and intermaxillary bones.

8. The foremost part of the head is formed by the extension of the facial disk forward; while this extension is going on, the cerebral lobes change their position from beneath the optic lobes to one in front of them.

9. Two anal fins, one quite large and the other very small, are developed, but both are afterwards wholly absorbed.

10. The dorsals change position from the middle to the end of the tail. At the time of hatching, however, there is still a slender terminal portion of the tail, which is afterwards either absorbed or covered up by the enlarged dorsals, as they extend backward.

EXPLANATION OF THE FIGURES.

Fig. 1. Egg case, one half of the natural size, linear measurement; *a*, pouch for the yelk; *b b'*, openings for the inward and outward flow of water; *c*, spur; *d*, membrane formed by the interweaving of the lateral filaments.

Fig. 2. Eel-shaped embryo, connected with the yelk by a slender umbilical cord; *a*, omphalo-mesenteric artery; *c*, dorsals; *d*, anal.

Fig. 3. Under side of the yelk of the preceding specimen; *a*, the continuation of the artery seen in fig. 2, and connecting with the triangular terminal sinus.

Fig. 4. A more advanced embryo, showing at *a* and *b* the pectorals and ventrals; *d*, the temporary anal.

Fig. 4*. Head of the preceding enlarged; *b*, first branchial arch, without fringes; *d*, projection of the optic lobes; *e*, projection of the cerebral lobes; the open space between the first branchial arches is the mouth.

Fig. 5. Side view of the same; *a*, first branchial fissure, largest at its outer end; this enlarged portion corresponds with the future spiracle; *b*, the inner end; the first arch is in front of this fissure; *b'*, the second fissure, in front of which is the second arch, bearing a fringe; *c*, nasal fossa; *d*, projection of the optic lobes; *e*, cerebral lobes.

Fig. 6. A shark-shaped embryo; *c*, dorsals; *d d'*, anals. In this figure the embryo is represented as twisted on the yelk, through half a circle, consequently the artery is directed backward instead of forward.

Figs. 7, 8, 9. A more advanced embryo, seen from the side, from above, and from below; *a*, fig. 7, artery; *a a'*, figs. 8 and 9, pectoral fin; *b*, figs. 7 and 9, facial disk; *c*, figs. 8 and 9, ventrals; *c*, fig. 7, dorsals; *d*, figs. 7 and 9, anal; *e*, figs. 8 and 9, gill fringes.

Fig. 10. Head of figure 6, enlarged; *a*, nasal lobe; *b*, facial disk; *c*, upper lip.

Fig. 11. Newly-hatched skate; *a*, yelk-sack in the cavity of the abdomen, connecting with the intestine, *b*; *c*, embryonic portion of the tail which disappears in the adult; this corresponds with all that is behind the dorsals in the preceding figures.

Fig. 1.

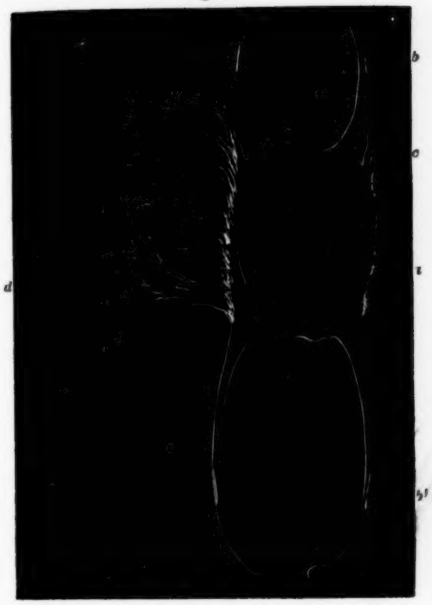


Fig. 2.

Fig. 6.

Fig. 3.

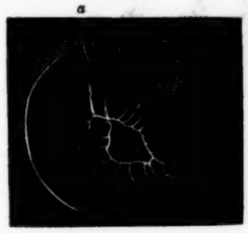


Fig. 4.

Fig. 4ᵃ.

Fig. 7.

Fig. 11.

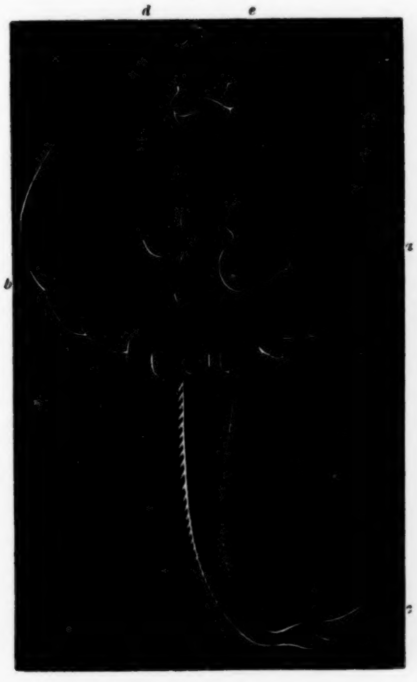


Fig. 9.

Fig. 8.

Fig. 5.

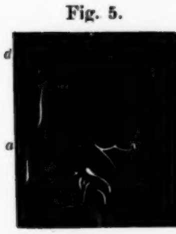


Fig. 10.

J. W. del.

IV.

On the Construction of Hooped Cannon; being a Sequel to a Memoir "On the Practicability of Constructing Cannon of Great Caliber, &c." published in the VIth Volume of the Memoirs of the Academy.

By DANIEL TREADWELL,

LATE RUMFORD PROFESSOR IN HARVARD UNIVERSITY.

Communicated April 12th, 1864.

ALTHOUGH a great improvement has been made in the construction of cannon by the adoption of the principle, and imperfectly the mechanical form, indicated by me in a specification of June 19th, 1855, which was immediately afterwards expanded into the memoir published in the VIth Volume of the Academy's Memoirs;* still many points in the theory of that construction remain not only unperfected, but almost unexamined.

It is my purpose, therefore, in this paper, to investigate several important properties and laws which are inherent in the materials of which the gun described in my former memoir is constructed; and from this investigation I shall endeavor to draw such instruction as will enable us, if not to perfect, at least to understand and improve, the theory of construction. The investigation will be founded almost entirely upon certain peculiarities in the nature, character, and properties of the materials (wrought-iron, cast-iron, and steel) of which the guns, constructed upon the principle heretofore published by me, are formed.

With these preliminary remarks I enter at once into the proposed inquiry, leaving the development of the course to be pursued to appear as I proceed.

In the memoir of 1855, before referred to, in giving an account of the theory of hooping cannon, I inserted the following paragraph (p. 10):

* The memoir in the VIth Volume must be considered as a sequel to, and a further development of, the principles contained in a publication, in the form of a pamphlet, made by me in 1845. In this pamphlet, not only the principles, but the method of construction since followed by Armstrong and others, are fully pointed out.

"There may, at the first view, seem to be a great practical difficulty in making the hoops of the exact size required to produce the necessary compression. This would be true if the hoops were made of cast-iron, or any body which fractures when extended in the least degree beyond the limit of its elasticity. But wrought-iron and all malleable bodies are capable of being extended without fracture much beyond their power of elasticity. They may, therefore, be greatly elongated without being weakened. Hence we have only to form the hoops *small in excess*, and they will accommodate themselves under the strain without the least injury."

And again, in a note, I said: "Mr. Barlow does not limit the application of his investigation to any kind of material, but it is evident that his conclusions are not applicable to any *malleable* metal like bronze; for in a cylinder constructed of hoops of this material the inner hoops may be elongated by the pressure acting as a *crushing* force, and by this means be enlarged without any diminution of tenacity. Perhaps some kinds of soft cast-iron may accommodate themselves to an enlargement in the same way. But with hard crystalline cast-iron, no actual displacement of the constituent particles can take place without fracture; and although the effect of the fluid as a crushing force may act as an auxiliary to the strain, as any estimate of its amount would be a mere guess, I shall not attempt any modification of Mr. Barlow's conclusion, when applied, as in this case, to hard cast-iron gun-metal."

However important I might have considered the effects of the crushing force, and the partial or imperfect malleability of cast-iron, by which the gun may be permanently distended, a further examination of the subject has convinced me, that Mr. Barlow's theory must be in all cases modified and limited by the elongation or yielding from this malleability under the crushing pressure of the fluid; and, in many cases, as where the material is bronze or wrought-iron, the whole theory must be discarded as inapplicable. To show this, I will state the following experiment. I took a ring or hoop of wrought-iron, made up of four concentric rings, one placed over another, after one of the methods practised by me in making my wrought-iron guns in 1840–1843. These rings, when welded together, formed a hollow cylinder 1 inch long, having an internal diameter of $1\frac{1}{2}$ inches, and an external diameter of 3 inches; consequently its walls were $\frac{3}{4}$ of an inch thick. This cylinder, after being smartly hammered or sledged when cold, was subjected to distension by driving into it a conical plug or pin, by blows with a heavy sledge. By this means the inside diameter was increased to $2\frac{1}{8}$ inches. This distension, from $\frac{12}{8}$ to $\frac{17}{8}$, was far from rupturing the ring, although it produced a

great number of minute fissures upon the outside, while the inside did not show the least sign of crack or flaw.

I should remark, that this ring was made of very tough Norway iron; but, although I made several others in the same way, and of common English as well as of American iron, none of them broke under the strain before a distension of $\frac{1}{10}$ of their inside diameter; and, in all cases, the fracture commenced upon the outside and worked gradually inward to the caliber.

Another thing, worthy of all attention, was this: The end of each cylinder or ring showed, after welding, the thickness of each of the several concentric rings of which it was formed; and after the distension, the greater diminution of thickness in the inner and smaller rings was very apparent; thus showing how much greater was their distension or elongation, circumferentially, than that of the rings outside of them; and thus furnishing an experimental exemplification and corroboration, if such corroboration were required, of the fact first geometrically demonstrated by Barlow, and upon which he founded his theory explaining the weakness of cast-iron hollow cylinders when exposed to an internal pressure. Now, although the fact is to be received as he has demonstrated it, yet it becomes evident that the theory and formulas founded upon it must be limited, rigidly, to unmalleable bodies, and is in nowise applicable to cylinders of wrought metal, like the rings or hoops experimented upon by me. For, to bring a case under the conditions or facts supposed to operate in that theory, the fracture must begin upon the inside, which is supposed to be distended, like a rod strained by a suspended weight. But, in my experiments, not only was the innermost part of the cylinder subjected to the straining force of the conical pin, tending to rupture the whole thickness of the cylinder, but the inner portion of it, to a certain depth outward, was placed between two opposing forces, viz. the pressure of the conical pin in one direction, and the binding strength of the external portion of the cylinder in the other. Between these two forces it was crushed or pressed and extended laterally, and thus made thinner and longer, as a bar or sheet of metal is under a hammer, or between the rollers of a mill. Under these conditions it could not suffer fracture; for, to fracture a body its integrant atoms or molecules must be separated; but in this case they were pressed more closely together. This crushing pressure of the conical plug differed in no essential form from that produced by fired gunpowder. So the fracture commencing upon the outside of the ring is similar to that made in the bursting of bronze guns, which always commences upon the outside. The same fact was observed

by me, twenty years ago, in the trial, to extremity, of two 32-pounder wrought-iron guns. In both of these, the fracture began upon the outside and worked slowly inwards.

The preceding statement cannot fail, I think, to convince any one that Mr. Barlow's theory is wholly inapplicable to guns made of wrought-iron, or any like malleable material, and, indeed, is to be applied, in its complete and unlimited extent, only to such materials as highly hardened steel, glass, and those crystalline or wholly unyielding bodies, in which the ultimate particles or molecules are incapable of being made to change place permanently in relation to each other, but in which the limit of elasticity ends in complete separation or fracture. When applied to hollow cylinders made of substances of this latter kind, it is probably true to the letter. But what is cast-iron? And are we to be guided by Barlow's theory in computing the strength of cannon made of this material? Believing, as I do, that most kinds of cast-iron are, to some though a very limited extent, malleable, or at least that they admit of some small permanent change of form without fracture, we ought not, in my judgment, to apply Barlow's theory, without some modification to express the strength of guns made of such material, as they really possess greater strength than the formula given by that theory assigns them; though for many of the harder and completely crystalline kinds of iron we must consider it applicable, as a safe, if not an entirely accurate, guide for practical purposes.

Following this property of malleability, from the cast-iron, or body, of the gun to the wrought-iron or steel hoops with which the body is encircled and compressed, let us next see what method of constructing the hoops should be adopted to obtain the greatest strength to the gun.

It is a fact well known to all smiths, or actual workers in the metals, and to many engineers, whose knowledge is often derived principally from books, that all the metals, by being subjected for a considerable time to hammering, rolling, or wire-drawing, acquire a great increase of elasticity and hardness. Indeed, if any of these processes be carried beyond a certain extent, the metal loses its malleability and ruptures or cracks under the continued operation. The hardness and elasticity thus induced are, however, easily destroyed, and the original malleability is restored by simply subjecting the hardened metal to heat, which should be considerably below its melting point. For tin, it is said that the heat of boiling water is sufficient for the purpose. But for gold, silver, copper, and iron, the heat

of about 1000° is required to produce this *annealing* effect. For iron, it should be carried to a full red heat, whatever temperature that may be.

Now, for the purpose of ascertaining, with some degree of precision, the difference in hardness, tenacity, and elasticity between a piece of iron subjected to various degrees of heat from 400° up to that which produces a thoroughly annealed state, that is, a full red heat, and the same iron after having been subjected to some one of the hardening processes before mentioned, I have made a great many experiments upon iron wire of various sizes, and in various states as produced by previous working and heating. I will now relate the mode of making a few of these experiments and the results obtained from them, which results were in accordance with numerous others obtained by the same method of operating.

These experiments were made upon pieces of iron wire about fifteen feet long and of different sizes. The instrument for performing the experiments consisted of a long horizontal frame, to one end of which was affixed a strong steelyard, which was bent into the form of a bell-crank; and the shorter arm of which was vertical, while the longer arm, upon which hung the poise, was horizontal. One end of the wire to be experimented upon was connected with the shorter arm by being turned a few times about a ring which was connected with the arm by a free joint. The other end of the wire was fastened, by similar means, to a strong bolt fixed to the frame at a distance of fifteen feet from the steelyard. Connected by cramping it with the wire, near the end last described, was a stiff wooden rod, which lay upon the frame, and passed, by the side of the wire and parallel with it, to near that end which was connected with the steelyard. To the neighbouring end of the wooden rod was fixed a smooth tin plate, about one foot long and five inches wide. This plate lay upon the frame, immediately under the wire and nearly in contact with it; and upon the surface was registered, by a fine needle-point, the changes in the length of the wire under different tensions. To do this, a short straight-edge, or ruler, was firmly cramped to the wire, directly over the register-plate, so that, when a line was drawn upon the register-plate by the needle, which was laterally pressed against the edge of the ruler, its direction would be across the register-plate, or at a right angle with the axis of the wire. The distance from the point upon the wire where the rod connected with the register-plate was cramped to it, to the point where the straight-edge or ruler was cramped, was exactly 140 inches; and it will be seen that, by the arrangement here described, whatever yielding or springing

might take place in the frame, in the bolts, in the steelyard, or in the wire itself outside of the 140 inches comprised between the points at which the rod of the register-plate and the straight-edge were respectively cramped, could not affect the accuracy of the measure of any change in the wire between those points, and that the straight-edge could not change its place upon the register-plate in the direction of the length of the wire, unless the length of the wire itself was changed in an equal degree.

The following four experiments, made on wire of ordinary quality, from the same hank, will give sufficient warranty to the conclusions afterwards drawn from them. The results here given are, as I have before remarked, altogether in accordance with other results obtained by the same mode of operating upon other wires.

The wire used in the experiments now given was $\frac{100}{1075}$ of an inch in diameter, and, consequently, the area of its cross-section contained .006789 of a square inch. Having taken a piece of this wire about sixteen feet long, hard as it came from the draw-plate, I straightened it and fixed it upon the testing apparatus in the manner before described; and, after cramping to it the register-plate and the ruler, I commenced the operation upon it by letting the steelyard draw upon it with a weight of 10 pounds, for the purpose of taking out the sag, and to bring all the bearings into place. I then drew a line, directed by the straight-edge, as before described, upon the register-plate. This line is shown in *Figure* 1, marked *Zero A*. I then placed the poise of the steelyard so as to give to the wire a tension of 40 pounds. This weight elongated the 140 inches of wire, carrying the ruler over the register-plate the distance shown by the interval between the short line and the zero line at the left-hand end of the latter and immediately under it; and this short line (seen above the number 40) was then made by the needle-point, guided by the ruler. The poise was then removed from the steelyard, when the wire returned to its original length, as was shown by the ruler again coinciding with the zero line. The poise of the steelyard was then placed so as to strain the wire with 80 pounds, and the distance of the short line (seen above the number 80) from the zero line, shows the lengthening of the wire under this strain. On removing this weight, the wire again returned to its original length; and, on repeating the operation so as to give the wire the several strains of 120, 160, 200, and 240 pounds, the several elongations, shown in the figure by the distances of the short lines (seen over these numbers) from the zero line, were produced; but from each of these elongations the wire recovered its original

Fig. 1. WIRE, hard, as it came from the draw-plate; 140 inches long, $\frac{100}{1075}$ths of an inch diameter. Area, .006789 square inches.

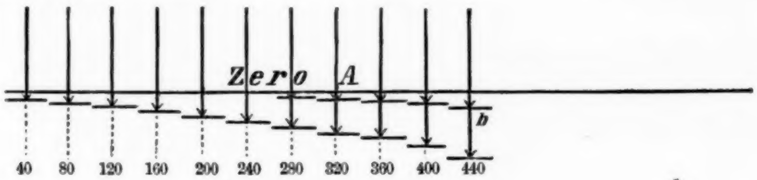


Fig. 2. SAME WIRE, after the above strain.

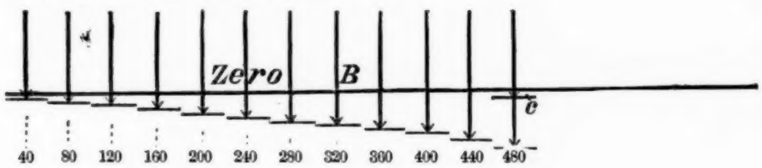
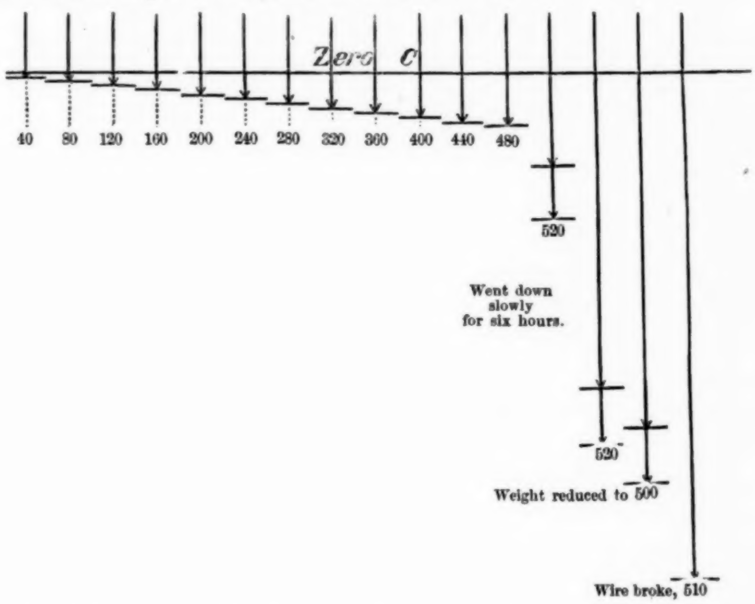


Fig. 3. SAME WIRE, after being heated, when under a strain of 40 pounds, so as to burn oil upon it, through its whole length. Temperature estimated at 850°.



The wire being 140 inches = 14,000 hundredths of an inch, we have the first permanent elongation with the load of 280 pounds = 41,241 pounds per square inch section; and this elongation is only $\frac{1}{14000}$th of the length. With the same strain the permanent elasticity is $\frac{15}{100}$ths of an inch, or $\frac{15}{14000}$ths $= \frac{1}{933}$d of the length of the rod.

original length on being released from the strain. On carrying the strain, however, to 280 pounds, the wire became permanently elongated, as was shown, on its release from the strain, by the small quantity denoted by the distance from the zero line of the short line immediately under it; while the permanent elasticity was shown by the distance of the two short lines from each other (as seen over the number 280). The operation was then continued by successively giving to the wire, and again removing from it, the loads of 320, 360, 400, and 440 pounds; and the effects are graphically shown in the figure by the distances between the zero line and the nearest short lines, which distances represent the permanent elongations, while the distances between the short lines themselves represent the permanent elasticity, after the strain of each load.

The register-plate was then moved upwards a little upon the rod, and a new zero line, B, (*Figure* 2) was marked upon it. With this I began as before, by placing upon it a strain of 40 pounds, and then, in succession, 80, 120, 160, 200, 240, 280, 320, 360, 400, and 440 pounds; removing each in like succession, to see if any permanent elongation would be produced by this repetition of the strain. As might have been expected, none whatever was produced, but the ruler returned in each case to the zero line, as shown in the figure. But, on increasing the strain to 480 pounds, the elongation shown at *c* was produced.

Knowing that the strain had reached nearly the limit of the strength of the material, I then subjected the wire to the heat of two large pieces of iron, heated to a glowing red, and passed, one above and one below, in contact with the wire. By this means the wire was heated to such a degree that oil burned freely, on being dropped upon it, through its whole length. The wire was kept connected, in its place, with the apparatus during this heating operation, and it was likewise kept straight by a tension of 40 pounds from the steelyard. The temperature to which it was raised could not have been less than 850°. After it was cold, the register-plate and the ruler, which had been removed, were readjusted upon it, and a new zero line, *C*, was marked. (See *Figure* 3.) The weights were again applied to and removed from the steelyard as before, and the effect produced by each strain was marked, as shown in the figure, above the respective numbers. No permanent elongation was produced up to the strain of 480 pounds, the same result that was before reached. The elasticity, therefore, remained unimpaired by the heating. But, this load of 480 pounds being very near the tensile strength of the wire, on increasing it to 520 the great stretch shown

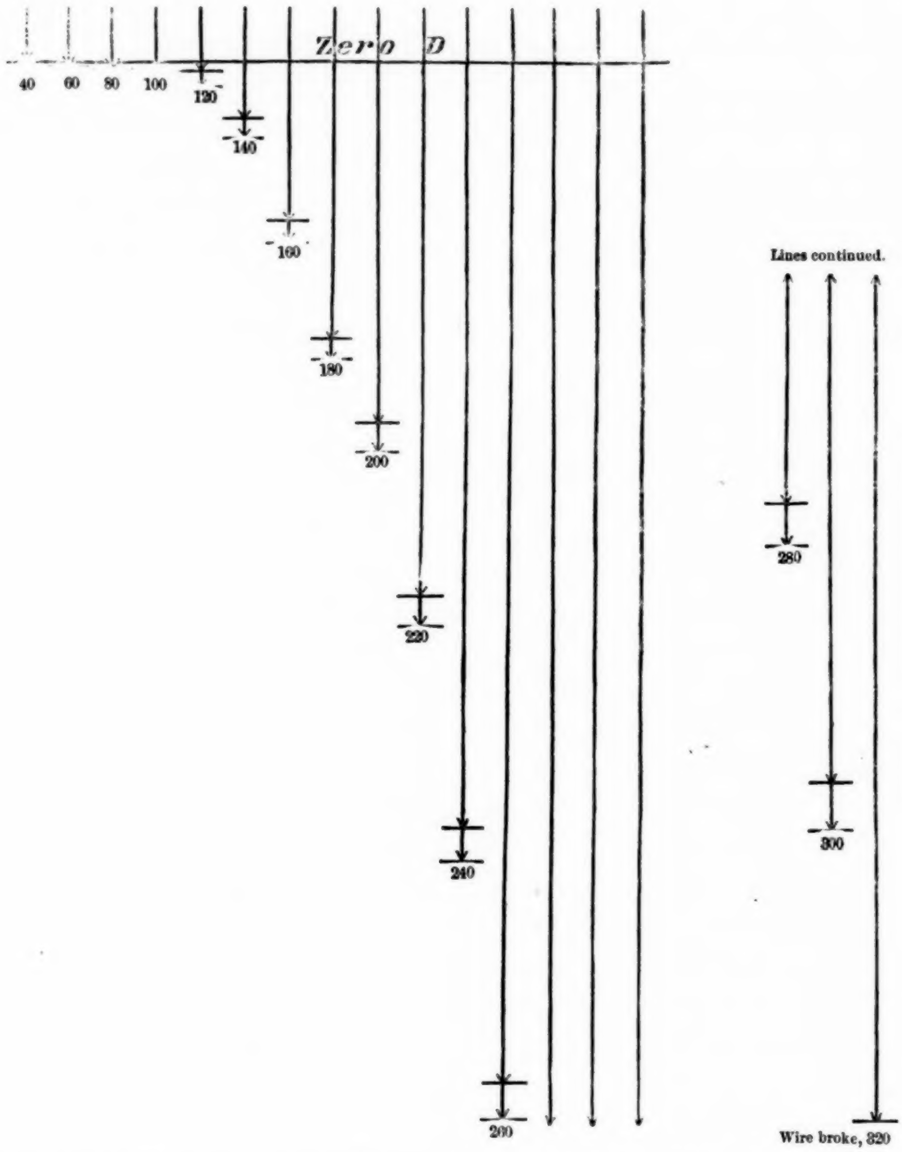
shown in the figure, above this number and immediately below the zero line, was produced. Still, on taking off the load, the wire exhibited its old elasticity, and this even a little increased. This load, replaced and continued for six hours, carried the elongation much further, as shown in the figure; and so nearly was the breaking-point reached, that the elongation continued even after the load was reduced to 500 pounds, and the wire finally broke after it had been elongated the $\frac{1}{51}$st part of its length under a strain of 510 pounds, thus showing a tensile strength of 75,120 pounds to the square-inch area.* The permanent elasticity had reached, just before the fracture, or when the load was 480 pounds (equal to 70,700 pounds per inch area), $\frac{3}{10}$ of an inch, or the $\frac{1}{466}$th (.00214) part of the length of the wire. Other experiments were made by exposing the wire, which was the subject of them, to the heat of melted lead. Here, in two cases, the wire was passed slowly under the surface of lead which was kept very much above its melting temperature. These wires, when tried afterwards in the testing machine, showed their previous elasticity unimpaired.

The fourth experiment that I shall relate, was made upon a piece of wire from the same hank with that before used, after it had been annealed to a full red heat. This, after having been straightened, was placed upon the testing machine, and subjected to a succession of strains, commencing at 40 pounds, and increasing by steps of 20 pounds each (instead of the 40 pounds before used) up to 320 pounds, under which strain the wire broke; thus showing an ultimate tenacity of 47,128 pounds per square inch, or, allowing for the diminution of the area of the wire by the elongation previous to breaking, about 51,000 pounds, instead of 75,120 pounds, as given in the former case. The permanent elasticity had reached, just before the fracture, or when the load was 300 pounds upon the wire (or 44,186 pounds per square inch), $\frac{26}{100}$ of an inch, or the $\frac{1}{538}$th part of the original length of the wire. The effects of the various increasing strains are fully exhibited in *Figure* 4 (next page), which (as is also true of the preceding figures)

* The following numbers represent the weight upon the wire when reduced, or computed, for a square bar of one-inch section:—

40 = 5,891	200 = 29,458	360 = 53,025	520 = 76,592
80 = 11,783	240 = 35,350	400 = 58,917	560 = 82,433
120 = 17,675	280 = 41,241	440 = 64,808	
160 = 23,596	320 = 47,128	480 = 70,700	

Fig. 4. WIRE from same hank after it had been fully annealed.



With this soft wire, the permanent stretch commences at 100 pounds = **14,648 pounds strain per square inch section**; and the permanent elasticity left after this strain is only $\frac{5}{100}$ths **of an inch, or** $\frac{1}{2800}$**th part of the length of the wire.**

figures) shows a true copy of the lines made upon the register-plate. By this it will be observed that the permanent elongation commenced at a strain of 100 pounds, and increased rapidly, at each successive increase of weight, until it reached a length of $10\frac{1}{4}$ inches, or the $\frac{10}{136}$th part of the length of the wire.

Now, on comparing the results shown in all these figures, we see that these experiments demonstrate with some degree of precision several physical facts, all of which are of high importance in the construction of cannon upon the principle pointed out in the Memoir to which this is a sequel. These facts are:—

First, That with a piece of iron hardened by compression and tension, in the condition of hard wire, the amount of permanent elongation is far smaller than the permanent elasticity up to near the breaking-point, and also that the permanent elongation does not begin until about one-half of the breaking strain is applied.

Second, That the part of the elongation, or stretch, which is within the elastic power of the wire, increases very regularly under equal increments of strain; thus exhibiting the truth of the maxim, *Ut tensio, sic vis*—As the stretch, so the strain. But the permanent elongations made by the same increments of strain, especially when near the breaking of the wire, are entirely at variance with this maxim. This will be seen in *Figure* 4, where an increment of 20 pounds to an existing strain of 120 pounds, produces a permanent stretch of $\frac{5}{16}$ths of an inch, while the same increment of 20 pounds, when the wire was under a strain of 280 pounds, increased the length, permanently, full $1\frac{1}{2}$ inches.

Third, That, when the material has been subjected to a strain of a given amount (say 440 pounds, as in *Figure* 1), the repeated application of a strain within that amount produces no further permanent elongation.

Fourth, That the subjecting of the same material to a heat sufficient to burn oil in contact with it (supposed in this case to be 800° Far., at least), will not impair its elasticity.

Fifth, That, when the iron is annealed, the permanent elongation commences at a comparatively low strain, and that its extent is very large in proportion to the elasticity of the iron, which shows how inappropriate is the use, upon a cast-iron body, of a hoop that has been heated to an annealing temperature; as it must be loosened, or suffer the cast-iron to break within its grasp, before a strain upon it up to half its tensile strength shall be reached.

Guided by the conclusions derived from the preceding experiments, I will now proceed to compare, with such precision as the knowledge thus opened to me

permits, two guns constructed upon the principles and after the proportions given in the Memoir of 1855, both of these guns being supposed to be of the same size, and made of the same quality of iron throughout, but differing only in this, viz., that, in the one, the hoops are put upon the body in an annealed state; and, in the other, in such a state of hardness, produced by cold hammering and stretching (as hereafter described), as shall bring the iron, as near as may be, to the state of the wire represented in *Figure* 3.

These guns may be shortly described as having a caliber of 14 inches diameter; bodies of cast-iron, 7 inches thick in the reinforce, so as to make the external diameter of the body there 28 inches; and a covering of wrought-iron hoops in two layers, having together an equal thickness of 7 inches. The strength of these cast-iron bodies, as shown in detail in the former Memoir, if made of cast-iron of 30,000 pounds tensile strength, when reduced acccording to Mr. Barlow's formula (which is recognized as sufficiently perfect for a practical guide), will be 210,000 pounds for each inch in length.

Now, let us suppose one of these bodies to be hooped with two layers of hoops, $3\frac{1}{2}$ inches each in thickness, these hoops being made of wide bars of wrought-iron, coiled like a ribbon wound upon a block, and in this state the coils being welded so as to form one ring, or hollow cylinder, or hoop.* The hoops, being thus formed and properly forged to shape and size, are supposed to be left in an *annealed* state; and, after being bored and finished to .001, .002, or even .003 of their internal diameter less than the part of the body that they are to enclose, we will suppose them to be heated and put in their place, where they cool and compress the cast-iron, being themselves at the same time strained and stretched by the resistance of the enclosed body. Now, whatever proportions this compression and this stretch may bear to each other, it must be evident, from an inspection of *Figure* 4, that, when the strain upon the hoops, from the shrinking, reaches 17,675 pounds per square inch (equal to 120 pounds upon the wire), they will receive a decided permanent elongation. Let us suppose, then, that the hoops are grasping the body with this force of 17,675 pounds per square inch, at the instant when the fired gunpowder has distended the cast-iron to its normal diameter. We see by the

* A more full account of this method of forming rings may be found in the pamphlet published by me in 1845, or in "English Printed Specifications," No. 10,013; enrolled in July, 1844; printed in 1854.

Figure 4, that by a further distension of the body the resistance of the hoops will be very slowly increased. What will this resistance amount to, when the cast-iron is distended to its breaking point? Although we cannot determine with any great accuracy how far the cast-iron can be distended before fracture, we may, I think, be very certain that a fracture would be produced by repeated firing under an enlargement of $\frac{1}{1000}$ part of its external diameter. But it will be seen by the figure, that a strain upon the wire of 160 pounds, or 23,596 pounds per inch section, produces an elongation of the wire of .9 of an inch, or $\frac{1}{155}$ part of its length; and it must be evident, that, long before this elongation and distension in the hoops are reached, the cast-iron must give way and the gun be destroyed. But, even allowing the gun to hold together up to the strain of 23,596 pounds per square inch upon the cross section of the hoop, we have the following computation of the strength of the gun, for each inch in the length of the reinforce: Cast-iron body, 210,000 pounds per square inch; wrought-iron hoops, 23,596 pounds per square inch, and, as both sides give 14 inches thickness, $14 \times 23{,}596 = 330{,}344$ pounds for each inch in length, and $210{,}000 + 330{,}344 = 540{,}344$ pounds for the strength through each inch in the length of the reinforce of the gun of these dimensions and proportions.

Let us next suppose a gun to be constructed, in size and material, like that just given, but having this single difference in the method of preparing the wrought-iron hoops: that instead of placing them upon the gun in an annealed state, such as is represented by the wire from which *Figure* 4 was formed, they shall be subjected to a process of cold hammering and stretching, so as to bring them into the same condition, as near as attainable, with that of the wire used in making *Figures* 2 and 3.

Computing the strength of a gun covered with hoops brought into this state of hardness and elasticity, we have the diameter of the body, as before, 28 inches. Let the hoops be .001 part of their diameter less than the body, or 27.972 inches.

The hoops thus made, and expanded by heat, and placed upon the body, will, when cold, compress the body to a diameter somewhere between 28 and 27.972 inches, — the exact degree of compression depending upon the power of the body to resist compression, and that of the hoops to resist distension; but, when the force of the fired gunpowder is exerted upon the caliber, and the external diameter of the body is distended to its normal dimension of 28 inches, the power of the hoops to resist further distension will become 35,350

pounds for every inch area of their cross-section (this being equal to 240 pounds' strain upon the wire).

From this point the distending force of the gunpowder will be resisted, both by the hoops and by the body; and, if we suppose the cast-iron body, in this as in the last case, to be fractured only after a distension of $\frac{1}{1000}$th part of its normal external diameter, we shall find that that point will not be reached until a strain of more than 70,700 pounds per square inch (shown by 480 pounds upon the wire) has been exerted upon the hoops. Then, taking the body, as before, to resist with a force of 210,000 pounds per inch in length, we have, as the whole strength of the gun: Body 210,000, and hoops $\cdot 14 \times 70{,}700 = 989{,}800$, making together 1,199,800 pounds for each inch of its length,—decidedly more than twice the strength shown in the former case, where the hoops were annealed before being put in place.

In this statement, I have taken the comparative diameters of the body and the hoops at 28 and 27.972 inches. Now this difference is so small, that it cannot be produced in practice with geometrical precision; nor is this necessary; all that is required being, that the difference shall not be *less* than that here given, though a deviation by which the difference of diameters shall be twice, or even thrice, as great as this, will not affect injuriously the construction.

Thus, suppose the hoops, instead of being 27.972 inches, be made 27.916 inches, in diameter. A heat of 800°, to which the hoops may be heated without affecting their elasticity, will expand them to 28.064 inches, thus giving a margin of .064 or about $\frac{1}{16}$th of an inch, for play, and imperfect workmanship, when the hoops are run on to their places. In this case, although the first compression of the body would be greater than if the hoops were made of the exact size assigned, yet the first discharge of the powder would, by a little permanent elongation of the hoops, bring them to the true diameter, without enlarging them beyond their elastic limits.

In the preceding computations of strength, I have confined myself to that manifestation of it which preserves the gun from longitudinal fracture. But a gun may be fractured transversely, or diagonally, as well as longitudinally, although I have heretofore fully proved, that, if made of a material which has an equal strength in each direction, the gun, or any hollow cylinder, has a vastly greater power of resisting cross, than longitudinal, fracture. This is likewise applicable to any diagonal fracture. But as these guns owe their superiority in a great degree to their being formed, in part at least, of fibrous wrought-

iron, the direction of the fibres being in the circumference of the gun, and moreover not perfectly integrated with the cast-iron as making one piece with it, it becomes necessary to consider the resistance to cross-fracture.

By a recurrence to the former Memoir, it will be seen that the cast-iron body alone, if possessed of a tensile strength of 30,000 pounds per inch, may be relied upon for preserving the gun from cross-fracture. But I cannot say that I have always been without some shade of doubt, whether the cast-iron, when exposed to the crushing force between the fired powder and the hoops, would exhibit the same resistance to cross-fracture that it would when free from this condition. It was in some degree to guard against any defect that might possibly arise from this source, that I proposed that the hoops be made in two layers, and be fitted to the body and to each other, by a screw-thread. In this case the screw was not to exercise its usual function of a mechanical power; but to serve, by the interlocking of the threads upon the body with those of the hoops, to so cramp the two together, that the body could not be fractured crosswise without either stripping the thread through a space equal to at least half the length of the hoop, or fracturing the hoop crosswise before or at the instant when the body gave way. Now, to strip the thread of a screw through half the length of the hoop would require a force sufficient to make a shear cut, through a section of metal equal to at least one-third the internal surface of the hoop. The inner surface of the inner hoop, being $28 \times 3.14 = 88$ inches in circumfrence and 15 inches long, gives a surface of 1,320 inches, one-third of which is 440 inches. To strip or cut through a screw-thread forming a section of this magnitude, taking each inch to require but even 30,000 pounds, demands a force of 13,200,000 pounds, while, as is shown in the former Memoir, the whole force of the charge tending to produce cross-fracture is but 4,896,000 pounds, being the pressure of 32,000 pounds per square inch upon 153 inches, — the area of the caliber. The other alternative, that of fracturing one of the hoops at its weakest point, that is, where it breaks joint with two of the hoops of the other layer, and where, of course, one thickness alone gives its support against cross-fracture, furnishes the following computation: The area of the cross-section of the inner and smaller hoop, contains 346 square inches, which, giving the iron, in this its weakest direction, a tensile strength of 40,000 pounds per inch, shows that a force of 13,840,000 pounds will be required to tear the hoop asunder.

The preceding computations, therefore, place it beyond doubt, that, even allowing that the lateral pressure of the fired powder upon the cast-iron body of the gun may impair the tensile strength of the body in resisting cross-fracture, yet, under this condition, and thus admitting as a truth that of which we have no evidence, we see that we may rely with perfect confidence upon the strength of the hoops alone, when secured to the gun by the screw-thread, as described in the former Memoir, to preserve the gun from cross-fracture.

In the Specification and Memoir before mentioned, I propose to form the screw "of about eight threads, each thread taking about one-eighth of an inch space, so that one turn advances each thread one inch," and "to make the threads of the female screws sensibly finer than those of the male, to draw by the shrink, the inner rings together endwise." The advantage of this form of construction will appear in this: that by the rapid advance of the hoop to its place the shrinkage from cooling during its passage over the body will be avoided; while the dividing of the inch space of the spiral into several parts, enables us to give a great bearing surface to very shallow threads.

I give, in *Figure* 5, a drawing of the threads as I would form them for

Fig. 5.

a six-threaded screw. They have an .18 inch pitch, and a depth of .04 in., being .11 in. thick at the root or bottom, and .07 in. breadth upon the face. Threads of this shape may be more easily and exactly made than any other, as a large part of the surfaces left by the boring and turning tools requires no change from the screw-tool, but remains and forms the flat faces of both the male and female screws. By this means the gauged sizes and requisite diameters of both the body and the hoops are more easily ascertained and preserved, when the screw-threads are formed.

The depth of the threads given in this figure must be ample; for, as the threads, when once interlocked and in place, are kept in contact by the shrinkage of the hoops and the distension of the gunpowder, the idea of the outer threads slipping and riding over the inner ones, like a loose nut upon a screw bolt, is simply preposterous.

A mechanical equivalent for the screw-threads may be found in small circular prominences formed upon one surface, to fit into corresponding grooves

upon the opposite surface. The principal objection to this mode of cramping or interlocking the surfaces will be found in the necessity of heating the hoop to a much higher temperature than is required with the screw-thread, in order to expand it so that it may pass to its place upon the gun. These prominences and grooves, doubtless, might serve a better purpose than the mere roughness left by the turning tools, as now often used. I am confident, however, that no device can be made, superior to the screw-threads, and nothing better than this is needed.

The benefit, moreover, to be derived from making "the threads of the female screws sensibly finer than those of the male," must, I think, be evident after a little examination. By making this difference the $\frac{1}{1000}$th part, or perhaps a little more (that is, by making 1000 turns of the spiral of the female to occupy a shorter space upon the hoop by a $\frac{1}{1000}$th part than the same number of turns do upon the body), they will be more nearly equal when the hoop is expanded by heat, than if they had been formed of equal fineness or pitch. The hoop will, therefore, go more readily to its place when expanded, from bearing this finer thread. When the first layer is shrunk in its place, each hoop will be under a lengthwise strain; and, again, when the second layer is shrunk upon the first, the first layer, and, under it, the body of the gun, will be drawn together lengthwise, and thus the body will be guarded from cross-fracture, as it is guarded from longitudinal fracture by the circumferential strain of the same hoops.

Having thus exhibited the principles which should direct us in the construction of hooped cannon, and the experiments by which these principles are come at, I now proceed to describe the method of forging the hoops, and of giving to them that combination of hardness, elasticity, and tenacity, which has been shown to be so important to the strength of the cannon.

To construct one of the hoops for a cannon of the size before mentioned, that is, of 14-inch caliber, the hoop having, when finished, 27.972 inches' internal diameter, and being $3\frac{1}{2}$ inches thick, and 15 inches long (or broad), I take a flat bar, say 14 inches wide, from half an inch to an inch thick, and of such length that, when wound into a coil, it shall form the thickness required for the hoop, after allowing for the waste in welding, forging, and finishing. After its ends have been scarfed to a long wedge form, it is to be heated to a low red heat, and then wound upon a cylinder of say 25 or 26 inches diameter, as a ribbon is wound upon a block. Next, it is to be heated in a proper furnace to a good welding heat, and then, being placed upon an arbor, or mandrel, of

about 25 or 26 inches' diameter, and between proper dies, setts, or swages, it is to be completely welded, or the several layers or coils are to be made to form one piece. This may be done by compressing it with the swages, by a hydrostatic press, or by a steam hammer. After it is properly welded and condensed in this way, and has cooled as low as 600°, it is to be placed upon a cold arbor, or mandrel (shown, in section, at *A*, *A*, *Figures* 6 and 7), which is supported at both its ends by the upright studs of the heavy iron frame *B*, *B*. It is then to be hammered by the steam hammer *C*, until its internal diameter is enlarged to about 27 inches. The last part of the hammering is to be performed after the hoop has become cold. Instead of operating in this way with the steam hammer, we may produce the same effect upon the hoop by a rolling-mill, in which the operating part of the rollers is made to project beyond the housings, or frame.

After the hoop has been condensed and stretched in this way, it is next to be placed upon an annular anvil, *D*, *D* (*Figures* 8 and 9), and the segmental swages or blocks, *E*, *E*, are to be adjusted within it. These segments form a cylinder upon their outer surface, but inside they form a hollow cone. A solid conical plug, *F*, is fitted to be driven into this hollow cone within the swages. With this arrangement, the whole being under the drop or steam hammer *C*, the plug is driven by repeated blows into the hollow cone, by which operation the hoop is stretched sufficiently to destroy all conflicting strains or tensions that might have been produced in it by the hammering. The strain is thus reduced to a circumferential direction, and the hoop put as near as possible into the condition of the hard wire (as shown in *Figure* 2), after it had been subjected to the first series of strains (as shown in *Figure* 1).

The hoop may be stretched by this last operation the $\frac{1}{100}$th part of its diameter, and, if it is made of very soft and tough iron and has not been hammered very hard, much more than this quantity. The extent, however, to which this hammering and cold stretching may be carried, must depend upon the quality of the iron and the heating and working to which it has been previously subjected. It will be well, when the stretching is commenced, to have the hoop warmed up to 200° or 300°.

After the hoop has been prepared in this way by cold hammering and stretching, it is to be bored and turned; and, whether it is to be fixed to the gun by a screw-thread, or by any equivalent, it is to be carefully and equably heated to such a temperature (but never up to an annealing heat), as shall expand it sufficiently, and, in this state, is to be placed upon the gun.

In all

Fig. 6.

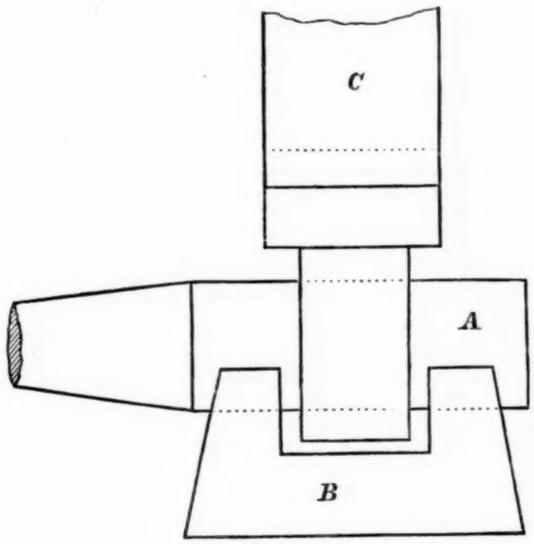


Fig. 7.

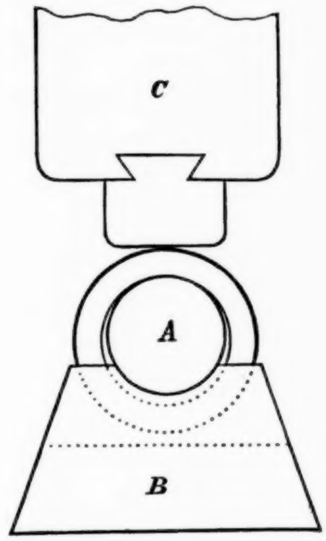


Fig. 8.

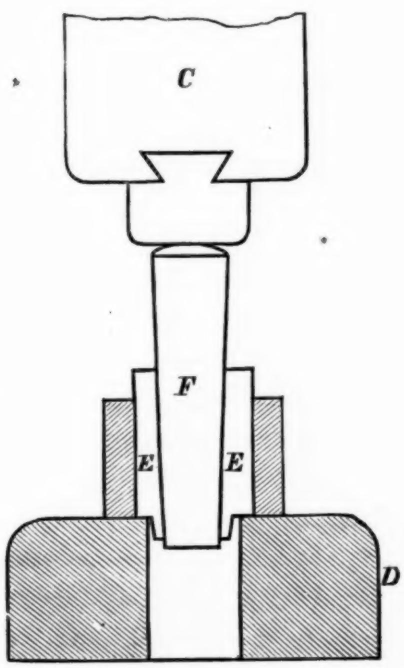
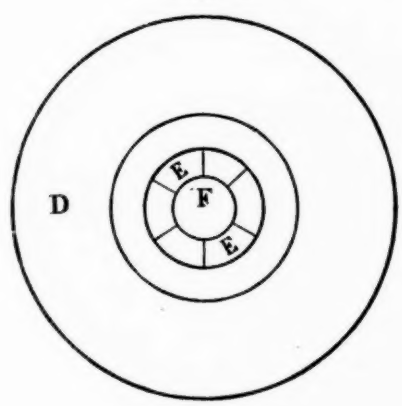


Fig. 9.



In all the preceding computations of the force which the cannon is required to resist (both in this paper and in that to which this is intended as a sequel), I have considered the powder, when fired, as acting by a pressure generated with inconceivable rapidity, and with an intensity sufficient to produce the required velocity in the missile; — this velocity being produced by the pressure alone. I am fully aware, however, that the force thus produced, almost instantaneously, from a single point within the gun, does, and must, throw a shock upon, and a vibration through, the whole mass, the destructive effect of which must be provided against in addition to that of the mere pressure of the fired powder, if that pressure be supposed to act as the pressure acts in the hydrostatic press, for example, where it is raised and communicated slowly and gently to its object, thus producing its motion without violence or shock. Although we are without the knowledge requisite to subject to a rigorous computation the destructive effect of this shock and vibration from the discharge, yet it is necessary that a sufficient strength should be provided in the gun to resist it. Nor are we without the light of experience to direct us to this end; for, although it has not yet been determined, by direct experiment, what strength is required in a gun of say 14-inch caliber, in addition to that thrown upon it by the pressure of the charge, in order to withstand the sudden shock and vibration before mentioned, yet we have direct experiments which have determined this element in guns of smaller caliber. Thus the cast-iron 32-pounder, $6\frac{1}{4}$ inch caliber, if made of good iron, and in the usual proportions, that is, with walls of one caliber in thickness, has been proved, by the experience of ages, to be quite reliable for long continued use with service charges.

Now, it was shown, in my former Memoir, that a maximum pressure, from the fired powder, of 920 atmospheres, will give a velocity of 1,600 feet a second to a 32-pound shot; and, further, that, computing the strength of the gun from the tenacity of the iron, taken at 30,000 pounds per inch, it is capable of resisting a force of 1,333 atmospheres; or, the strength of the gun is to the maximum pressure of the powder as 144 : 100. Hence, we have an excess of 44 per cent., which has proved sufficient to sustain all the extra violence from the shock and vibration occasioned by the suddenness of the discharge, from the heat, and from all other adventitious causes. It is furthermore shown, in the same Memoir, that a spherical shot of 14 inches' diameter will receive a velocity of 1,600 feet a second, if fired from a cannon, the bore of which is 112 inches long from the seat of the ball to the muzzle, under a maximum pressure of 2,133 atmospheres. But we have seen, that a 14-inch gun,

constructed with hoops as herein described, will sustain a pressure of more than double that required to throw the ball with 1,600 feet initial velocity. There can be no doubt, then, that, as 44 per cent. above the necessary powder pressure has, through long experience, proved sufficient, in the 32-pounder, to provide for the contingent strains from shock and vibration, 100 per cent. must be more than sufficient to provide against the same contingencies in the 14-inch gun; and, indeed, that 14 inches does not approach the size to which guns may be safely trusted, if constructed upon the principles, and in the manner, herein laid down.

Although it is hardly to be expected that the preceding method of cold working will impart to the hoops, if made of common iron, the elasticity and tenacity possessed by the wire used in the experiments herein related, yet, by the use of iron of superior quality, I think that that standard may be reached. But, should it be found, in the end, that 10 per cent. must be deducted from the tenacity of the wire, in computing that of the hoops, we shall still find the gun constructed in this way, for all that I can see to the contrary, *more than twice as strong as any hooped gun ever yet constructed, of the same materials, weight, and dimensions*; and, by the use of iron of a somewhat steely character, or of some of the *low* steels, the standard of the strength of the wire may be much surpassed.

I cannot conclude this paper without observing, that, although in the Memoir formerly published no particular method of hardening the hoops was pointed out, and thus the process of cold hammering and stretching was omitted, still it was always my intention, whenever I should undertake the manufacture of hooped cannon, to prepare the hoops by some process of condensing and hammer hardening. So fully was I impressed, from my experience in the working of iron, with the importance of thus preparing the hoops, that, in 1862, when I had made an arrangement, with the Massachusetts Committee on the Defence of the Ports nad Harbors of the State, for the manufacture of two hundred large cannon (an arrangement which was entirely approved by the Executive government of the State, and which failed to be consummated only by the rejection of the appropriation bill, in the Senate, by a majority of one), I visited several of the large machine-shops in the vicinity to find where I could best procure the construction of the steam hammer and tools for performing the operation herein described. My ideas (which were not then very definite) of the importance of subjecting the hoops, to be used upon cannon, to this condensing and hardening process, have been fully confirmed and defined by the experiments herein detailed; and the conclusions that I have drawn from these experiments will, I think, be

assented to by any practical engineer who may take the pains to examine them. Indeed, it seems to me remarkable, that, with all the attention that has been given to the subject of hooped cannon in Europe, as well as in this country, for several years past, the cause of the great defect, which it has been one object of this paper to point out and remedy, does not seem to have been discussed nor seen, although the defect itself has been made known by the bursting of such guns in so many instances as to have shaken, if it has not destroyed, the confidence of artillerists in them, when used with heavy charges. To avoid this defect, resort has been had to the use of *low* cast-steel, under the name of homogeneous iron; or to an adoption of the manufacture with wrought-iron, after the method invented and practised by me more than twenty years ago, and which I afterwards improved into the simpler and cheaper form so fully described in these Memoirs.

I may also observe, with regard to the theory of the strength of hollow cylinders, when exposed to a bursting force, that many changes to the original formulas of Mr. Barlow have been proposed as expressing more exactly the physical conditions of these cylinders. These changes, however ingenious or learned, are of very trifling practical importance in the manufacture of cannon. The omission of Mr. Barlow to consider the pressure, as acting as a crushing force upon the internal portion of the cylinder, and thus as aiding, to some unascertained extent, its action as a distending force, in rupturing the walls of the cylinder, was, I believe, first made known by me in my former Memoir. So, also, no writer or engineer has yet, so far as I know, perceived or shown, that the theory of the strength of hollow cylinders, as now generally adopted, is wholly inapplicable to cylinders or hoops made of malleable materials, such as wrought-iron or bronze; for the reason, that the inner portion of such a cylinder will, as shown by my experiments herein detailed, be permanently elongated or stretched circumferentially, without being ruptured or weakened until after the outside has given way; a fact entirely at variance with the foundation of the theory. For, the assumption, on which the whole theory rests, requires that the fracture shall first take place upon the inside of the cylinder; an assumption that can only be true in fact, when the cylinder is formed of a material which is unmalleable, that is, incapable of being elongated or stretched to any considerable extent beyond the limits of its elasticity.

V.

Discussion of the Observations of the Great Comet of 1858, *with the Object of determining the most probable Orbit.*

By G. W. HILL.

Communicated by T. H. Safford, April 12, 1864.

The interesting physical aspect of this comet attracted the attention of astronomers to it in an unusual degree, a large part of which was expended in obtaining observations for position. Consequently, we have a large mass of material for determining its orbit, not a little of which is of very good quality. Added to this, the quite long period of the apparition of the comet (nine months), would enable us to obtain the elements with considerable precision. Moreover, hints were thrown out that some other force besides gravity might effect its motion. Although these seem to have had no foundation other than the fact that the orbits derived from three normals did not well represent the intermediate observations, yet it is a matter of some interest to clear up the suspicion.

As the first step in the work, I determined to reduce the observations to uniformity, in respect to the places adopted for the comparison stars; which last I proposed to derive from all the material accessible to me. The desirableness of this course is evident when we consider that the observers at Bonn, Kremsmünster, Ann Arbor, and the two observatories in the southern hemisphere reobserved their comparison stars, in consequence of which their observations agree much better among themselves; while the rest contented themselves with places from Lalande, Bessel's Zones, or the British Association Catalogue, and their results exhibit larger probable errors. And as the comet was observed nearly simultaneously in Europe, the same comparison star was frequently used by a dozen observatories for the same night's work; and thus the stars of the latter class of observatories mentioned above are often found among those reobserved by the former. The result of this labor has convinced

me that it has not been wasted; the good effect is apparent, particularly in the Liverpool and Gottingen observations.

A catalogue of all the stars used for comparison having been formed, the following authorities were consulted for material:—

Baily's Lalande, Piazzi, Bessel's Zones (Weisse's Reduction), Struve Catalogus Generalis, Taylor, Rümker, Argelander's Southern Zones (Oeltzen), Robinson's Armagh Catalogue, Johnson's Radcliffe Catalogue, Greenwich Twelve Year and Six Year Catalogues, Mädler, Greenwich Observations, 1854–60, Henderson Edinburg Observations, Challis Cambridge Observations, Leverrier Paris Observations, 1856–59.

Leverrier commenced, in 1856, to reobserve the stars of Lalande; hence quite a number of the stars the observers had taken from this source, were found in the Paris Observations. The searching them out and reducing them entailed considerable labor. In addition to the material before mentioned, that furnished by the observatories at which the comparison stars were reobserved, was, of course, not omitted.

All this material was reduced to 1858.0, and to the standard of Wolfer's Tabulæ Reductionum, by applying the systematic corrections given by Auwers, in Astr. Nachr., No. 1300, with the modifications suggested by Mr. Safford, in No. 1368. The systematic corrections for Robinson are found in Astr. Nachr. No. 1408. Also, the following, kindly furnished by Mr. Safford, were employed:

	R. A.	DEC.
Greenwich Six Year Catalogue, ...	$+0^s.017$	
Greenwich Observations, 1854–60, ...	$+0\ .027$	$+0''70$
Paris Observations, 1856–59, ...	$+0\ .056$	$+0\ .19$

In a few cases, mostly Piazzi stars, where the observations indicated proper motion, it was taken into account. With regard to the stars used in the southern observations, those common to the northern being excepted, they were retained without change, or when the same star had been used at both observatories, the observations were combined, allowing a weight of 3 to the Cape and of 2 to the Santiago observation. However, the place of the Santiago star No. 57, equivalent to Cape No. 95, is wrong, seemingly an error of reduction; hence the Cape place has been adopted. And Santiago, No. 49, differing 7″.5, in declination, from its equivalent, Cape No. 87, the Cape declination appearing the better, has been retained.

Number.	α 1858.0.	δ 1858.0.	Number.	α 1858.0.	δ 1858.0.
	h m s	° ′ ″		h m s	° ′ ″
1	9 11 35.277	+25 0 59.98	57	10 47 51.964	+34 15 49.07
2	9 23 19.992	25 2 12.96	58	10 52 35.910	35 13 36.25
3	9 25 52.434	24 5 6.93	59	10 56 35.054	35 7 11.70
4	9 29 26.949	25 1 49.81	60	10 59 36.820	35 36 32.64
5	9 29 41.987	25 18 21.93	61	11 0 45.884	35 29 1.63
6	9 30 47.635	26 34 35.94	62	11 1 29.939	37 4 43.39
7	9 32 23.230	26 38 48.99	63	11 1 58.610	35 40 37.52
8	9 33 27.857	26 33 26.79	64	11 2 24.790	36 6 12.73
9	9 37 42.094	27 41 55.28	65	11 4 16.855	35 46 40.87
10	9 37 47.087	24 25 33.49	66	11 4 37.860	35 33 27.80
11	9 38 33.273	27 34 38.43	67	11 10 16.610	36 13 5.54
12	9 38 42.482	27 48 43.19	68	11 10 48.100	33 52 6.00
13	9 44 17.169	28 26 25.61	69	11 11 4.961	36 15 52.34
14	9 45 49.118	28 21 41.66	70	11 13 48.264	36 25 24.60
15	9 45 51.470	27 57 29.24	71	11 14 24.766	36 6 48.63
16	9 46 34.633	28 1 10.89	72	11 17 49.011	35 56 46.68
17	9 48 45.001	28 46 15.38	73	11 19 30.334	36 32 58.19
18	9 49 3.501	29 14 1.93	74	11 20 16.048	36 9 7.63
19	9 50 10.023	29 15 28.20	75	11 22 8.300	36 25 12.10
20	9 51 24.453	30 19 26.44	76	11 27 39.248	36 11 24.50
21	9 53 8.109	29 27 50.85	77	11 28 14.746	36 42 40.78
22	9 56 54.727	30 26 9.00	78	11 29 52.400	36 23 30.10
23	9 58 59.212	30 12 16.94	79	11 30 28.154	36 23 31.78
24	10 3 36.641	30 50 50.42	80	11 31 6.811	36 23 1.60
25	10 6 0.703	32 7 41.13	81	11 33 33.925	35 0 12.15
26	10 6 56.647	32 10 17.05	82	11 38 8.137	36 40 53.08
27	10 8 9.965	30 0 58.15	83	11 41 22.161	35 37 17.78
28	10 9 27.359	31 35 38.88	84	11 42 18.698	35 43 13.14
29	10 9 50	31 8 36	85	11 48 39.684	36 7 52.28
30	10 10 27.200	31 19 36.16	86	11 48 57.507	36 14 16.51
31	10 12 33.258	32 8 25.35	87	11 54 23.109	36 50 12.34
32	10 12 45.450	31 22 26.94	88	11 55 23.490	36 31 5.11
33	10 14 12.375	32 15 26.28	89	11 57 25.064	36 21 29.55
34	10 14 47.246	31 2 47.57	90	11 59 22.626	36 7 52.04
35	10 14 56.648	31 33 9.66	91	12 8 41	36 2
36	10 16 57.222	31 5 41.78	92	12 9 21.473	33 51 20.73
37	10 23 37.058	31 46 9.62	93	12 14 5.054	35 28 35.44
38	10 23 47.154	33 6 25.56	94	12 18 0.818	35 33 5.54
39	10 25 56.094	33 14 35.93	95	12 23 36.015	34 32 7.24
40	10 26 29.210	32 24 43.22	96	12 24 3.679	34 40 32.25
41	10 27 27.290	32 30 36.72	97	12 24 38.593	34 42 4.90
42	10 29 41.545	33 28 13.95	98	12 26 38.907	34 1 58.92
43	10 29 46.127	33 25 30.26	99	12 30 5.468	33 48 31.59
44	10 30 43.132	32 42 45.11	100	12 40 14.318	33 20 42.67
45	10 34 4.401	33 53 25.44	101	12 44 8.808	32 15 8.42
46	10 34 13.347	32 26 21.00	102	12 48 56.000	32 46 19.64
47	10 35	34 10	103	12 49 22.827	39 5 10.26
48	10 35 11.612	34 6 20.68	104	12 53 28.505	31 33 8.05
49	10 36 27.312	33 21 49.84	105	12 53 38.459	32 32 45.83
50	10 37 50.279	33 20 33.31	106	12 55 34.619	31 7 17.08
51	10 38 50.569	34 18 20.60	107	12 57 5.635	31 31 16.16
52	10 39 45.817	34 20 17.23	108	12 57 16	31 14
53	10 44 6.512	32 7 11.86	109	12 57 26.035	30 58 58.19
54	10 44 8.277	33 47 57.50	110	12 59 23.612	29 47 28.59
55	10 45 21.594	34 58 45.71	111	13 0 21.817	28 23 16.41
56	10 47 3.944	+34 47 31.18	112	13 2 21.623	+31 0 9.03

Number.	α 1858.0.	δ 1858.0.	Number.	α 1858.0.	δ 1858.0.
	h m s	° ′ ″		h m s	° ′ ″
113	13 2 45.467	+31 11 36.59	169	15 20 44.108	+ 0 23 21.61
114	13 7 53.393	30 9 19.48	170	15 23 56.400	− 0 14 16.79
115	13 9 5.077	30 5 55.53	171	15 30 20.818	3 7 57.60
116	13 10 14.299	29 47 44.00	172	15 33 46.943	3 31 59.42
117	13 12 20.788	29 18 25.90	173	15 37 0.458	− 3 23 9.11
118	13 18 20.109	24 35 44.78	174	15 37 16.575	+ 6 52 30.92
119	13 20 10.842	26 59 50.88	175	15 41 30.788	− 3 22 46.67
120	13 21 46.769	28 5 9.80	176	15 43 44.425	+ 4 54 29.08
121	13 22 2.800	29 11 20.02	177	15 44 11.680	− 7 36 47.93
122	13 23 8.620	28 24 36.87	178	15 44 33	6 53
123	13 23 45.303	28 23 16.90	179	15 46 54.770	7 40 54.50
124	13 25	28 20	180	15 52 4.738	6 53 37.22
125	13 30 3.869	26 36 19.12	181	15 52 26.783	6 42 53.62
126	13 33 22.650	26 38 49.62	182	15 53 7.954	8 0 23.10
127	13 37 33.182	26 0 5.60	183	15 55 1.103	10 13 57.47
128	13 40 7.651	26 24 59.35	184	15 56 33.959	10 58 40.88
129	13 44 19.729	24 20 51.41	185	16 0 21.617	13 22 56.05
130	13 45 56.310	24 15 58.17	186	16 0 41.310	9 42 57.87
131	13 46 12.072	24 2 8.33	187	16 2 59.572	14 0 27.35
132	13 46 46.651	24 51 40.80	188	16 3 6.618	13 36 59.00
133	13 51 39.354	24 38 30.49	189	16 4 24.098	13 22 3.31
134	13 51 59.705	22 23 26.35	190	16 4 41.570	10 6 50.01
135	13 54 25.217	22 39 58.50	191	16 5 42.980	12 40 2.27
136	13 55 20.693	22 14 33.72	192	16 5 59.266	16 22 13.69
137	14 7 56.650	19 9 59.50	193	16 6 12.084	13 37 42.35
138	14 9 11.160	19 55 24.82	194	16 6 29.082	10 3 1.35
139	14 9 23.644	19 34 29.31	195	16 6 59.660	14 16 29.96
140	14 11 14.667	19 6 2.59	196	16 8 32.770	13 17 23.70
141	14 13 2.053	16 57 35.06	197	16 10 3.544	13 5 23.42
142	14 17 27.790	16 55 11.16	198	16 11 34.281	16 8 19.19
143	14 20 0.953	17 3 22.25	199	16 14 44.967	16 40 51.94
144	14 21 31.064	16 45 49.85	200	16 20 9.964	15 53 23.81
145	14 23 11.387	16 50 40.76	201	16 23 0.959	16 17 57.48
146	14 28 12.905	13 43 16.55	202	16 23 43.537	21 9 29.66
147	14 33 46.108	13 52 14.60	203	16 30 18.736	18 32 9.72
148	14 33 55.070	14 8 48.84	204	16 34 32.419	21 29 43.20
149	14 34 22.174	14 20 23.45	205	16 34 36.256	21 4 1.58
150	14 34 54.307	12 16 29.77	206	16 37 12.274	18 52 12.21
151	14 39 4.926	13 42 18.62	207	16 40 7.990	21 41 3.72
152	14 42 33.357	10 38 27.29	208	16 41 6.008	24 23 11.15
153	14 42 47.985	10 47 39.30	209	16 41 7.071	21 35 54.42
154	14 44 10.403	10 18 35.81	210	16 41 50.606	24 15 51.12
155	14 44 35.229	10 35 48.49	211	16 52 37.309	26 25 39.09
156	14 51 54.337	7 10 15.57	212	16 53 3.366	27 43 31.09
157	14 57 3.620	6 3 17.78	213	16 53 10.777	13 20 26.52
158	14 58 1.431	7 15 39.93	214	16 55 4.596	28 2 57.96
159	14 58 12.582	6 51 17.26	215	16 55 31.065	28 22 0.37
160	14 59 35.593	6 19 28.56	216	16 57 44.663	28 3 54.25
161	14 59 57.698	6 54 50.52	217	16 58 39.016	27 55 53.96
162	15 0 33.375	6 49 9.03	218	16 59 24.776	27 54 39.07
163	15 4 21.645	3 22 6.91	219	17 5 10.403	29 52 34.77
164	15 5 11.046	7 10 34.03	220	17 5 37.478	29 41 14.99
165	15 8 54.017	6 59 39.72	221	17 6 47.473	30 2 30.62
166	15 12 35.554	+ 3 51 2.73	222	17 8 16.677	30 0 8.39
167	15 17 4.230	− 0 2 17.19	223	17 9 20.175	29 42 53.91
168	15 20 28.516	− 0 6 57.69	224	17 10 7.169	−31 12 16.83

Number.	α 1858.0.	δ 1858.0.
	h m s	° ′ ″
225	17 12 14.840	−31 25 56.61
226	17 13 4.999	31 26 22.48
227	17 17 17.581	32 50 3.02
228	17 19 44.388	32 52 53.83
229	17 23 0.424	34 10 1.01
230	17 23 54.929	34 16 20.54
231	17 29 16.449	35 21 48.14
232	17 31 4.961	35 33 46.60
233	17 33 12.654	36 52 6.28
234	17 34 26.669	36 42 1.87
235	17 40 15.178	37 28 49.04
236	17 41 32.567	37 45 43.95
237	17 44 36.555	38 35 8.95
238	17 45 57.092	38 38 45.01
239	17 50 27.549	39 13 45.64
240	17 50 39.138	39 39 2.66
241	17 54 38.324	40 38 8.40
242	17 55 11.130	40 26 50.86
243	18 2 23.867	41 44 28.49
244	18 5 14.187	41 56 26.36
245	18 5 36.073	43 12 19.51
246	18 7 1.414	42 30 48.85
247	18 7 5.615	42 15 28.83
248	18 8 31.282	42 20 5.76
249	18 10 43.566	43 49 49.10
250	18 10 52.913	43 1 59.55
251	18 11 7.779	42 37 40.29
252	18 12 9.145	42 59 37.64
253	18 12 36.869	42 39
254	18 13 58.237	44 10 30.94
255	18 18 12.611	44 14 43.36
256	18 18 54.896	43 55 46.90
257	18 21 39.243	44 41 8.96
258	18 27 50.763	45 34 44.15
259	18 33 11.685	46 18 24.55
260	18 35 45.167	46 43 42.28
261	18 36 12.466	46 31 17.59
262	18 41 53.432	46 45 22.59
263	18 43 23.840	47 26 22.74
264	18 44 10.419	47 49 47.56
265	18 44 28.573	47 47 17.12
266	18 46 26.978	47 45 18.57
267	18 46 31.226	47 34 3.31
268	18 48 13.570	48 9 23.70
269	18 49 54.520	48 28 21.45
270	18 52 54.066	48 54 32.28
271	18 53 55.641	48 36 18.11
272	18 54 10.142	48 51 11.31
273	18 56 41.428	49 14 21.17
274	18 59 22.395	49 32 0.29
275	19 3 52.330	49 46 24.01
276	19 5 52.858	50 13 41.25
277	19 6 44.688	49 42 18.51
278	19 12 11.637	50 30 20.14
279	19 14 33.567	50 46 57.51
280	19 19 10.963	−51 3 5.09
	h m s	° ′ ″
281	19 19 18.450	−51 16 7.09
282	19 23 22.183	50 51 50.34
283	19 23 34.615	51 34 45.35
284	19 26 53.202	51 45 7.15
285	19 29 44.020	51 51 59.82
286	19 30 15.827	51 50 49.35
287	19 30 35.565	52 5 43.17
288	19 33 0.830	52 8 8.42
289	19 33 15.841	52 16 22.64
290	19 34 26.520	52 21 40.38
291	19 38 11.319	52 25 21.16
292	19 39 33.763	52 35 4.72
293	19 40 57.485	52 47 37.75
294	19 42 1.927	52 40 19.22
295	19 45 4.527	53 10 20.41
296	19 45 36.756	53 4 53.37
297	19 50 33.215	53 21 50.75
298	19 50 43.777	53 12 39.53
299	19 56 47.461	53 30 37.25
300	19 57 18.613	52 58 52.81
301	20 0 15.512	53 45 3.25
302	20 2 33.653	54 1 31.30
303	20 5 17.695	54 11 0.36
304	20 6 48.770	54 14 53.47
305	20 9 15.916	54 29 49.18
306	20 11 41.803	54 42 28.61
307	20 15 45.780	54 16 41.37
308	20 16 25.769	54 39 2.12
309	20 17 13.505	54 45 46.37
310	20 18 46.916	55 33 8.81
311	20 19 7.030	55 2 3.04
312	20 21 57.156	54 59 26.98
313	20 22 58.968	54 56 2.85
	α 1859.0.	δ 1859.0.
314	20 25 26.390	55 3 20.79
315	20 27 7.650	55 18 29.95
316	20 27 13.290	55 24 33.43
317	20 31 23.834	55 36 23.14
318	20 33 40.930	55 36 2.15
319	20 34 34.990	55 41 47.50
320	20 38 21.560	55 43 23.42
321	20 39 58.730	55 53 23.14
322	20 43 12.100	56 6 49.19
323	20 44 29.930	55 59 27.04
324	20 45 35.640	55 45 12.47
325	20 47 1.230	56 14 47.53
326	20 47 55.460	56 20 9.53
327	21 1 2.480	57 5 12.43
328	21 2 14.520	57 5 6.31
329	21 4 50.880	57 8 13.18
330	21 8 3.260	57 18 3.90
331	21 10 45.750	57 12 15.77
332	21 11 6.720	57 26 34.46
333	21 12 39.970	−57 23 55.34

Number.	α 1859.0.	δ 1859.0.	Number.	α 1859.0.	δ 1859.0.
	h m s	° ′ ″		h m s	° ′ ″
334	21 14 20.600	—57 51 22.09	349	21 35 8.380	—58 41 34.64
335	21 18 19.350	57 45 21.06	350	21 37 51.400	58 40
336	21 20 26.300	57 29 5.95	351	21 40 18.106	58 57 17.50
337	21 20 48.940	57 46 28.59	352	22 8 41.970	60 32 20.95
338	21 21 55.210	57 55 14.83	353	22 8 48.310	60 57 36.81
339	21 23 0.430	58 0 17.76	354	22 9 36.790	60 49 14.47
340	21 23 22.890	57 42 4.33	355	22 11 13	61 8 11.00
341	21 25 2.280	58 0 4.65	356	22 12 6.080	60 39 16.90
342	21 28 35.340	58 20 30.78	357	22 16 36.870	61 5 50.15
343	21 29 54.810	58 4 24.05	358	22 18 40.420	61 17 31.70
344	21 30 52.172	58 22 23.85	359	22 21 12.300	61 13 40.79
345	21 32 9.160	58 15 1.52	360	22 23 53.980	61 32 27.09
346	21 33 18.394	58 32 14.83	361	22 25 40.970	61 40 32.46
347	21 33 39.750	58 0 28.20	362	22 27 25.610	61 43 53.15
348	21 33 57.470	—57 55 21.35	363	22 30 54.250	—61 57 58.99

The following are the authorities for the observations and the places of the comparison stars:—

ALTONA. Astr. Nachr., L. 187.

ANN ARBOR. Astr. Nachr., XLIX. 179. Brunnow's Astr. Notices, I. 6, 53.

ARMAGH. Monthly Notices, XIX. 305.

BATAVIA. Astr. Nachr., L. 107.

BERLIN. Astr. Nachr., XLVIII. 333, LI. 65.

BONN. Astr. Nachr., XLIX. 253, LI. 187.

BRESLAU. Astr. Nachr., L. 37.

CAMBRIDGE, ENG. Astr. Nachr., L. 243.

CAMBRIDGE, U. S. Astr. Nachr., LI. 273. Brunnow's Astr. Notices, I. 71.

CAPE OF GOOD HOPE. Mem. Astr. Soc., XXIX. 59-83. The observations were made with two different instruments; those made with the larger have been denoted in the list of observations which follows by "Cape 1," and those made with the smaller by "Cape 2."

CHRISTIANIA. Astr. Nachr., LII. 277.

COPENHAGEN. Oversigt kgl. danske Videnskabernes Selkabs, 1858.

DORPAT. Beob. Kaiserl. Sternw. Dorpat, Vol. XV. These observations are published in a crude form, and I was unable to reduce and use them, from a want of the instrumental constants.

DURHAM. Astr. Nachr., L. 11.

FLORENCE. Astr. Nachr., XLVIII. 347, 355, XLIX. 57, L. 97. The observation of October 13 is erroneous as regards the comparison star, which it seems should be Piazzi XV. 227.

GENEVA. Astr. Nachr., XLIX. 115, L. 21.

GÖTTINGEN. Astr. Nachr., XLIX. 235, L. 11.

GREENWICH. Greenwich Observations for 1858. Monthly Notices, XIX. 12.

KÖNIGSBERG. Astr. Nachr., L. 71, LIII. 289.

KREMSMÜNSTER. Astr. Nachr., XLIX. 68, 79, 257, LI. 23.

LEYDEN. Astr. Nachr., L. 157. The observer is mistaken in the comparison star of his last observation; it should be Weisse XV. 369.

LIVERPOOL. Astr. Nachr., XLIX. 267. Monthly Notices, XIX. 54.

MARKREE. Observations on Donati's Comet, 1858, at Markree.

PADUA. Astr. Nachr., XLVIII. 357.

PARIS. Annales de l'Observatoire Imperial, Paris. Tome XIV. Observations.

PULKOVA. Astr. Nachr., L. 307. Beobachtungen der Grossen Cometen 1858. Otto Struve.

SANTIAGO. Astr. Nachr., LIII. 131. Astr. Jour., VI. 100.

VIENNA. Astr. Nachr., XLVIII. 349, XLIX. 43, 53, L. 227, LII. 57.

WILLIAMSTOWN. Astr. Nachr., L. 7. As the latitude and longitude of the place are uncertain, I have not reduced these observations.

WASHINGTON. Astr. Nachr., XLIX. 55, 113, 363. Astr. Jour., V. 150, 158, 166, 180. The comparison star of October 1 is mistaken.

The typographical errors to be met with are so numerous I cannot undertake to mention them. To render the reduction of the comparison stars from mean to apparent place uniform, the elements of reduction in the British Nautical Almanac for 1858 were adopted as the standard; and the same will be used in reducing our normals from apparent to mean places. Consequently it becomes necessary to add to the observations in which the elements of the Berlin Jahrbuch were used, quantities easily obtained from this small ephemeris.

	R. A.	Dec.		R. A.	Dec.
June 15	+0''.09	+0''.18	Sept. 18	+0''.08	+0''.03
July 15	+0.02	+0.22	Oct. 3	+0.07	—0.04
Aug. 14	+0.03	+0.18	Oct. 18	+0.04	—0.19
Sept. 3	+0.05	+0.10	Nov. 2	+0.14	—0.23

For the reduction of the observations for parallax, and the computation of the perturbations, and for comparison, an ephemeris was computed from these elements published by Searle in the Astronomical Journal, V. 188, Searle's own ephemeris not being sufficiently exact for the purpose of comparison.

$$T = \text{Sept. } 29.75230 \quad \text{1858 Washington Mean Time.}$$

$$\left.\begin{aligned} \pi - \Omega &= 129^\circ\ 6'\ 24''.8 \\ \Omega &= 165\ \ 18\ \ 46.2 \\ i &= 116\ \ 57\ \ 46.1 \end{aligned}\right\} \text{Mean Equinox and Ecliptic 1858.0.}$$

$$\varphi = 85\ \ 21\ \ 21.2$$

$$\log q = 9.7622362.$$

In the following list the observations of the comet are given reduced for parallax, and are made to accord with the places of the comparison stars given in the foregoing catalogue. Gould's list of Longitudes (in the American Ephemeris) has been used in getting the Paris M. T. of Observation. The comparisons in the two last columns are Obs. — Cal. The declinations of the Southern observations have generally been reduced to the time of observing the right ascension; that observation of right ascension being selected which was nearest in time and which had the same comparison star.

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δ α	Δ δ
June 7.41071	Florence	141° 14′ 47″.79	+24° 21′ 54″.73	3	+21″.69	+ 6″.26
8.37659	"	141 15 36.99	24 27 52.30	10	+39.17	—15.66
9.42802	"	141 16 20.54	24 34 48.42	10	+27.71	— 7.33
10.39044	"	141 17 25.48	24 41 10.00	10	+23.44	+ 5.67
11.40973	"	141 19 3.43	24 47 35.12	1	+28.89	+ 5.05
12.37591	Padua	141 20 31.82	24 53 36.67	4	+11.55	+ 5.14
12.41803	Florence	141 20 21.71	24 53 56.68	1	— 3.35	+10.15
13.37729	Padua	141 22 34.98	24 59 27.99	4	+ 6.81	—14.10
13.40557	Florence	141 22 16.33	25 0 14.83	1	—15.72	+22.34
13.43268	Berlin	141 22 43.08	24 59 50.15	2	+ 7.30	—12.31
14.41069	"	141 24 58.54	25 5 52.65	2	— 0.89	— 7.69
14.41609	Vienna	141 25 15.40	25 5 55.69	2–5	+15.13	— 6.61
15.39007	Florence	141 28 20.08	25 11 23.30	5	+37.49	—31.86
15.40675	Vienna	141 27 58.18		2–5	+14.65	
15.44201	Berlin	141 27 36.29	25 12 2.13	5	— 9.96	— 4.80
16.39944	Kremsmünster	141 30 54.96	25 17 48.71	5	+10.41	— 8.58
16.41628	Berlin	141 30 39.41	25 17 49.11	5	— 8.36	—14.19
17.39261	Florence	141 34 31.47	25 23 26.85	5	+28.40	—23.47
19.37441	"	141 41 42.14	25 35 32.96	5	+12.05	+ 7.07
19.38451	Padua	141 42 8.91	25 35 39.64	5	+23.63	+10.24
28.38292	Florence	142 29 25.62	26 26 8.67	8	+26.28	— 6.86
28.61976	Cambridge, U. S.	142 30 24.43	26 27 36.77	6	— 7.25	+ 2.80
29.38224	Florence	142 35 56.97	26 31 43.74	8	+22.16	— 2.41
29.41947	Berlin	142 36 2.15	26 31 52.88	6	+13.31	— 5.57
30.37599	Florence	142 42 46.34	26 37 8.61	8	+22.98	— 5.44
30.38577	Vienna	142 42 24.72	26 37 20.26	8	— 2.72	+ 2.98
July 2.37816	Florence	142 56 57.17	26 48 14.56	7	+ 4.77	+ 1.86
8.38159	"	143 46 55.06	27 20 55.54	11	+34.65	— 9.74
9.38324	Vienna	143 55 34.04	27 26 44.86	12	+ 6.24	+ 6.85
9.60789	Washington	143 57 42.00	27 27 56.51	11	+ 9.40	+ 6.19
10.37333	Florence	144 5 0.20	27 32 2.33	11	+16.80	— 1.59
10.59343	Washington	144 6 50.62	27 33 16.24	9	+ 1.75	— 0.72
10.59343	"	144 6 59.45	27 33 18.41	11	+10.58	+ 1.45
11.59576	"	144 16 28.58	27 38 51.84	9	— 0.61	+ 1.42
12.37144	Florence	144 24 2.74	27 43 13.41	9	— 5.59	+ 3.93
13.37158	"	144 34 23.43	27 48 47.14	12	+10.19	+ 2.20
13.59089	Cambridge, U. S.	144 36 26.15	27 50 6.11	12	— 1.67	+ 7.56
14.36879	Florence	144 44 34.08	27 54 18.74	12	+ 3.24	— 2.50
14.58534	Washington	144 46 51.57	27 55 38.77	9	+ 4.72	+ 4.34
15.58781	Cambridge, U. S.	144 57 23.93	28 1 20.09	15	— 1.44	+ 4.97
15.58781	"	144 57 25.18	28 1 17.14	16	— 0.19	+ 2.02
15.58803	Washington	144 57 30.05	28 1 17.20	12	+ 4.55	+ 2.00
16.58028	"	145 8 17.33	28 6 59.62	12	+ 5.49	+ 5.02
17.58135	"	145 19 23.15	28 12 55.11	14	+ 4.72	+15.60
19.36496	Florence,	145 40 9.14	28 22 35.10	14	+26.92	—25.84
19.57128	Cambridge, U. S.	145 42 5.95	28 24 25.03	13	— 0.82	+11.59
19.57128	"	145 42 8.63	28 24 14.88	14	+ 1.86	+ 1.44
20.35855	Florence,	145 51 27.80	28 28 46.13	13	+ 4.39	— 5.81
21.58017	Washington	146 6 8.33	28 36 7.34	14	+ 1.28	+ 0.80
23.61560	Ann Arbor	146 31 25.02	28 48 22.98	17	— 2.55	— 0.64
24.58362	Washington	146 43 53.23	28 54 25.65	17	+ 0.71	+ 5.84
25.57816	"	146 56 54.02	29 0 37.18	17	+ 0.72	+ 7.35
27.57886	"	147 23 48.85	29 13 12.53	18	— 1.71	+ 5.10
28.57465	"	147 37 32.23	29 19 43.76	19	— 7.18	+12.22
29.57953	"	147 51 52.66	+29 25 49.46	21	+ 0.36	—14.60

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δα	Δδ
July 31.35674	Florence	148° 17′ 13″.40	+29° 37′ 38″.77	21	—28″.85	—12″.75
Aug. 4.35187	"	149 19 6.35	30 5 14.06	27	— 3.54	—14.07
4.37075	Berlin	149 19 22.65	30 5 46.12	23	— 5.38	+ 9.91
4.57000	Washington	149 22 39.36	30 7 9.00	23	— 0.37	+ 7.56
5.34162	Kremsmünster	149 35 2.22	30 12 41.42	23	— 8.19	+ 7.34
5.34827	Florence	149 35 27.94	30 13 3.26	23	+12.02	+26.29
5.54365	Cambridge, U. S.	149 38 31.44	30 14 9.93	20	+ 3.83	+ 8.10
5.54365	"	149 38 21.08	30 14 8.53	23	— 6.53	+ 6.70
6.34048	Florence	149 51 27.72	30 19 54.03	23	— 9.48	+ 4.37
7.36200	Berlin	150 8 43.81	30 27 30.63	22	— 4.35	+ 6.21
7.56564	Washington	150 12 8.39	30 28 59.20	23	— 7.85	+ 3.44
8.56004	"	150 29 24.36	30 36 40.41	24	— 0.50	+ 9.71
10.33796	Kremsmünster	151 1 3.61	30 50 22.10	24	+ 6.28	+11.63
10.35184	Berlin	151 1 5.40	30 50 24.07	24	— 6.98	+ 7.07
10.56020	Washington	151 4 50.17	30 52 5.23	24	— 9.79	+12.49
11.33986	Kremsmünster	151 19 15.59	30 58 12.55	24	+ 0.36	+ 6.60
12.33952	"	151 37 33.32	31 6 17.22	36	—20.96	+ 8.66
12.59010	Ann Arbor	151 42 30.69	31 8 5.47	34	— 7.89	— 5.36
13.58572	"	152 1 33.00	31 16 24.08	30–2	—10.57	+ 2.20
14.33189	Vienna	152 16 14.43	31 22 42.03	32	— 4.01	+ 6.51
14.34131	Kremsmünster	152 16 19.63	31 22 45.35	32	— 9.94	+ 5.09
14.37376	Copenhagen	152 17 0.51	31 23 22.17	32	— 7.45	+25.54
14.571921	Ann Arbor	152 21 1.85	31 24 37.64	32	— 1.10	+ 0.91
15.556670	Washington	152 40 53.49	31 32 59.51	35	+ 6.95	+ 0.20
15.578418	Ann Arbor	152 41 13.22	31 33 18.56	35	+ 0.24	+ 8.05
16.329858	Florence	152 56 21.66	31 39 25.74	35	—13.09	—14.24
16.367755	Copenhagen	152 57 18.67	31 40 24.83	32	— 3.02	+25.08
16.550079	Washington	153 1 12.45	31 41 39.36	35	+ 4.49	+ 4.30
17.327305	Vienna	153 17 15.93	31 48 22.98	25–6	— 7.69	— 1.78
17.335186	Kremsmünster	153 17 29.01	31 48 33.89	28	— 4.56	+ 4.95
17.360620	Copenhagen	153 18 35.17	31 48 40.92	37	+29.33	— 1.52
17.375546	"	153 19 4.92	31 48 50.15		+40.15	— 0.23
17.541171	Washington	153 22 1.02	31 50 19.63	35	+ 5.74	+ 1.18
17.568303	Ann Arbor	153 22 29.67	31 50 35.82	37	— 0.15	+ 2.92
18.320072	Vienna	153 38 49.51	31 57 23.25	35	+12.98	+ 7.12
19.346349	Berlin	154 1 6.13	32 6 48.19	31	+ 0.94	+13.47
19.376823	Cambridge, Eng.	154 1 46.76	32 6 38.02	46	+ 0.80	—13.45
19.544957	Washington	154 5 37.10	32 8 36.44	26	+ 6.69	+12.53
19.548659	Cambridge, U. S.	154 5 27.88	32 8 29.71	53	— 7.49	+ 3.71
20.544643	"	154 28 57.08	32 17 51.52	33	— 8.00	+12.26
20.546599	"	154 28 54.87	32 17 43.99	40	—12.90	+ 3.64
21.330193	Kremsmünster	154 46 5.90	32 25 20.05	41	— 8.55	+18.06
22.539509	Washington	155 14 45.23	32 36 45.15	41	—11.54	+10.71
22.577310	Ann Arbor	155 15 44.05	32 37 3.12	41	— 7.46	+ 6.82
23.341254	Königsberg	155 34 14.71	32 44 9.54	38	—15.77	—11.18
23.360472	Copenhagen	155 34 37.11	32 44 21.26	38	—21.84	—10.66
23.370056	Cambridge, Eng.	155 35 12.50	32 44 42.35	38	— 0.64	+ 4.82
23.379349	"	155 35 21.95	32 44 52.87	44	— 4.97	+ 9.89
23.545062	Washington	155 39 21.99	32 46 28.14	44	—11.08	+ 8.04
23.563000	Ann Arbor	155 39 52.38	32 46 40.21	44	— 7.40	+ 9.59
24.328091	Königsberg	155 59 7.10	32 54 9.92	38	— 5.00	+ 7.73
24.333372	Copenhagen	155 59 21.07	32 54 19.14	38	+ 0.92	+13.82
24.335916	"	155 59 27.58	32 54 7.43		+ 3.55	+ 0.60
24.538632	Washington	156 4 26.18	32 56 20.54	38	— 7.54	+13.21
25.304874	Vienna	156 24 20.98	+33 4 0.52	38	— 0.15	+14.69

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δα	Δδ
Aug. 25.320927	Vienna	156° 24′ 28″.28	+33° 3′ 58″.26	38	−18″.03	+ 2″.75
25.381750	Cambridge, Eng.	156 26 0.38	33 4 36.78	38	−21.40	+ 4.70
25.536968	Washington	156 30 19.48	33 6 21.39	38	− 6.71	+15.75
26.374066	Christiania	156 52 43.15	33 14 47.88	44	− 1.30	+14.43
26.378525	"	156 52 32.18	33 14 43.05	39	−19.49	+ 6.88
26.395693	"	156 53 13.79	33 14 54.46	38	− 5.70	+ 7.79
26.482289	"	156 55 36.26	33 15 48.95	38	− 3.68	+ 9.47
27.370067	Cambridge, Eng.	157 19 59.28	33 24 58.51	43	− 3.51	+14.01
27.380735	Christiania	157 20 16.24	33 25 3.85	43	− 4.31	+12.76
28.309822	Vienna	157 46 20.57	33 34 32.47	38	−16.50	+ 5.07
28.318270	Berlin	157 46 39.71	33 34 46.46	42	−11.92	+13.79
28.319291	Geneva	157 46 50.62	33 34 45.89	49	− 2.77	+12.59
28.322451	"	157 47 4.49	33 34 49.00	50	+ 5.65	+13.73
28.480204	Christiania	157 51 24.47	33 36 33.58	38	− 6.96	+19.89
30.299305	Kremsmünster	158 45 36.15	33 55 39.06	48	− 6.40	+20.21
30.304230	Vienna	158 45 47.70	33 55 36.62	51-2	− 3.95	+14.75
30.309523	Florence	158 45 37.69	33 55 25.96	54	−23.73	+ 0.73
30.377443	Cambridge, Eng.	158 48 3.62	33 56 17.78	45	− 3.26	+ 9.48
30.526796	Cambridge, U. S.	158 52 31.74	33 57 54.46	48	−12.05	+11.38
31.291317	Vienna	159 16 32.49		51-2	−12.39	
31.319005	Kremsmünster	159 17 33.52	34 6 20.04	48	− 4.31	+12.83
31.337849	Copenhagen	159 17 59.57	34 6 25.76	48	−14.33	+ 6.54
31.339817	"	159 18 2.98	34 6 36.56		−14.69	+16.08
31.551986	Ann Arbor	159 24 53.06	34 8 48.07	57	−12.42	+12.21
Sept. 1.295680	Florence	159 49 5.83	34 16 36.92	57	−14.72	+ 5.70
1.309567	Kremsmünster	159 49 37.11	34 16 54.23	51	−11.00	+14.08
1.320362	Bonn	159 50 1.29	34 17 0.04	51	− 8.25	+13.02
1.322480	Christiania	159 50 3.13	34 17 0.94	51	−10.62	+12.57
1.326603	Berlin	159 50 11.89	34 17 6.08	51	− 9.75	+15.17
1.540701	Ann Arbor	159 57 17.76	34 19 16.27	57	−10.16	+ 8.18
1.563647	"	159 58 12.94	34 19 38.10	51	− 1.96	+15.32
2.298887	Kremsmünster	160 23 1.61	34 27 23.80	51	− 7.84	+ 9.99
2.301560	Florence	160 22 49.02	34 27 29.68	57	−25.94	+14.16
2.305161	Vienna	160 23 9.90	34 27 32.33	56	−12.48	+14.50
2.327381	Geneva	160 24 1.89	34 27 48.29	57	− 6.35	+16.22
2.337187	Copenhagen	160 24 19.82	34 27 50.99	51	− 8.67	+12.64
2.350639	Königsberg	160 24 51.38	34 27 46.76	55	− 4.90	− 0.21
2.406874	Christiania	160 26 42.59	34 28 19.63	51	−10.03	− 3.37
2.418651	Pulkova	160 27 12.15	34 28 48.55		− 4.86	+18.00
2.427687	Christiania	160 27 32.91	34 28 48.24	57	− 2.83	+11.90
3.281247	Vienna	160 57 29.49	34 37 56.89	55	− 5.72	+14.13
3.292555	Florence	160 58 0.01	34 38 2.05	56	+ 0.58	+12.07
3.295584	Vienna	160 57 46.70	34 37 59.35	55	−19.27	+ 7.42
3.322590	Geneva	160 58 56.61	34 38 20.04	56	− 7.34	+10.85
3.530240	Washington	161 6 26.13	34 40 36.24	55	− 5.76	+14.37
4.276201	Kremsmünster	161 33 42.46	34 48 33.42	56	−10.63	+16.17
4.289568	"	161 34 16.06	34 48 39.87	56	− 6.91	+14.13
4.300379	Florence	161 34 36.28	34 48 49.44	56	−10.86	+16.83
4.308592	Christiania	161 34 57.50	34 48 48.59	56	− 8.02	+10.76
4.308932	Geneva	161 35 4.38	34 48 56.46	56	− 1.90	+18.41
4.311267	Berlin	161 35 0.39	34 48 51.87	56	−11.11	+12.34
4.316907	Geneva		34 48 52.29	55		+10.87
4.321872	"	161 35 25.19		55	−10.05	
4.421240	Christiania	161 39 10.86	34 49 59.11	56	− 7.32	+ 9.72
5.293191	Florence	162 12 25.51	+34 59 17.57	55	− 9.81	+17.00

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δ α	Δ δ
Sept. 5.383408	Armagh	162° 15′ 54″.08	+35° 0′ 15″.53	55	−12″.20	+18″.25
5.419101	Christiania	162 17 20.59	35 0 30.13	56	− 9.39	+10.39
5.419101	"	162 17 23.46	35 0 35.01	55	− 6.52	+15.27
5.529968	Washington	162 21 43.11	35 1 40.76	56	− 7.61	+11.49
5.537181	Ann Arbor	162 21 58.91	35 1 48.01	55	− 8.84	+14.21
5.654786	Durham	162 26 34.00	35 3 1.50	55	−11.85	+14.05
6.329700	Copenhagen	162 53 33.17	35 10 10.64		−17.06	+23.08
6.350846	"	162 54 8.16	35 10 6.70	58	−33.77	+ 6.06
6.364638	Armagh	162 55 13.80	35 10 26.90	55	− 1.88	+17.74
6.524524	Washington	163 1 26.35	35 12 8.28	56	−21.99	+20.40
6.544780	Ann Arbor	163 2 31.25	35 12 14.35	58	− 7.04	+13.99
6.917566	"	163 18 0.35	35 16 2.85	58	− 5.65	+13.70
7.315249	Berlin	163 34 46.30	35 20 3.87	58	− 6.76	+12.91
7.367969	Cambridge, Eng.	163 37 1.79	35 20 42.31	59	− 6.24	+19.48
7.514951	Washington	163 43 31.43	35 21 52.33	61	+ 5.57	+ 0.87
8.322259	Copenhagen	164 18 58.21	35 30 3.29		+ 9.08	+12.00
8.340315	Königsberg	164 19 28.57	35 30 10.23	60	− 8.99	+ 8.36
8.516888	Cambridge, U. S.	164 27 29.44	35 31 53.17	60	− 3.92	+ 8.14
8.516888	"	164 27 21.05	35 31 46.46	61	−12.31	+ 1.43
8.516888	"	164 27 25.09	35 31 57.43	66	− 8.27	+12.40
9.299974	Geneva	165 3 26.88	35 39 26.56	66	− 6.95	+12.82
9.302115	Florence	165 3 40.97	35 39 16.42	63	+ 1.10	+ 1.48
9.312209	Bonn	165 3 57.18	35 39 30.23	60	−11.08	+ 9.61
9.315659	Königsberg	165 4 9.73	35 39 38.55	60	− 8.24	+15.98
9.315932	Berlin	165 4 7.88	35 39 21.79	60	−10.86	− 0.93
9.319178	Paris	165 4 14.60	35 39 37.57	66	−13.28	+13.02
9.523602	Washington	165 13 57.13	35 41 33.91	61	− 9.16	+14.92
10.263712	Kremsmünster	165 49 36.73	35 48 19.58	65	−14.43	+16.55
10.277018	Vienna	165 50 23.97	35 48 24.58	65–6	− 6.50	+14.44
10.284499	Florence	165 50 43.39	35 48 29.91	65	− 9.20	+15.78
10.294314	Kremsmünster	165 51 7.95	35 48 35.62	65	−13.65	+16.26
10.306418	Berlin	165 51 50.46	35 48 42.52	65	− 6.96	+16.70
10.307120	Copenhagen	165 52 3.80	35 48 36.08		+ 4.30	+ 9.89
10.325063	Königsberg	165 52 42.14	35 48 50.29	65	−10.49	+14.54
10.325709	Paris	165 52 32.51	35 48 45.94	65	−22.03	+ 9.85
10.354888	Armagh	165 54 12.98	35 49 1.87	65	− 8.07	+10.27
10.520362	Washington	166 2 38.05	35 50 44.35	65	+ 4.00	+25.32
10.628708	Bonn	166 7 46.05	35 51 32.36	65	−13.14	+16.62
11.280961	Kremsmünster	166 41 6.09	35 57 2.34	64	− 9.66	+14.46
11.301907	Geneva	166 42 6.50	35 57 16.88	65	−14.52	+17.61
11.319423	Paris	166 43 10.02	35 57 29.26	69	− 5.64	+22.33
11.321783	Copenhagen	166 43 13.97	35 57 16.04	65	− 9.05	+ 7.94
11.325972	Königsberg	166 43 29.17	35 57 29.14	65	− 6.92	+18.96
11.411657	Pulkova	166 47 59.70	35 58 6.22		− 4.47	+13.85
12.261121	Kremsmünster	167 33 18.94	36 4 47.28	69	−10.38	+14.42
12.286131	Christiania	167 34 43.59	36 5 1.27	68	− 7.88	+17.13
12.286559	Florence	167 34 44.73	36 4 53.86	67	− 8.15	+ 9.55
12.293542	Königsberg	167 34 53.84	36 5 9.51	69	− 7.21	+22.06
12.295472	Geneva	167 35 12.02	36 4 58.98	69	−10.15	+10.66
12.306918	"	167 35 50.51	36 5 3.35	71	− 9.32	+ 9.89
12.311009	Liverpool	167 35 58.70	36 5 11.76	62	−14.60	+16.46
12.321429	"	167 36 36.73	36 5 14.60	62	−10.88	+14.63
12.331857	"	167 37 12.38	36 5 20.44	62	− 9.58	+15.80
12.346679	Paris	167 38 6.03	36 5 28.54	69	− 4.79	+17.28
12.354098	Armagh	167 38 15.70	+36 5 30.97	69	−19.59	+16.39

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δα	Δδ
Sept.12.411405	Pulkova	167° 41′ 25″.95	+36° 5′ 53″.48		−18″.72	+13″.41
12.519159	Washington	167 47 33.62	36 6 47.30	69	−11.68	+19.75
12.526442	Ann Arbor	167 47 58.52	36 6 45.74	69	− 8.03	+15.00
12.536618	"	167 48 33.25	36 6 48.84	67	− 7.21	+13.66
12.612848	Bonn	167 52 41.84	36 7 22.26	69	−13.13	+13.93
13.277211	Kremsmünster	168 30 27.71	36 11 58.86	69	−12.00	+15.48
13.277425	Vienna	168 30 32.55	36 11 56.74	69	− 7.90	+13.28
13.291278	Königsberg	168 31 19.32	36 12 1.52	71	− 9.26	+12.59
13.291733	Christiania	168 31 15.14	36 12 2.23	62	−15.02	+13.12
13.294974	Geneva	168 31 27.37	36 12 6.30	69	−14.06	+15.92
13.305238	Berlin	168 31 59.11	36 12 8.35	69	−18.00	+13.93
13.317599	Paris	168 32 50.64	36 12 18.72	69	− 9.48	+19.45
13.328567	Leyden	168 33 27.24	36 12 15.34	70	−11.07	+11.78
13.329885	Copenhagen	168 33 27.66	36 12 12.04		−15.24	+ 7.96
13.333017	Christiania	168 33 44.19	36 12 20.78	68	− 9.62	+15.47
13.335091	Cambridge, Eng.	168 33 50.03	36 12 22.04	71	−11.05	+15.93
13.371491	Leyden	168 35 52.93	36 12 37.07	69	−15.03	+16.77
13.516267	Washington	168 44 23.18	36 13 30.19	69	−12.22	+14.31
13.525041	Ann Arbor	168 44 54.92	36 13 33.94	69	−11.36	+14.72
14.273293	Kremsmünster	169 29 51.01	36 17 57.33	69	− 6.96	+13.63
14.288013	Königsberg	169 30 44.22	36 18 2.07	72	− 7.87	+13.57
14.295328	Vienna	169 31 1.74	36 18 3.61	67–9	−17.26	+12.73
14.307231	Geneva	169 31 50.02	36 18 7.03	69	− 9.23	+12.28
14.307231	"	169 31 48.81	36 18 5.89	70	−10.44	+11.14
14.316540	Leyden	169 32 34.48		74	− 2.62	
14.318520	Copenhagen	169 32 27.58	36 18 13.68		−16.83	+15.27
14.323235	Paris	169 32 52.48	36 18 17.61	69	− 9.41	+17.69
15.294684	Vienna	170 34 17.06	36 22 47.86	72	− 8.36	+11.71
15.299126	Königsberg	170 34 31.52	36 22 58.52	74	−11.24	+21.30
15.299256	Geneva	170 34 26.31	36 22 49.14	69	−16.96	+10.88
15.299256	"	170 34 26.10	36 22 49.20	70	−17.17	+10.94
15.308400	"	170 35 3.64	36 22 43.84	73	−15.32	+ 4.37
15.316643	Liverpool	170 35 38.37	36 22 56.37	81	−13.78	+14.90
15.324451	Berlin	170 36 3.78	36 22 59.39	75	−17.88	+16.04
15.332976	Cambridge, Eng.	170 36 43.46	36 22 59.71	74	−11.53	+14.31
15.337503	Leyden	170 36 58.28	36 22 56.14	73	−14.41	+ 9.66
15.330548	Liverpool	170 36 37.19	36 23 0.77	81	− 8.31	+15.96
15.340177	Cambridge, Eng.	170 37 7.72	36 22 59.88	75	−15.43	+12.76
15.344453	Liverpool	170 37 35.46	36 23 5.47	81	− 4.41	+17.33
16.282016	Kremsmünster	171 40 22.08	36 26 5.74	80	−10.77	+15.72
16.289858	Florence	171 40 42.92	36 26 10.11	78	− 3.41	+18.94
16.294017	Christiania	171 41 9.40	36 26 6.29	62	−13.14	+14.51
16.305316	Königsberg	171 41 56.83	36 26 3.94	76	−12.52	+10.53
16.349513	Leyden	171 45 3.20	36 25 58.38	76	− 9.59	− 1.30
16.354993	Cambridge, Eng.	171 45 28.16	36 26 17.25	76	− 7.41	+16.81
16.366305	Leyden	171 46 4.63	36 26 15.05	77	−17.98	+13.03
16.411922	Pulkova	171 49 22.50	36 26 19.33		−10.14	+11.15
16.527175	Ann Arbor	171 57 20.10	36 26 36.40	78	−14.97	+13.66
16.548921	"	171 58 52.17	36 26 40.60	80	−14.29	+15.28
16.638835	Bonn	172 5 18.22	36 26 47.11	76	− 7.51	+11.66
17.259056	Vienna	172 49 50.32	36 27 36.18	82	− 8.84	+16.79
17.264848	Kremsmünster	172 50 9.59	36 27 36.31	82	−15.02	+16.64
17.282358	Königsberg	172 51 27.98	36 27 39.12	80	−13.64	+18.84
17.291647	Vienna	172 52 6.45	36 27 32.04	76	−16.05	+11.53
17.306382	Copenhagen	172 53 6.85	+36 27 38.35	78–80	−20.55	+17.43

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δ α	Δ δ
Sept. 17.329512	Christiania	172° 55′ 11″.24	+36° 27′ 43″.33		+ 1″.84	+21″.81
17.359841	Cambridge, Eng.	172 57 24.53	36 27 35.67	79	+ 1.18	+12.48
17.412500	Pulkova	173 1 3.30	36 27 37.43		—13.25	+14.35
17.532051	Ann Arbor	173 9 55.97	36 27 41.20	80	—13.00	+17.41
17.550766	"	173 11 18.70	36 27 39.60	78	—13.94	+15.88
18.287944	Königsberg	174 7 41.96	36 26 52.95	82	— 5.55	+10.13
18.317231	Copenhagen	174 9 47.28	36 26 36.12	82	—17.56	— 3.46
18.315490	Paris	174 9 50.67	36 27 5.47	82	—15.38	+25.92
18.319006	Copenhagen	174 10 16.79			+ 3.62	
18.319751	Liverpool	174 10 4.71	36 26 54.20	84	—11.95	+14.91
18.328100	"	174 10 44.18	36 26 50.64	84	—11.69	+12.29
18.336446	"	174 11 25.44	36 26 46.37	84	— 9.63	+ 8.98
18.351046	Markree	174 12 34.18	36 26 29.05	82	— 9.53	— 6.62
18.413297	Pulkova	174 17 27.00	36 26 37.87		—10.07	+ 9.83
18.424440	Christiania	174 18 19.92	36 26 48.38		— 9.80	+21.78
18.534230	Ann Arbor	174 26 57.07	36 26 28.80	80	—13.13	+17.32
18.545534	Christiania	174 27 44.11	36 26 24.43	82	—19.88	+14.61
19.272473	Kremsmünster	175 26 52.76	36 23 57.00	82	—13.13	+17.81
19.286802	Christiania	175 28 3.28	36 23 47.34	83–4	—14.10	+12.02
19.289696	Geneva	175 28 16.40	36 23 47.67	86	—15.42	+13.14
19.289697	Florence	175 28 16.03	36 23 46.81	86	—17.29	+12.36
19.312751	Paris	175 30 10.86	36 23 35.74	86	—18.14	+ 7.51
19.387398	Armagh	175 36 59.67	36 23 13.49	82	+18.68	+ 6.36
19.515182	Washington	175 47 20.76	36 22 40.93	82	— 4.56	+12.25
19.531160	Ann Arbor	175 48 29.01	36 22 36.21	86	—17.26	+12.55
20.252856	Florence	176 51 5.56	36 17 55.79	86	— 4.42	+ 9.65
20.265575	Kremsmünster	176 52 6.63	36 17 54.15	85–6	—10.89	+13.83
20.288793	Christiania	176 54 6.19	36 17 44.26	84	—14.68	+14.67
20.310506	Paris	176 55 56.55	36 17 32.58	86	—20.02	+13.09
20.344173	Durham	176 59 4.23		86	—11.91	
20.360918	Markree	177 1 4.66	36 17 10.97	85–6	+19.05	+15.35
20.362970	Durham		36 17 8.06	86		+13.43
20.470112	Christiania	177 10 18.96	36 16 12.75	84–7	—12.33	+10.69
20.510140	Washington	177 14 0.80	36 15 52.79	85	— 6.24	+11.01
20.510962	Cambridge, U. S.	177 13 56.28	36 15 55.65	86	—15.20	+14.28
20.514399	Ann Arbor	177 14 13.81	36 15 53.71	86	—16.22	+14.10
20.515778	Washington	177 14 31.25	36 15 53.84	86	— 6.23	+18.42
20.337975	Göttingen	176 58 20.29	36 17 15.66	86	—17.46	+ 8.64
20.647851	Durham	177 26 20.58		86	—12.96	
20.650006	Berlin	177 26 35.43	36 14 41.65	86	— 9.98	+13.48
20.663728	Durham		36 14 36.18	86		+15.39
20.680067	"	177 29 15.12		86	—14.02	
20.690908	Armagh	177 30 10.59	36 14 16.78	85	—17.72	+10.86
21.299000	Geneva	178 26 46.31	36 8 7.33	86	— 8.18	+18.35
21.299037	Bonn	178 26 39.87	36 8 3.23	88	—14.82	+14.28
21.311531	Königsberg	178 27 56.29	36 7 51.27	88	— 9.39	+11.08
21.303351	Liverpool	178 27 8.29	36 8 1.77	92	—10.92	+15.80
21.317379	Göttingen	178 28 8.88	36 7 55.28	87	—30.04	+19.07
21.325561	Christiania	178 29 7.94	36 7 51.07	87	—17.50	+20.55
21.326150	Göttingen	178 29 1.55		85	—27.25	
21.319350	Liverpool	178 28 41.22	36 7 52.96	92	— 8.91	+18.11
21.326150	Göttingen	178 28 56.49		86	—32.31	
21.328914	Leyden	178 29 34.02	36 7 45.85	89	—10.51	+17.68
21.334417	Copenhagen	178 30 4.85	36 8 1.66	88	— 8.99	+37.35
21.335339	Liverpool	178 30 13.96	+36 7 44.14	92	— 7.24	+20.48

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δ α	Δ δ
Sept.21.335411	Berlin	178° 30′ 9″.55	+36° 7′ 38″.80	85	−11″.95	+15″.17
21.337062	Copenhagen	178 30 18.26	36 7 43.82		−12.65	+21.26
21.350418	Armagh	178 31 34.04	36 7 20.35	86	−12.93	+ 7.27
21.351112	Markree	178 31 35.38	36 7 20.64	85–6	−15.54	+ 8.05
21.364569	Cambridge, Eng.	178 33 34.25	36 7 18.18	85	+26.73	+15.09
21.375570	Leyden	178 33 51.15	36 7 10.58	86	−19.23	+15.29
21.471664	Christiania	178 43 5.72	36 5 55.02	86	−14.71	+ 9.06
22.287329	Kremsmünster	180 3 16.71	35 54 35.24	90	−12.99	+15.55
22.290700	Göttingen	180 3 28.68	35 54 35.97	90	−21.42	+19.49
22.294876	Bonn	180 3 56.12	35 54 25.39	90	−19.26	+12.90
22.305836	Göttingen	180 5 8.42	35 54 13.96	89	−13.35	+11.95
22.357669	Markree	180 10 12.18	35 53 14.67	90	−24.22	+ 2.69
23.266456	Kremsmünster	181 45 16.71	35 36 36.83	93	−10.04	+13.25
23.296747	Copenhagen	181 48 34.97	35 35 45.14	93	− 7.21	− 0.33
23.302829	Vienna	181 49 2.03		93	−19.44	
24.270615	"	183 36 39.67	35 12 43.13	94	−11.82	+12.79
24.279692	Liverpool	183 37 45.87	35 12 32.54	92	− 8.00	+16.83
24.290996	Königsberg	183 39 0.60	35 12 10.37	93	−12.01	+12.92
24.290137	Liverpool	183 38 55.72	35 12 14.66	92	− 9.98	+15.83
24.300586	"	183 40 7.05	35 11 57.07	92	−10.56	+15.16
24.332837	Greenwich	183 43 43.65	35 11 6.04		−16.20	+16.63
24.341926	Cambridge, Eng.	183 44 49.63	35 10 44.54	93	−12.93	+10.00
24.422078	Christiania	183 53 54.89	35 8 45.68	92–3	−22.26	+23.65
24.423542	Pulkova	183 54 10.95	35 8 29.39		−16.36	+ 9.80
25.261189	Kremsmünster	185 33 31.30	34 42 52.75	95	−13.68	+14.55
25.272704	Göttingen	185 35 5.21	34 42 29.31	97	−13.14	+14.41
25.275727	"	185 35 19.06	34 42 27.18	96	−21.39	+18.36
25.292244	Königsberg	185 37 26.04	34 41 49.07	95	−15.31	+13.75
25.285921	Liverpool	185 36 40.26	34 42 2.64	98	−14.79	+14.49
25.292886	"	185 37 27.31	34 41 49.79	98	−18.74	+15.78
25.299857	"	185 38 20.05	34 41 35.14	98	−17.07	+15.30
25.314011	Berlin	185 40 15.72	34 41 8.62	97	− 5.15	+17.63
25.325785	Cambridge, Eng.	185 41 23.38	34 40 48.39	95	−23.87	+21.46
25.352642	Greenwich	185 44 52.20	34 39 47.44		−12.29	+15.24
25.489007	Cambridge, U. S.	186 1 44.71	34 35 5.39	95	− 6.54	+18.45
25.520255	Ann Arbor	186 5 36.19	34 33 59.05	96	− 6.93	+18.58
25.648289	Durham	186 21 26.18	34 29 18.03	95	−11.63	+14.59
25.667479	"	186 23 47.69	34 28 37.93	95	−13.85	+16.65
26.298863	Geneva	187 44 6.99	34 3 46.23	98	−18.40	+11.15
26.306382	Bonn	187 45 11.32	34 3 29.86	99	−12.62	+13.69
26.484109	Christiania	188 8 21.71	33 55 50.83	98	−13.95	+ 9.52
26.527307	Washington	188 14 4.57	33 53 58.36	99	−11.54	+10.03
26.531339	Ann Arbor	188 14 32.82	33 53 52.01	99	−15.15	+14.33
26.322762	Durham	187 47 18.92		98	−12.67	
26.339268	"	187 49 25.98		98	−14.35	
26.346136	"		34 1 52.60	98		+16.76
27.242312	Kremsmünster	189 50 3.90	33 20 26.11	100	−15.03	+15.02
27.278053	Florence	189 54 52.02	33 18 29.13	100	−21.23	+ 6.36
27.279344	Bonn	189 55 13.95	33 18 31.70	100	− 9.94	+12.86
27.286224	Geneva	189 56 5.07	33 18 6.62	100	−15.56	+10.26
27.293169	Liverpool	189 56 45.38	33 17 50.51	103	−32.62	+13.76
27.320928	Cambridge, Eng.	190 0 55.60	33 16 25.90	100	−11.51	+14.06
27.360644	Christiania	190 6 22.42	33 14 25.88	100	−13.25	+16.27
27.368878	Armagh	190 8 5.00	33 12 51.72	100	+21.11	+ 7.56
28.259639	Kremsmünster	192 13 38.59	+32 24 1.27	105	−14.00	+13.79

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δα	Δδ
Sept. 28.279187	Bonn	192° 16′ 33″.12	+32° 22′ 51″.95	102	− 9″.72	+15″.84
28.292041	Paris	192 18 24.52	32 22 7.66	105	−10.37	+18.61
28.299010	Copenhagen	192 19 25.03	32 22 47.15	102	−10.64	+83.66
28.307004	"	192 20 17.27			−28.15	
28.326205	Durham	192 23 30.97	32 20 2.45	101	− 2.10	+19.01
28.506643	Washington	192 49 51.07	32 9 57.40	105	− 7.10	+90.12
29.260515	Kremsmünster	194 42 44.06	31 17 38.66	104	−15.74	+17.75
29.263858	Vienna	194 43 20.04	31 17 22.20	112–3	−10.41	+15.78
29.268433	Königsberg	194 43 55.92	31 17 3.25	109	−16.49	+16.68
29.281328	Leyden	194 46 4.00		107	− 6.72	
29.284939	Florence	194 46 20.95	31 15 58.15	106	−22.91	+23.32
29.293825	Göttingen	194 47 12.72	31 14 27.20	108	−52.74	+31.07
29.303599	Vienna	194 49 25.47	31 14 26.76	104	− 9.77	+13.28
29.323978	Geneva	194 52 22.65	31 12 49.99	104	−10.75	+ 1.24
29.498277	Cambridge, U. S.	195 19 17.55	31 0 5.74	112	−15.54	+16.24
29.505309	Washington	195 20 39.97	30 59 33.17	112	+ 1.62	+15.39
29.708542	Greenwich	195 52 4.65	30 43 56.43		− 9.47	+11.88
30.207585	Pulkova	197 10 52.77	30 3 30.84	115	−15.05	+12.96
30.227222	"	197 14 4.05	30 1 54.13	115	−12.25	+15.75
30.257998	Kremsmünster	197 18 58.29	29 59 16.75	114	−13.70	+14.94
30.263155	Liverpool	197 19 53.58	29 58 48.95	111	− 8.03	+13.44
30.270043	Geneva	197 20 44.43	29 58 7.41	114	−14.94	+ 7.07
30.273627	Liverpool	197 21 33.97	29 57 55.89	111	− 8.37	+13.88
30.284093	"	197 23 11.36	29 57 0.83	111	−11.71	+12.36
30.295506	Geneva	197 24 59.97	29 56 4.30	110	−13.00	+14.33
30.314046	Markree	197 27 57.13	29 54 40.09	116	−14.49	+25.37
30.320418	Armagh	197 28 27.61	29 53 52.46	114	−45.43	+10.53
30.335823	Cambridge, Eng.	197 31 27.60	29 52 40.33	116	−14.03	+17.84
30.342848	Armagh	197 32 10.53	29 52 7.19	116	−38.89	+20.97
30.501681	Washington	197 58 19.42	29 38 11.42	116	− 8.92	+16.37
30.533553	"	198 4 1.45	29 35 4.29	117	+22.99	− 1.49
30.323109	Durham	197 29 30.83		116	− 8.17	
30.334899	"		29 52 43.32	116		+16.06
Oct. 1.268006	Kremsmünster	200 4 28.15	28 26 19.30	122	−13.99	+10.99
1.295157	Greenwich	200 9 6.30	28 23 40.45		− 8.55	+13.98
1.300854	Göttingen	200 9 54.20	28 23 4.11	122	−17.99	+11.72
1.300854	"	200 9 56.05	28 23 12.04	123	−16.14	+19.65
1.311247	Geneva	200 11 43.22	28 22 0.09	122	−13.36	+ 8.83
1.312189	Leyden	200 11 57.08		124	− 8.79	
1.350841	Christiania	200 18 34.24	28 18 7.24	121	− 2.79	+16.49
1.507696	Washington	200 44 39.17	28 2 23.24	120	−20.96	+26.08
2.263880	Königsberg	202 54 21.88	26 40 19.60	125	− 9.63	+18.78
2.264704	Vienna	202 54 35.63	26 40 0.05	126	− 8.62	+ 7.72
2.268121	Kremsmünster	202 55 2.46	26 39 43.59	126	−14.25	+12.60
2.285332	Greenwich	202 58 7.20	26 37 48.19		− 8.83	+15.30
2.286363	Geneva	202 58 16.03	26 37 35.34	128	−10.74	+ 9.53
2.287888	Leyden	202 58 36.73		126	− 5.95	
2.300320	Christiania	203 0 39.16	26 36 4.47	125	−13.12	+14.64
2.300320	"	203 0 42.22	26 36 3.14	126	−10.06	+13.31
2.304222	Geneva	203 1 16.27	26 35 31.18	125	−16.69	+10.39
2.304203	Florence	203 1 24.76	26 35 33.99	125	− 9.04	+11.60
2.509941	Washington	203 37 20.32	26 11 41.67	127	−13.33	+20.39
3.257362	Christiania	205 49 35.43	24 39 9.16	118	−11.51	+21.07
3.270639	Kremsmünster	205 51 58.36	24 37 15.00	132	−11.16	+10.48
3.272642	Vienna	205 52 16.67	+24 36 58.69	130	−14.36	+ 9.83

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δα	Δδ
Oct. 3.284526	Bonn	205° 54′ 28″.85	+24° 35′ 30″.35	129	− 9″.85	+14″.36
3.290934	Geneva	205 55 35.48	24 34 31.04	133	−12.08	+ 5.19
3.308712	"	205 58 49.85	24 32 15.86	130	− 8.82	+ 9.33
3.530868	Washington	206 38 46.64	24 2 50.88	131	− 7.97	+11.12
4.224497	Berlin	208 44 54.47	22 25 47.75	134	− 7.10	+13.51
4.243681	Vienna	208 48 26.32	22 23 13.59	134	− 6.14	+27.13
4.251518	Liverpool	208 49 55.98	22 21 51.55	138	− 2.64	+13.74
4.258040	Vienna	208 50 59.54	22 20 51.68	135	−10.81	+11.04
4.262837	Kremsmünster	208 51 55.80	22 20 11.16	134	− 7.31	+12.59
4.265508	Liverpool	208 52 31.20	22 19 47.54	138	− 1.30	+12.42
4.279468	"	208 55 1.54	22 17 46.22	138	− 4.51	+13.71
4.283145	Göttingen	208 55 45.53	22 17 23.01	136	− 0.99	+22.82
4.287557	Leyden	208 56 37.45	22 16 33.54	136	+ 2.39	+12.16
4.299512	Durham	208 58 50.63		136	+ 4.00	
4.303897	Geneva	208 59 18.72	22 14 4.14	134	−16.18	+ 6.66
4.308288	Durham		22 13 32.19	136		+13.42
4.316911	"	209 2 0.96		136	+ 2.78	
4.334335	Markree	209 5 7.55	22 9 33.66	136	− 2.52	+ 4.91
5.204358	Berlin	211 46 12.01	19 55 14.35	138	+ 1.71	+ 4.58
5.204722	Vienna	211 46 5.12	19 55 21.54	138	− 9.23	+15.30
5.205456	Pulkova	211 46 13.80	19 55 12.84	138	− 8.74	+13.68
5.217774	Königsberg	211 48 32.30	19 53 12.25	138	− 7.91	+12.39
5.218247	Pulkova	211 48 39.50	19 53 7.27	138	− 6.00	+12.01
5.219959	Vienna	211 48 59.87	19 52 49.10	138	− 4.78	+10.46
5.234935	Göttingen	211 51 42.14	19 50 27.26	138	− 9.90	+13.85
5.249079	Breslau	211 54 28.07	19 47 58.98	138	− 2.12	+ 2.97
5.253226	Geneva	211 55 4.97	19 47 24.09	138	−11.59	+13.40
5.254982	Christiania	211 55 26.91	19 47 14.03	138	− 9.27	+15.42
5.275980	Bonn	211 59 23.37	19 43 47.78	138	− 7.65	+13.59
5.285601	Greenwich	212 1 9.15	19 42 12.83		− 9.49	+12.44
5.285703	Markree	212 1 17.04	19 42 15.95	138	− 2.75	+16.56
5.291148	Armagh	212 2 17.13	19 41 13.71	138	− 3.60	+ 7.46
5.299001	Geneva	212 3 38.23	19 40 2.00	139	−10.33	+12.40
5.302480	Cambridge, Eng.	212 4 21.40	19 39 23.24	138	− 6.07	+ 7.63
5.313001	Durham	212 6 20.02		139	− 5.20	
5.324932	"		19 35 48.01	139		+12.00
5.506095	Washington	212 42 19.31	19 5 58.72	140	−10.13	+11.89
5.513707	Ann Arbor	212 43 46.90	19 4 42.00	137	− 7.94	+11.00
6.223085	Berlin	214 56 56.76	17 3 5.99	143	− 4.30	+12.95
6.243362	Christiania	215 0 43.33	16 59 31.04	143	− 6.57	+13.15
6.245121	Breslau	215 1 0.08	16 59 5.37	143	− 9.68	+ 6.18
6.259660	Göttingen	215 3 45.60	16 56 40.65	143	− 8.24	+15.76
6.268458	Geneva	215 5 21.56	16 55 2.18	142	−11.60	+11.08
6.274580	"	215 6 31.88	16 53 58.80	141	−10.39	+12.86
6.276532	Göttingen	215 6 52.90		144	−11.40	
6.281182	Copenhagen	215 8 6.09	16 52 47.79		+ 9.30	+12.14
6.283378	Leyden	215 8 13.91		143	− 7.67	
6.292062	Kremsmünster	215 9 48.62	16 50 49.68	144	−11.01	+10.00
6.295746	Paris	215 10 30.22	16 50 16.69	142	−10.98	+16.28
7.223698	Berlin	218 5 29.88	13 59 20.99	148	− 4.97	+ 8.83
7.233783	Vienna	218 7 27.10	13 57 27.63	149	− 1.74	+10.48
7.243249	Kremsmünster	218 9 8.42	13 55 40.48	148	− 7.50	+11.42
7.245732	Vienna	218 9 37.35	13 55 12.90	149	− 6.61	+12.24
7.246070	Breslau	218 9 31.48	13 55 2.98	147–9	−16.35	+ 6.16
7.262217	Kremsmünster	218 12 44.56	+13 52 3.43	148	− 5.82	+10.75

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δ α	Δ δ
Oct. 7.283573	Geneva	218° 16′ 46″.97	+13° 47′ 59″.50	146	− 4″.87	+11″.53
7.285147	"	218 16 58.14	13 47 42.69	151	−11.50	+12.74
7.291893	Göttingen	218 18 16.98	13 46 28.98	148	− 8.93	+16.28
7.301455	Florence	218 20 4.61	13 44 24.33	146	− 9.40	+ 1.19
7.307955	Durham	218 21 19.72	13 43 39.05	146	− 7.77	+30.43
7.319152	Markree	218 23 26.57	13 41 6.23	151	− 7.51	+ 6.05
7.326498	Armagh	218 24 50.37	13 39 49.65	151	− 6.74	+13.78
8.228454	Breslau	221 14 27.66	10 42 54.76	152–3	− 6.84	+25.18
8.231321	Kremsmünster	221 15 4.35	10 42 8.57	152	− 2.40	+13.61
8.243545	Bonn	221 17 19.32	10 39 36.69	154	− 4.85	+ 9.15
8.264194	Göttingen	221 21 14.02		155	− 2.27	
8.266683	Paris	221 21 41.47	10 35 3.07	152	− 2.80	+14.90
8.266998	Liverpool	221 21 35.40	10 34 51.86	150	−12.42	+ 7.52
8.275565	Göttingen		10 33 7.11	155		+ 6.28
8.276015	Florence	221 23 17.71	10 32 56.76	152	−11.46	+ 1.37
8.279737	Altona	221 23 58.29	10 32 12.74	154	−12.70	+ 2.31
8.279866	Leyden	221 24 12.91		152	+ 0.49	
8.294975	Liverpool	221 26 57.68	10 29 16.75	150	− 4.56	+10.59
8.296998	Armagh	221 27 26.35	10 28 51.13	152	+ 1.39	+ 9.44
8.298771	Cambridge, Eng.	221 27 34.94	10 28 30.68	154	− 9.95	+ 9.42
8.311355	Leyden		10 25 59.79	152		+11.83
8.347951	Markree	221 36 20.06	10 18 31.69	154	−37.30	+ 7.08
8.994392	Batavia	223 37 18.09	8 6 22.90		−14.23	+19.45
9.231500	Christiania	224 21 25.29	7 16 55.62	174	− 4.68	+10.06
9.233137	Pulkova	224 21 41.92	7 16 33.77	158	− 6.23	+ 8.73
9.253497	Göttingen	224 25 39.65	7 12 15.59	159	+ 5.57	+ 5.52
9.263202	Altona	224 27 9.79	7 10 16.79	159–62	−11.95	+ 8.30
9.266902	Königsberg	224 27 50.13	7 9 31.58	159	−12.66	+ 9.47
9.268608	Greenwich	224 28 17.10	7 9 10.90		− 0.68	+10.16
9.275645	Geneva	224 29 38.63	7 7 40.58	165	− 1.14	+ 8.04
9.277736	Göttingen	224 30 7.98	7 7 15.21	159	+ 5.03	+ 8.87
9.282223	Paris	224 30 48.43	7 6 17.56	156	− 4.29	+ 7.49
9.282566	Geneva	224 30 43.79	7 6 7.43	164	−12.73	+ 1.66
9.312163	Cambridge, Eng.	224 36 20.97	6 59 59.69	161	− 3.69	+ 5.21
9.495923	Washington	225 10 50.23	6 20 51.92	157	+31.34	−31.71
9.496575	Ann Arbor	225 10 26.00	6 21 20.16	160	− 0.09	+ 4.63
10.224865	Christiania	227 23 39.55	3 47 17.21	176	−20.35	+ 3.77
10.254573	Kremsmünster	227 28 53.28	3 40 59.55	166	− 6.49	+ 5.32
10.267580	Altona	227 31 12.00	3 38 15.43	166	− 9.19	+ 7.15
10.309878	Armagh	227 38 46.19	3 29 15.99	163	−14.63	+ 7.59
10.987956	Batavia	229 40 28.31	1 4 11.49		−16.69	−28.46
11.238792	Kremsmünster	230 25 8.58	0 11 16.82	169	− 4.78	+ 1.73
11.266800	Vienna	230 29 55.06	0 5 27.48	168	−14.97	+ 9.92
11.274023	Cape 1	230 31 11.64	0 3 52.07	168	−14.85	+ 6.91
11.274066	Greenwich	230 31 24.90	0 3 49.50		− 2.05	+ 3.69
11.296548	Armagh	230 35 27.26	+ 0 0 6.50	167	+ 2.42	+50.76
11.297047	Leyden	230 35 7.51	− 0 1 3.06	168	−22.62	+ 5.38
11.310513	Markree	230 37 48.17	0 3 57.06	168	− 4.36	+ 3.21
11.313423	Cambridge, Eng.	230 38 18.92	0 4 37.88	170	− 4.38	− 0.49
12.269342	Altona	233 24 24.30	3 26 22.83	171–3	+ 3.31	+ 4.99
12.270434	Cape 1	233 24 0.80	3 26 34.39	172	−31.39	+ 6.08
12.280254	Paris	233 26 8.65	3 28 33.88	175	− 4.17	+10.85
12.982945	Batavia	235 24 35.70	5 54 11.79		− 6.64	− 0.68
13.213628	Pulkova	236 2 43.19	6 40 46.99	181	−12.35	+24.10
13.255055	Geneva	236 9 39.78	− 6 49 28.88	180	− 5.28	+ 5.65

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δα	Δδ
		° ′ ″	° ′ ″		″	″
Oct. 13.277955	Liverpool	236 13 34.28	− 6 54 6.25	182	+ 3.14	+ 6.41
13.287219	Florence	236 14 0.26	6 54 37.67	180	+ 6.15	+ 3.25
13.288445	Liverpool	236 15 19.72	6 56 14.72	182	+ 5.08	+ 5.27
13.488816	Washington	236 48 8.43	7 36 41.60	177	+ 4.97	− 0.12
13.508620	Ann Arbor	236 51 16.00	7 40 41.80	179	− 1.20	− 2.08
14.234268	Geneva	238 47 40.71	10 3 39.86	186	− 4.92	+ 0.52
14.256363	"	238 51 12.38	10 8 0.59	190	− 2.61	− 1.21
14.256363	"	238 51 9.36	10 7 56.95	194	− 5.63	− 4.85
14.261041	Kremsmünster	238 51 55.57	10 8 50.99	194	− 3.73	+ 0.04
14.271791	Cape 1	238 53 35.95	10 10 50.70	183	− 5.11	+ 4.95
14.287875	"	238 56 7.05	10 13 58.57	183	− 6.17	+ 3.38
14.528756	Ann Arbor	239 34 4.89	11 0 10.88	184	+ 5.45	+ 2.62
15.239372	Geneva	241 22 54.89	13 12 46.38	197	− 6.90	+ 4.10
15.240280	Vienna	241 23 7.16	13 12 59.52	188	− 2.84	+ 0.88
15.241461	Göttingen	241 23 24.24	13 13 8.86	191	+ 3.55	+ 4.49
15.243991	Kremsmünster	241 23 37.26	13 13 39.29	197	− 6.32	+ 0.75
15.247077	Göttingen	241 24 14.36	13 14 21.93	189	+ 2.84	− 7.09
15.256351	Florence	241 25 31.52	13 15 55.77	196	− 3.90	+ 0.56
15.256407	Liverpool	241 25 34.18	13 15 57.40	213	− 1.77	− 0.45
15.259106	Geneva	241 25 59.61	13 16 21.25	185	− 0.75	+ 5.25
15.286250	Cambridge, Eng.	241 29 56.64	13 21 30.43	196	− 9.08	− 7.24
15.316533	Cape 1	241 34 25.22	13 26 51.51	193	−13.86	+ 2.14
15.499105	Washington	242 1 57.44	13 59 51.88	195	− 1.47	+20.70
15.503998	Ann Arbor	242 2 46.28	14 0 45.35	187	+ 2.56	− 0.05
16.231845	Breslau	243 49 15.80	16 9 2.62	200	+ 0.19	−47.67
16.234474	Bonn	243 49 35.41	16 8 44.87	199	− 2.88	− 2.97
16.234960	Berlin	243 49 43.56	16 8 48.85	198	+ 1.21	− 1.96
16.240516	Vienna	243 50 31.08	16 9 49.33	200	+ 0.70	− 5.59
16.242347	Göttingen	243 50 43.61	16 10 9.11	199	− 2.56	− 6.59
16.245888	Altona	243 51 20.48	16 10 28.13	199	+ 3.80	+10.62
16.257474	Florence	243 53 1.93	16 12 51.81	201	+ 5.43	−14.54
16.270196	Cambridge, Eng.	243 54 41.20	16 14 42.87	201	− 4.86	+ 4.44
16.271260	Paris	243 54 55.08	16 14 55.92	201	− 0.15	+ 2.24
16.277275	Cape 1	243 55 45.29	16 15 58.81	192	− 1.70	+ 0.82
16.287908	Armagh	243 57 24.89	16 16 35.73	201	+ 6.42	+72.43
16.288004	Cape 2	243 57 13.13	16 17 47.31	201	− 6.17	+ 1.83
16.292798	Markree	243 58 0.03	16 18 41.71	201	− 0.48	− 3.67
16.296702	Cape 1	243 58 31.73	16 19 17.80	192	− 2.38	+ 0.08
16.318776	Cape 2	244 1 33.08	16 23 0.71	201	−10.66	+ 2.06
16.501781	Washington	244 27 47.24	16 53 51.26	199	− 0.90	+ 2.28
17.224979	Vienna	246 8 24.59	18 51 37.14	203	− 0.96	+ 2.68
17.237201	"	246 10 5.33	18 53 39.33	203	− 3.73	+ 0.43
17.258608	Geneva	246 12 59.23	18 57 1.17	206	− 5.03	+ 1.73
18.000873	Batavia	247 52 5.86	20 50 46.13		−14.59	+ 9.25
18.229243	Vienna	248 21 57.50	21 24 30.24	205	− 8.58	+ 5.21
18.241802	Geneva	248 23 37.72	21 26 26.89	202	− 5.93	− 0.43
18.259592	"	248 25 58.24	21 28 54.26	209	− 3.49	+ 6.80
18.270142	Cape 1	248 27 23.57	21 30 31.30	204	+ 0.03	+ 1.92
18.285490	Cape 2	248 29 21.63	21 32 45.94	207	− 0.86	+ 1.21
18.285490	"	248 29 21.93	21 32 42.07	209	− 0.56	+ 5.08
18.285797	Cape 1	248 29 26.42	21 32 44.76	204	+ 1.54	+ 1.35
18.299314	Cape 2	248 31 6.06	21 34 45.36	207	− 3.49	+ 2.28
18.299314	"	248 31 7.26	21 34 42.86	209	− 2.29	+ 4.78
18.305006	Cape 1	248 31 53.66	21 35 35.47	204	+ 0.05	+ 1.74
18.322705	Cape 2	248 33 53.28	−21 38 9.85	207	−17.23	+ 1.34

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δ α	Δ δ
Oct. 18.322705	Cape 2	248° 33′ 54″.33	—21° 38′ 7″.77	209	—16″.18	+ 3″.42
19.002366	Batavia	250 0 7.86	23 13 49.14		— 1.78	+ 3.58
19.452976	Cambridge, U. S.	250 55 27.49	24 14 13.54	210	+ 1.62	+ 4.77
19.494892	Washington	251 0 32.11	24 19 46.55	208	+ 1.94	+ 1.80
20.454666	Cambridge, U. S.	252 53 30.05	26 20 15.44	211	+ 1.39	+ 5.44
21.264748	Cape 2	254 24 12.23	27 54 18.61	212	— 3.14	+ 2.07
21.264748	"	254 24 15.98	27 54 18.64	214	+ 0.61	+ 2.04
21.275041	Cape 1	254 25 20.30	27 55 28.28	216	— 2.70	+ 1.42
21.278415	Cape 2	254 25 43.22	27 55 49.45	212	— 1.95	+ 2.88
21.278415	"	254 25 39.32	27 55 53.33	214	— 5.85	— 1.00
21.291348	"	254 27 0.37	27 57 19.44	212	— 9.70	— 0.48
21.291348	"	254 27 5.32	27 57 24.08	214	— 4.75	— 5.12
21.291348	"	254 27 2.92	27 57 20.73	218	— 7.15	— 1.77
21.304881	Cape 1	254 28 34.31	27 58 49.18	216	— 4.56	+ 0.33
21.305007	Cape 2	254 28 34.76	27 58 46.55	212	— 4.94	+ 3.80
21.305007	"	254 28 25.41	27 58 50.29	214	—14.29	+ 0.06
21.312209	Cape 1	254 29 23.87	27 59 37.80	217	— 3.06	+ 0.69
21.331513	"	254 31 31.57	28 1 45.93	217	— 1.83	+ 1.44
21.500843	Ann Arbor	254 49 54.78	28 20 33.27	215	— 2.26	— 4.99
22.239002	Florence	256 7 51.54	29 39 14.04	223	—17.10	—35.31
22.278702	Cape 2	256 12 11.95	29 42 41.71	221	— 3.20	+ 0.20
22.278702	"	256 12 10.36	29 42 40.11	222	— 4.79	+ 1.80
22.278702	"	256 12 11.41	29 42 41.16	223	— 3.74	+ 0.75
22.284269	Cape 1	256 12 49.03	29 43 16.82	220	— 1.12	— 0.42
22.289345	Cape 2	256 13 23.57	29 43 44.51	221	+ 1.90	+ 2.94
22.289345	"	256 13 20.18	29 43 45.91	222	— 1.49	+ 1.54
22.289345	"	256 13 23.03	29 43 43.77	223	+ 1.36	+ 3.68
22.290592	Cape 1	256 13 29.05	29 43 52.60	219	— 0.36	+ 0.65
22.299591	Cape 2	256 14 22.83	29 44 47.54	221	— 2.45	+ 2.54
22.299591	"	256 14 21.09	29 44 48.28	222	— 4.19	+ 1.80
22.299591	"	256 14 18.69	29 44 48.80	223	— 6.59	+ 1.28
22.309611	"	256 15 25.94	29 45 48.96	221	— 1.50	+ 2.34
22.309611	"	256 15 25.70	29 45 49.95	222	— 1.74	+ 1.35
22.309611	"	256 15 29.45	29 45 50.73	223	+ 2.01	+ 0.57
22.312775	Cape 1	256 15 45.68	29 46 10.47	219	— 1.38	+ 0.14
22.318450	"	256 16 18.42	29 46 46.33	220	— 3.82	— 1.08
22.319024	Cape 2	256 16 17.61	29 46 45.72	221	— 8.19	+ 3.03
22.319024	"	256 16 10.47	29 46 47.92	222	—15.33	+ 0.83
22.319024	"	256 16 1.92	29 46 42.31	223	—23.88	+ 6.44
22.483971	Ann Arbor	256 33 22.80	30 3 31.72	221	+ 0.15	— 5.17
23.267205	Cape 2	257 51 40.72	31 19 14.31	224	—11.29	— 2.77
23.267205	"	257 51 53.62		226	+ 1.61	
23.280606	"	257 52 53.95	31 20 26.12	224	—16.88	+ 0.29
23.280606	"	257 52 43.45	31 20 26.39	226	— 6.38	+ 0.02
23.288695	"	257 53 48.86	31 21 9.07	224	— 9.51	+ 2.67
23.288695	"	257 53 57.11	31 21 12.89	226	— 1.26	— 0.15
23.289249	Cape 1	257 53 56.37	31 21 13.60	225	— 4.25	+ 1.24
23.295526	Cape 2	257 54 21.53	31 21 52.66	224	—16.96	— 2.76
23.295526	"	257 54 30.23	31 21 50.95	226	— 8.26	— 1.05
23.302760	"	257 55 8.13	31 22 29.71	224	—12.86	+ 0.57
23.302760	"	257 55 26.13	31 22 28.43	226	+ 5.14	+ 1.85
23.309794	Cape 1	257 55 58.76	31 23 10.07	225	— 3.52	— 0.54
23.327418	"	257 57 41.92	31 24 48.95	225	— 3.74	— 1.20
24.270223	"	259 27 32.31	32 48 36.57	227	— 1.42	+ 5.01
24.291839	"	259 29 32.03	—32 50 31.24	227	— 2.03	+ 0.85

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δ α	Δ δ
Oct. 24.297234	Cape 2	259° 29′ 49″.22	—32° 51′ 1″.49	227	—14″.85	— 1″.86
24.310027	"	259 31 11.13	32 52 4.42	227	— 4.07	+ 0.48
24.310027	"	259 31 16.53	32 52 6.29	228	+ 1.33	— 1.39
24.310359	Cape 1	259 31 13.53	32 52 5.50	227	— 3.51	+ 1.09
24.320564	Cape 2	259 31 42.71	32 53 5.54	227	—31.04	— 6.94
24.320564	"	259 31 52.61	32 53 1.05	228	—21.14	— 2.45
24.330842	"	259 32 59.55	32 54 0.14	227	—11.29	— 9.20
24.330842	"	259 33 9.30	32 53 48.45	228	— 1.54	+ 2.49
25.227393	Florence	260 53 53.91	34 6 59.10	229	—15.89	— 2.01
25.270231	Cape 1	260 57 54.27	34 10 16.66	229	— 1.94	+ 1.42
25.271213	Cape 2	260 57 57.74	34 10 19.08	229	— 3.65	+ 3.58
25.282375	"	260 58 58.75	34 11 12.24	229	— 6.54	+ 2.67
25.291788	Cape 1	260 59 46.79	34 11 57.90	229	— 3.15	+ 1.03
25.293562	Cape 2	260 59 59.88	34 12 3.38	229	+ 0.58	+ 3.85
25.304389	"	261 0 50.73	34 12 54.18	229	— 5.64	+ 3.63
25.315282	"	261 1 46.80	34 13 45.56	229	— 6.95	+ 3.09
25.318415	Cape 1	261 2 5.27	34 14 1.35	230	— 4.99	+ 1.91
26.278599	Cape 2	262 24 18.84	35 25 45.54	232	— 1.14	— 7.08
26.284208	Cape 1	262 24 45.07	35 26 1.73	231	— 3.00	+ 0.82
26.293092	Cape 2	262 25 33.24	35 26 38.90	231	+ 0.71	+ 1.72
26.302328	Cape 1	262 26 16.43	35 27 19.84	231	— 2.28	+ 0.33
26.314169	Cape 2	262 27 11.32	35 28 10.74	231	— 6.59	+ 0.09
26.326702	"	262 28 10.19	35 28 58.26	231	— 5.34	+ 1.76
27.280555	Cape 1	263 45 43.72		234	— 5.26	
27.284622	Cape 2	263 45 51.16	36 34 26.62	233	—17.17	+ 6.40
27.290788	Cape 1	263 46 31.03	36 34 55.06	234	— 6.50	+ 2.22
27.302058	Cape 2	263 47 16.51	36 35 37.46	233	—14.68	+ 4.15
27.309387	Cape 1	263 47 59.50	36 36 7.06	234	— 6.51	+ 3.35
27.317984	Cape 2	263 48 44.14	36 36 36.09	233	— 2.71	+ 8.08
28.281355	"	265 2 58.83	37 37 9.95	236	—12.43	+ 7.24
28.291188	Cape 2	265 3 51.38	37 37 48.73	236	— 4.37	+ 4.07
28.296537	Cape 1		37 38 9.03	236		+ 3.13
28.317088	"	265 5 48.72	37 39 21.58	235	— 4.09	+ 4.86
28.326112	"	265 6 30.33	37 39 57.60	236	— 3.24	+ 1.42
29.272940	"	266 16 5.94	38 34 46.18	237	— 5.26	+ 0.76
29.280063	Cape 2	266 16 37.16	38 35 8.57	237	— 4.75	+ 2.16
29.291846	Cape 1	266 17 28.09	38 35 49.21	237	— 4.61	+ 0.85
29.291909	Cape 2	266 17 24.19	38 35 49.05	237	— 8.79	+ 1.22
29.304695	"	266 18 16.89	38 36 32.27	237	—11.16	+ 0.63
29.307344	Cape 1	266 18 35.52		237	— 3.93	
29.331186	"	266 20 15.91	38 37 59.43	238	— 6.15	+ 1.65
30.301428	Cape 2	267 28 16.21	39 29 46.29	239	— 4.66	— 0.52
30.311740	Cape 1	267 28 57.38	39 30 14.23	240	— 5.90	+ 3.28
30.318273	Cape 2	267 29 22.79	39 30 30.26	240	— 7.29	+ 7.34
30.328224	Cape 1	267 30 5.90	39 31 6.00	240	— 5.04	+ 2.19
30.530435	Santiago	267 43 57.03	39 41 20.96	240	— 0.09	+ 3.60
31.295670	Cape 1	268 34 49.84	40 18 43.51	242	+ 0.67	+ 2.96
31.298280	Cape 2	268 34 53.81	40 18 50.34	241	—12.70	+ 8.71
31.306177	Cape 1	268 35 53.96	40 19 35.56	242	— 6.42	+ 2.56
31.319299	Cape 2	268 36 17.65	40 19 54.19	241	—11.41	+ 4.72
31.328081	Cape 1	268 36 56.93	40 20 20.59	242	— 6.60	+ 3.30
31.551379	Santiago	268 51 36.28	40 30 49.08	241	+ 0.85	+ 4.52
31.568757	"	268 52 32.10		241	—10.74	
Nov. 2.369050	Cape 2	270 44 42.82	—41 50 6.22	243	— 7.41	+ 0.79
2.516770	Santiago	270 53 37.97		244	— 2.09	

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δα	Δδ
Nov. 3.277366	Cape 1	271° 38′ 12″.72	—42° 25′ 55″.35	247	— 6″.45	+ 2″.12
3.278903	Cape 2	271 38 19.65	42 25 58.20	248	— 4.84	+ 2.80
3.293061	"	271 39 11.22	42 26 30.51	248	— 2.38	+ 2.93
3.309289	Cape 1	271 40 0.63	42 27 7.70	246	— 9.23	+ 2.87
3.317423	"		42 27 24.68	246		+ 4.50
3.536300	Santiago	271 53 6.60	42 35 42.91	251	— 6.38	+ 2.80
3.624757	"	271 58 11.93	42 38 59.81	251	— 4.32	+ 4.40
3.630955	"	271 58 26.58		253	—10.33	
4.276278	Cape 2	272 34 43.01	43 2 49.28	252	—15.65	+ 0.53
4.288704	Cape 1	272 35 31.25	43 3 12.07	250	— 8.89	+ 4.31
4.298756	Cape 2	272 36 7.70	43 3 37.54	252	— 5.98	+ 0.32
4.303762	Cape 1	272 36 22.17	43 3 43.33	250	— 8.22	+ 5.22
4.535387	Santiago	272 49 6.15	43 11 55.03	245	—13.43	+ 4.29
5.284601	Cape 2	273 29 48.34	43 37 27.19	254	—14.84	+ 8.22
5.309511	"	273 31 5.05	43 38 16.31	254	—18.25	+ 8.87
5.542071	Santiago	273 43 38.20	43 46 3.74	249	— 9.54	+ 2.16
6.064230	Batavia	274 11 8.70	44 2 27.45	254	—14.63	+31.54
6.284801	Cape 2	274 22 42.76	44 9 49.79	256	— 2.12	+ 1.59
6.290045	Cape 1	274 22 50.80	44 9 57.28	255	—10.37	+ 3.92
6.304504	Cape 2	274 23 38.85	44 10 30.04	256	— 7.22	— 1.77
6.305375	Cape 1	274 23 37.36	44 10 25.97	255	—11.42	+ 3.93
6.532629	Santiago	274 35 19.30	44 17 27.91	254	—12.16	+ 4.14
7.280508	Cape 2	275 13 15.71	44 39 53.10	257	—10.70	+ 6.01
7.292731	"	275 13 57.76	44 40 14.11	257	— 2.37	+ 6.49
7.303347	"	275 14 19.35	44 40 33.40	257	—15.66	+ 5.83
7.309731	Cape 1	275 14 42.67	44 40 44.70	257	—11.50	+ 5.77
7.344515	"	275 16 28.11		257	—10.70	
9.323115	Cape 2	276 51 59.05	45 36 11.76	258	—29.63	+ 5.23
9.335328	Cape 1	276 52 32.98	45 36 31.33	258	—30.19	+ 4.69
9.335882	Cape 2	276 52 51.74	45 36 26.63	258	—12.93	+10.22
9.352831	Cape 1	276 53 38.95		258	—13.47	
9.518349	Santiago	277 1 19.81	45 41 16.09	258	—17.66	+ 2.74
9.543581	"	277 2 33.13	45 41 56.89	258	—15.04	+ 0.70
11.282643	Cape 1	278 21 46.97	46 24 14.60	259	—13.47	+ 3.24
11.297551	"	278 22 26.77	46 24 34.49	259	—13.42	+ 4.06
11.314855	"	278 23 12.24		259	—14.09	
11.538432	Santiago	278 33 5.98	46 30 10.01	261	—14.46	+ 0.97
12.285219	Cape 1	279 5 46.91	46 46 51.73	260	—12.96	+ 2.35
12.301348	Cape 2	279 6 37.03	46 47 11.97	262	— 4.75	+ 3.34
12.304447	Cape 1	279 6 35.60	46 47 16.71	260	—14.23	+ 2.68
12.324950	Cape 2	279 7 23.58	46 47 40.41	262	—19.49	+ 5.92
14.318331	Cape 1	280 31 36.40	47 29 6.64	263	—17.17	+ 4.81
14.329283	"	280 32 4.18	47 29 20.97	263	—16.49	+ 3.44
14.522421	Santiago	280 40 14.47	47 33 8.22	267	—20.02	+ 3.53
14.532038	"	280 40 19.50		263	—21.55	
15.295493	Cape 1	281 11 29.82	47 47 55.49	265	—15.34	+ 4.58
15.298458	Cape 2	281 11 39.45	47 48 0.71	263	—12.89	+ 2.60
15.313592	"	281 12 9.87	47 48 14.55	266	—19.11	+ 5.81
15.323250	Cape 1	281 12 35.12	47 48 26.81	265	—17.24	+ 4.42
15.534608	Santiago	281 20 53.70	47 52 21.16	264	—28.95	+ 6.74
16.327195	Cape	281 52 37.43	48 6 48.25	268	—18.66	+ 5.70
16.335931	"		48 6 57.27	268		+ 6.05
17.301841	"	282 30 47.62	48 23 49.49	269	—15.39	+ 5.35
17.338512	"	282 32 11.74		269	—16.47	
17.542119	Santiago	282 39 55.49	—48 27 57.07	269	—24.64	+ 2.37

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δ α	Δ δ
Nov. 18.318259	Cape	283° 9′ 44″.52	−48° 40′ 46″.30	271	−18″.12	+ 4″.96
18.545262	Santiago	283 18 3.81	48 44 21.34	271	−35.40	+ 9.56
19.306291	Cape	283 47 1.08	48 56 29.14	272	−14.78	+ 6.31
19.324727	"	283 47 43.46	48 56 45.77	272	−13.71	+ 6.91
19.342447	"	283 48 22.28		272	−13.98	
19.526554	Santiago	283 55 16.12		272	−12.36	
19.537712	"	283 55 33.25	49 0 6.19	270	−20.14	+ 4.49
20.298313	Cape	284 23 44.33	49 11 39.33	273	−16.03	+ 3.67
20.312796	"	284 24 14.57	49 11 51.54	273	−17.71	+ 4.42
20.327914	"	284 24 50.91		273	−14.69	
20.553316	Santiago	284 32 58.99	49 15 25.53	273	−22.99	+ 4.57
21.310521	Cape	285 0 38.45	49 26 24.96	274	−15.20	+ 5.20
21.325433	"	285 1 9.24	49 26 38.00	274	−16.76	+ 4.96
21.342125	"	285 1 45.37		274	−16.83	
22.337353	"	285 37 22.65	49 40 44.27	275	−21.80	+ 8.54
22.354590	"	285 38 2.02		275	−19.26	
22.543740	Santiago	285 44 41.29	49 43 38.53	277	−23.56	+ 3.35
24.536028	"	286 54 22.66	50 9 38.78	276	−30.78	+ 6.39
26.31198	Cape	287 55 18.16	50 31 18.06	278	−22.57	+ 4.39
26.33066	"	287 55 54.92	50 31 29.58	278	−23.80	+ 6.08
27.30696	"	288 28 48.68	50 42 45.26	279	−23.98	+ 8.32
27.33047	"	288 29 33.10		279	−26.84	
27.54048	Santiago	288 36 27.31	50 45 34.04	282	−34.51	− 1.72
29.31176	Cape	289 35 21.13	51 4 49.73	280	−24.75	+ 6.31
29.33565	"	289 36 5.80		280	−27.23	
30.55715	Santiago	290 16 4.11		281	−25.75	
Dec. 2.31849	Cape	291 13 3.71	51 35 19.27	283	−20.32	+ 5.70
2.34125	"	291 13 45.93		283	−21.91	
2.54202	Santiago	291 20 2.94	51 37 26.70	283	−31.14	+ 7.61
3.31831	Cape	291 44 55.07	51 44 51.74	284	−27.05	+ 5.31
3.32911	"	291 45 16.05	51 44 59.38	284	−24.77	+ 3.18
3.34460	"	291 45 44.62	51 45 8.83	284	−27.72	+ 3.03
3.53935	Santiago	291 51 52.46	51 46 39.08	284	−31.91	+22.20
3.55992	"	291 52 26.16		285	−37.45	
3.56003	"	291 52 31.83	51 47 0.59	286	−31.99	+12.27
4.31389	Cape	292 16 29.01	51 54 3.78	285	−29.96	+ 6.83
4.33605	"	292 17 14.40	51 54 16.34	285	−26.67	+ 6.42
4.54287	Santiago	292 23 40.65	51 56 9.68	286	−32.91	+ 6.07
5.31285	Cape	292 48 5.50	52 3 6.04	287	−24.65	+ 4.85
5.33431	"	292 48 48.04	52 3 15.81	287	−22.62	+ 6.54
6.30326	"	293 19 5.90	52 11 49.21	288	−28.04	+ 3.25
6.30754	"	293 19 16.67	52 11 47.18	289	−25.30	+ 7.50
6.32438	"	293 19 43.38	52 11 58.79	288	−32.07	+ 5.17
6.32800	"	293 19 55.45	52 12 0.96	289	−24.86	+ 4.35
6.34854	"	293 20 33.79		288	−25.11	
6.35494	"	293 20 44.35		289	−26.57	
6.55325	Santiago	293 26 48.31	52 13 50.46	289	−33.43	+11.52
6.57384	"		52 14 7.61	289		+ 5.00
7.55036	"	293 57 46.77	52 22 21.15	290	−39.39	+ 9.40
8.30847	Cape	294 21 25.76	52 28 45.08	291	−30.71	+ 3.81
8.33255	"	294 22 5.14		291	−36.04	
8.55917	Santiago	294 28 59.09	52 30 48.53	292	−42.62	+ 4.01
9.55143	"	294 59 40.93	52 38 54.12	292	−36.59	+ 0.57
10.31458	Cape	295 23 9.19	52 44 56.24	294	−34.61	+ 1.95
10.32712	"	295 23 34.77	−52 45 2.94	294	−32.09	+ 1.17

Paris M. T. of Observation. 1858.	Place of Observation.	α	δ	Number of Comp. Star.	Δ α	Δ δ
Dec. 10.34029	Cape	295° 23′ 59″.59	−52° 45′ 8″.07	294	−31″.50	+ 2″.26
10.35130	"	295 24 16.74		294	−34.60	
10.52539	Santiago	295 30 41.38	52 46 48.94	293	−39.31	+ 6.28
11.31853	Cape	295 54 1.61	52 52 45.94	294	−25.15	+ 1.23
11.34878	"	295 54 53.01		294	−29.17	
11.55938	Santiago	296 1 4.80	52 54 45.46	300	−42.99	− 7.29
12.30595	Cape	296 24 3.24	53 0 11.88	296	−28.91	+ 6.88
12.31831	"	296 24 26.16	53 0 20.06	296	−28.55	+ 4.30
13.55581	Santiago	297 2 23.83	53 9 38.42	295	−22.23	+ 3.52
14.31744	Cape	297 24 59.08	53 15 8.23	298	−30.73	+ 3.29
14.32767	"	297 25 13.25	53 15 11.01	297	−35.11	+ 4.83
14.35668	"	297 26 9.74		297	−31.17	
14.36330	"	297 26 22.17		298	−30.75	
15.55895	Santiago	298 2 7.26	53 24 2.99	297	−48.06	+ 2.54
16.56203	"	298 32 19.10	53 31 4.23	299	−44.47	+ 4.32
19.32976	Cape	299 55 11.85	53 49 59.06	301	−37.07	+ 1.23
19.34409	"	299 55 35.13		301	−39.42	
20.55650	Santiago	300 31 24.36	53 58 6.62	302	−55.79	+ 0.19
21.31330	Cape	300 54 16.50	54 3 0.84	302	−32.96	+ 1.97
21.32952	"	300 54 45.80	54 3 7.25	302	−32.56	+ 1.87
21.56149	Santiago	301 1 16.78	54 4 41.22	302	−54.82	− 1.99
22.31768	Cape	301 24 3.54	54 9 30.07	303	−34.06	+ 1.13
22.33961	"	301 24 44.02	54 9 38.56	303	−32.59	+ 0.95
22.56760	Santiago	301 31 19.79	54 11 13.49	303	−40.57	− 6.57
23.32459	Cape	301 53 58.69	54 15 53.71	304	−28.92	+ 1.73
23.34512	"	301 54 35.10	54 16 2.22	304	−28.19	+ 1.02
24.32131	"	302 23 22.88	54 22 6.89	305	−34.35	+ 4.53
24.34167	"	302 24 1.64	54 22 13.51	305	−31.72	+ 5.55
27.32038	"	303 51 51.23	54 40 36.95	308	−40.07	+ 2.01
27.34108	"	303 52 25.23	54 40 44.18	308	−42.71	+ 2.31
27.57012	Santiago	303 59 16.34	54 42 20.15	306–7	−36.92	−10.40
28.32310	Cape	304 21 29.39	54 46 41.24	309	−36.00	+ 1.08
28.34646	"	304 22 11.90		309	−34.79	
28.57124	Santiago	304 28 48.20	54 48 9.68	309	−36.09	+ 2.09
29.31586	Cape	304 50 41.74	54 52 35.74	311	−39.23	+ 3.40
29.33171	"	304 51 16.86		311	−32.12	
29.55979	Santiago	304 57 49.17	54 54 22.27	312–3	−43.08	−15.87
30.33436	Cape	305 20 43.20	54 58 39.86	312	−38.14	+ 2.62
30.34829	"	305 21 3.13	54 58 45.87	312	−42.86	+ 1.57
30.35422	"		54 58 47.25	312		+ 2.29
30.55880	Santiago	305 26 54.52	55 0 5.65	312	−63.50	− 3.44
30.57658	"		54 59 57.69	312		+10.83
31.32622	Cape	305 49 58.84	55 4 35.18	313	−37.07	− 0.88
31.34421	"	305 50 28.26	55 4 39.50	313	−37.67	+ 0.80
31.56071	Santiago	305 56 41.16	55 5 49.08	312–10	−47.31	+ 7.64
1859.						
Jan. 1.33308	Cape	306 19 32.34	55 10 24.57	314	−40.72	+ 4.03
1.34183	"		55 10 32.06	314		− 0.38
2.55729	Santiago	306 55 32.95	55 17 58.57	315	−42.76	−21.28
3.56307	"	307 25 5.29	55 23 12.00	316	−47.20	+15.74
3.58347	"		55 23 47.17	316		−12.34
4.32549	Cape	307 47 42.29	55 27 51.49	317	−36.56	+ 1.00
4.33941	"	307 48 0.56	55 27 58.46	317	−43.42	− 1.14
5.32749	"	308 17 8.43	−55 33 41.49	318	−41.41	− 2.06

Paris M. T. of Observation. 1859.	Place of Observation.	α	δ	Number of Comp. Star.	Δ α	Δ δ
Jan. 5.33642	Cape		—55° 33′ 43.″59	318		— 1.″07
5.56881	Santiago	308° 24′ 10.″86	55 35 19.42	317	—45.″28	—16.58
5.59200	"		55 34 58.82	317		+12.04
6.32873	Cape	308 46 36.88	55 39 23.62	319	—42.15	+ 1.59
6.34175	"	308 47 3.12	55 39 29.44	319	—38.92	+ 0.26
7.56549	Santiago	309 23 0.93	55 46 34.02	320	—44.16	— 2.62
8.56182	"	309 52 19.96	55 52 38.89	321	—46.78	—24.67
10.56311	"	310 51 23.65	56 4 8.39	322	—43.70	—26.07
11.32404	Cape	311 13 47.59	56 8 3.68	323	—46.77	+ 0.31
11.33871	"	311 14 17.25	56 8 7.51	323	—43.11	+ 1.53
11.34569	"		56 8 10.78	323		+ 0.66
11.56671	Santiago	311 20 47.05	56 9 46.64	322–4	—56.99	—18.84
12.33199	Cape		56 13 49.44	325		+ 1.38
13.33141	"	312 13 4.69	56 19 35.81	326	—45.95	— 0.68
13.34791	"	312 13 44.50	56 19 42.70	326	—35.49	— 1.68
20.31958	"	315 40 16.81	57 0 9.91	328	—38.18	— 3.34
20.56512	Santiago	315 47 24.00		327	—49.91	
21.31852	Cape	316 9 55.20	57 6 1.66	329	—43.44	— 2.39
21.32957	"	316 10 12.30	57 6 6.83	329	—46.10	— 3.65
21.33482	"		57 6 8.27	329		— 3.23
21.57642	Santiago	316 17 45.04	57 8 5.68	327	—34.39	—35.18
22.57076	"	316 47 11.78	57 13 8.83	330	—46.21	+15.02
24.32387	Cape	317 39 31.81	57 23 56.51	332	—47.94	— 5.21
24.33599	"	317 39 58.63	57 24 0.42	332	—42.92	— 4.76
24.35405	"	317 40 24.57	57 24 13.69	332	—49.38	— 8.39
24.56212	Santiago	317 46 42.79	57 25 41.62	332	—44.61	—24.56
24.58272	"		57 25 23.07	334		+ 1.41
25.33812	Cape	318 9 58.12	57 30 2.55	333	—43.18	— 5.29
25.57425	Santiago	318 16 45.58		334	—60.23	
26.57324	"	318 46 52.91		334	—50.63	
28.32678	Cape	319 39 44.87	57 48 16.15	335	—41.70	— 6.20
28.34785	"	319 40 22.93	57 48 22.77	335	—40.63	— 5.03
28.57594	Santiago	319 47 6.75		334	—50.06	
29.32330	Cape	320 9 42.05	57 54 24.91	337	—46.43	— 5.37
29.33166	"	320 10 0.52	57 54 26.33	337	—43.07	— 3.68
29.55787	Santiago	320 16 30.68	57 55 15.35	338	—62.44	+31.59
29.57943	"		57 56 15.51	336		—20.53
30.32274	Cape	320 39 50.59	58 0 37.29	339	—48.44	— 4.22
30.33277	"	320 40 10.98	58 0 43.76	339	—46.19	— 6.93
31.33586	"	321 10 33.11	58 7 1.22	341	—44.81	— 6.40
31.57392	Santiago	321 17 53.76	58 8 42.23	343–0	—36.78	—17.24
Feb. 1.57647	"	321 48 9.28		345	—45.36	
1.57647	"	321 48 16.62		343	—38.02	
2.32352	Cape	322 10 51.71	58 19 42.48	342	—44.52	— 9.14
2.33020	"	322 11 5.56	58 19 46.24	342	—42.84	—10.37
2.58642	Santiago	322 18 45.70	58 21 28.47	345–8	—50.20	—13.82
2.58642	"	322 18 48.15	58 21 37.72	344	—47.75	—23.07
3.31838	Cape	322 41 10.40	58 26 5.90	344	—42.29	— 7.89
3.32531	"	322 41 20.64	58 26 8.90	344	—44.73	— 8.19
3.58027	Santiago	322 48 55.33	58 27 50.85	346–7	—21.99	—10.99
4.31867	Cape	323 11 35.16	58 32 36.46	346	—47.76	— 8.12
4.32665	"	323 11 48.53	58 32 39.42	346	—49.00	— 7.96
4.56046	Santiago	323 18 56.08		346	—49.82	
5.32129	Cape	323 42 16.59	58 39 12.94	349	—44.69	— 9.73
5.33415	"	323 42 44.72	—58 39 20.67	349	—40.20	—12.37

Paris M. T. of Observation. 1859.	Place of Observation.	α	δ	Number of Comp. Star.	Δα	Δδ
Feb. 5.55918	Santiago	323° 49′ 20″.06	—58° 41′ 2″.20	350–45	—57″.99	—24″.75
7.56469	"	324 50 58.45		351	—50.91	
8.32536	Cape	325 14 25.25	58 59 18.46	351	—48.58	— 9.06
21.32089	"	332 1 46.90	60 33 49.94	352	—47.86	— 9.99
21.32628	"	332 1 58.20	60 33 54.71	352	—46.92	—12.23
21.54674	Santiago	332 8 57.59	60 35 35.05	356–3	—50.79	— 9.39
22.32145	Cape	332 33 54.08	60 41 44.39	354	—44.30	—14.10
22.54558	Santiago	332 40 56.92	60 43 7.19	356–5	—53.32	— 9.13
23.54858	"	333 13 31.86		353	—35.53	
24.54393	"	333 45 32.03	60 59 13.05	353	—44.85	+ 0.85
25.31888	Cape	334 10 43.88	61 5 46.15	357	—40.53	—15.09
25.32777	"		61 5 50.04	357		—14.63
26.30820	"	334 42 49.77	61 13 52.40	358	—45.94	—15.06
26.31610	"	334 43 6.12	61 13 56.77	358	—45.04	—15.52
27.31914	"	335 15 56.44	61 22 17.03	359	—40.83	—17.15
28.32881	"	335 48 59.67	61 30 48.03	360	—45.06	—20.45
Mar. 1.30403	"	336 21 4.81	61 39 5.74	361	—47.70	—22.22
1.31416	"	336 21 25.32	61 39 5.96	361	—47.26	—17.26
1.53531	Santiago	336 28 34.91		360	—54.24	
2.30038	Cape	336 54 6.52	61 47 32.30	362	—44.02	—16.36
2.30915	"	336 54 26.91	61 47 35.01	362	—41.08	—14.53
4.31626	"	338 1 22.54	—62 5 13.02	363	—37.19	—22.37

In the next place we proceed to the computation of the perturbations produced by the five large planets, from Venus to Saturn inclusive. The perturbations by Mercury were neglected, as, from the rapid motion of this planet, the intervals of time in the computation of the disturbing forces would require much reduction, with consequent increase of labor, while a rough estimate of the change produced in the comet's geocentric place showed it could not at any time much exceed 0″.1. To render the integration possible it was necessary to adopt different intervals of time in the calculation of the disturbing force in different parts of the orbit; the near approach of the comet to Venus, in October, required them to be made as short as one day. The unit of time for the forces given below is however uniformly the same, being ten days. The unit of length is a unit in the seventh decimal place. The forces and perturbations belong to the usual system of rectangular equatorial co-ordinates; and the constants in the integration have been so taken, that the perturbations are the deviations of the comet from its osculating orbit of Oct. 2.

Washington Mean Noon. 1858.	X	Y	Z	δx	δy	δz
May 30	+ 2.282	— 4.081	— 2.414	+112.76	+ 59.52	—136.88
June 9	2.812	3.449	2.334	85.88	63.08	115.11
19	3.199	2.776	2.256	62.38	62.78	95.60
29	3.376	2.054	2.163	42.59	59.27	78.27
July 9	3.320	1.282	2.013	26.64	53.25	63.03
19	3.099	— 0.474	1.813	14.44	45.47	49.72
29	2.756	+ 0.333	1.613	5.76	36.72	38.15
Aug. 8	2.286	1.077	1.434	+ 0.23	27.74	28.12
18	1.697	1.712	1.295	— 2.58	19.26	18.99
28	0.998	2.180	1.210	3.23	11.88	11.91
Sept. 2	0.604	2.346	1.192	2.94	8.74	8.91
7	+ 0.195	2.420	1.197	2.38	6.04	6.29
12	— 0.200	2.432	1.235	1.69	3.82	4.11
17	0.564	2.340	1.326	0.99	2.11	2.38
22	0.831	2.148	1.534	0.43	0.91	1.11
27	0.898	1.914	1.929	0.10	0.22	0.30
Oct. 2	0.635	1.698	2.668	0.00	0.00	0.00
7	— 0.062	1.486	4.166	0.06	0.21	0.38
8	+ 0.123	1.346	4.829	0.08	0.30	0.56
9	0.322	1.149	5.643	0.10	0.40	0.79
10	0.621	0.883	6.837	0.12	0.51	1.08
11	1.163	0.561	8.706	0.14	0.63	1.43
12	2.122	0.264	11.645	0.14	0.76	1.88
13	3.894	0.286	16.416	0.12	0.89	2.43
14	7.326	1.587	24.376	— 0.07	1.02	3.16
15	14.196	7.291	37.424	+ 0.06	1.17	4.12
16	26.874	26.084	54.274	0.34	1.40	5.47
17	39.597	65.057	54.093	0.88	1.91	7.33
18	31.127	83.863	—16.538	1.80	3.06	9.70
19	12.813	60.792	+10.765	3.04	5.00	12.23
20	+ 2.934	36.632	15.493	4.41	7.54	14.67
21	— 1.001	22.390	13.417	5.82	10.45	16.95
22	2.572	14.485	10.690	7.23	13.58	19.08
27	3.790	2.506	3.916	14.00	30.47	28.37
Nov. 1	3.829	+ 0.055	2.246	20.22	47.65	36.45
6	3.789	— 1.173	1.496	25.87	64.25	43.91
16	3.605	2.718	0.744	35.42	94.40	58.03
26	3.216	3.816	0.312	42.52	119.44	72.07
Dec. 6	2.684	4.661	+ 0.029	47.26	138.61	86.62
16	2.065	5.287	— 0.158	49.93	151.42	102.02
26	1.410	5.714	0.268	51.01	157.58	118.44
1859.						
Jan. 5	0.766	5.957	0.309	51.07	156.92	135.99
15	— 0.174	6.037	0.286	50.67	149.42	154.69
25	+ 0.332	5.983	0.209	50.34	135.18	174.48
Feb. 4	+ 0.723	5.831	— 0.092	50.56	114.40	195.30
14	0.985	5.616	+ 0.053	51.67	87.37	217.00
24	1.115	5.377	0.205	53.90	54.41	239.42
Mar. 6	+ 1.117	—5.149	+ 0.353	57.35	+ 15.84	262.41
16				+ 61.99	— 28.03	—285.79

In forming the normals, the following system of weights was used; the weight being given, not to each observation as published by the observer, but to the result of all

the observing in a single night with one comparison star, or with all the stars when they were compared with a single observation of the comet.

The Weight 4 to	The Weight 3 to	The Weight 2 to	The Weight 1 to
Ann Arbor,	Berlin,	Cambridge, Eng.,	Altona,
Bonn,	Cambridge, U. S.,	Christiania,	Armagh,
Cape 1,	Geneva,	Durham,	Batavia,
Greenwich,	Königsberg,	Santiago, Filar Microm.,	Breslau,
Kremsmünster,	Paris,	Vienna,	Copenhagen,
Liverpool,	Göttingen.	Leyden,	Florence,
Pulkova, Mer. Obs.		Pulkova, Ring Microm.,	Markree,
		Cape 2.	Padua,
			Washington,
			Santiago, Ring Microm.

An examination of the Santiago Ring Micrometer Observations shows that when the comet was observed in the northern half of the ring the resulting place is too far to the north, and when in the southern half too far to the south; which is to be explained by a personal equation in estimating the time of ingress and egress of the comet. I have endeavored to eliminate this source of error by applying a constant correction to the declinations obtained from the northern half of the ring, and the same with a contrary sign to those obtained from the southern half. A comparison of the observations gives $\mp 14''.85$ for this correction. To the right ascensions it appears necessary to add the quantity $+2''.35 \sec.\delta$: this was obtained by a comparison with the Cape observations. The normals for convenience are reduced to the nearest Washington Mean Noon, equivalent to $0^{d}.220526$ Paris Mean Time.

		App. α	App. δ	Cor. to Comp. Ephem. $\Delta\alpha$	Cor. to Comp. Ephem. $\Delta\delta$	Normal formed from Observations between
1858.	June 14	141° 24′ 27″.15	+25° 4′ 45″.26	− 2″.98	− 5″.78	June 7 — June 19
	July 13	144 32 38.66	27 47 54.78	− 2.23	+ 0.62	June 28 — July 31
	Aug. 11	151 16 58.36	30 57 14.37	− 4.89	+ 5.48	Aug. 4 — Aug. 16
	Aug. 23	155 31 25.80	32 43 18.71	− 6.25	+ 8.59	Aug. 17 — Aug. 28
	Sept. 5	162 9 37.57	34 58 27.81	− 8.42	+12.96	Aug. 30 — Sept. 11
	Sept. 17	172 46 57.63	36 27 32.59	−12.67	+14.44	Sept. 12 — Sept. 22
	Sept. 28	192 7 59.94	32 26 23.74	−12.43	+14.21	Sept. 23 — Oct. 3
	Oct. 8	221 13 0.02	+10 44 14.24	− 5.19	+ 8.99	Oct. 4 — Oct. 14
	Oct. 19	250 27 5.84	−23 43 25.28	+ 0.41	+ 0.50	Oct. 15 — Oct. 25
	Nov. 1	269 34 10.88	41 1 15.00	− 7.15	+ 3.30	Oct. 26 — Nov. 7
	Nov. 16	281 48 26.58	48 4 54.63	−16.49	+ 4.70	Nov. 9 — Nov. 22
	Dec. 1	290 37 35.66	51 24 31.43	−25.62	+ 5.38	Nov. 24 — Dec. 6
	Dec. 16	298 22 14.09	53 28 43.21	−34.40	+ 2.16	Dec. 8 — Dec. 24
1859.	Jan. 3	307 15 6.95	55 21 28.01	−40.33	+ 0.54	Dec. 27 — Jan. 13
	Jan. 30	320 36 49.56	58 0 1.20	−44.14	− 6.47	Jan. 20 — Feb. 8
	Feb. 26	334 40 0.58	−61 13 10.20	−43.65	−16.18	Feb. 21 — Mar. 4

The following remarks must be made with regard to the composition of these normals.

June 14. The right ascension is the mean of four Berlin observations; the rest are so discordant that no confidence can be placed in them.

July 13. This normal is formed from the Berlin, Cambridge, and Ann Arbor observations, the others being rejected. The Washington observations, although more concordant at this time than they are generally, yet differ from the observations which should be considered the best, and on trial it has proved impossible to satisfy them along with the other normals.

Oct. 19. The right ascension of this normal has proved most refractory; when formed from all the material, it could not possibly be represented within 2″.5, and much experimenting showed that a curve drawn through the adjacent normals would leave this one distant from it by about that quantity. This difference seeming altogether too large to be admitted in a normal having so much weight, some means must be adopted for ameliorating it.

As a more careful scrutiny of the observations showed that those made with small telescopes, especially those made at the Cape with the small instrument, had produced this deviation, I reluctantly set them aside; and the right ascension given above is the result of the Berlin, Bonn, Cambridge, U. S., and Ann Arbor observations.

By subtracting the reductions given below we obtain the co-ordinates of the comet referred to the mean equinox and equator of 1858.0, and freed from perturbations.

	Aberration.		Reduction from 1858.0.		Perturbations.			
	$\Delta\alpha$	$\Delta\delta$	$\Delta\alpha$	$\Delta\delta$	$\Delta\alpha$	$\Delta\delta$	α 1858.0	δ 1858.0
1858. June 14	− 2″.20	− 5″.26	+31″.32	− 3″.43	−0″.86	−0″.71	141° 23′ 58″.89	+25° 4′ 54″.66
July 13	8.58	4.72	37.80	5.93	0.51	0.47	144 32 9.95	27 48 5.90
Aug. 11	13.36	5.78	43.55	9.07	0.26	0.28	151 16 28.43	30 57 29.50
Aug. 23	15.79	6.25	45.52	10.68	0.16	0.19	155 30 56.23	32 43 35.83
Sept. 5	20.37	5.51	47.12	12.86	0.10	0.12	162 9 10.92	34 58 46.30
Sept. 17	28.74	− 0.22	47.06	15.65	0.04	0.05	172 46 39.35	36 27 48.51
Sept. 28	38.30	+15.97	43.23	18.90	0.00	0.00	192 7 55.01	32 26 26.67
Oct. 8	35.41	38.05	40.05	19.27	−0.01	0.02	221 12 55.39	+10 43 55.48
Oct. 19	27.95	30.56	48.53	14.35	+0.04	0.42	250 26 45.22	−23 43 41.07
Nov. 1	22.57	15.90	60.30	8.58	0.52	1.17	269 33 32.63	41 1 21.15
Nov. 16	20.43	9.26	69.54	− 3.84	1.11	1.41	281 47 36.36	48 4 58.64
Dec. 1	21.23	6.56	76.45	+ 0.11	1.53	1.43	290 36 38.91	51 24 36.67
Dec. 16	23.27	5.43	82.19	3.83	1.79	1.38	298 21 13.38	53 28 51.09
1859. Jan. 3	26.29	5.16	88.00	8.24	1.92	1.23	307 14 3.32	55 21 40.18
Jan. 30	30.51	6.31	94.21	14.70	1.72	0.94	320 35 44.14	58 0 21.27
Feb. 26	−35.09	+ 8.86	+96.56	+20.85	+0.90	−0.61	334 38 58.21	−61 13 39.30

In forming equations of condition from these normals, it will be advantageous to use residuals from elements nearer the truth than those of Searle. The following elements, computed from three provisional normals, embracing the whole period of the comet's apparition, will serve this purpose.

$$T = 1858, \text{ Sept. } 29.971007, \text{ Paris Mean Time.}$$

$$\left.\begin{aligned} \omega &= 129^\circ\ 6'\ 39''.40, \\ \Omega &= 165\ \ 19\ \ 10.67, \\ i &= 116\ \ 58\ \ 10.87, \end{aligned}\right\} \text{Mean Equinox and Ecliptic, 1858.0.}$$

$$\begin{aligned} \varphi &= 85\ \ 2\ \ 43.72, \\ \log q &= 9.7622760, \\ \log a &= 2.18982, \\ P &= 1926^y.3. \end{aligned}$$

The places of the Sun used will be taken from Hansen and Olufsen's *Tables du Soleil*, substituting, however, the Pulkova constants of nutation and aberration. A comparison of the Greenwich observations of the Sun, for 1858–59, shows a pretty good representation of observation by these tables. And the small differences that remain may be much modified by the introduction of corrections peculiar to the observer and the instrument. And knowing the difficulty that attends the consideration of this matter, I do not propose to inquire further into it.

The following are the equations of condition that result from the above normals. The logarithms of the coefficients are given instead of the coefficients themselves, and the variations of the elements are supposed to be expressed in seconds of arc, $0^d.0001$ in δT being equivalent to $1''$, and 0.00001 in $\delta \log q$ and δe; the right-hand members are $\Delta\alpha \cos\delta$ and $\Delta\delta$.

Equations from the Right Ascensions.

$\delta \log q$	δe	δT	$\delta \omega$	δi	$\delta \Omega$		Weight.
−9.8662	+9.3875	+8.7810	−9.2176	−9.2190	+9.8566	= +1″.17	0.12
−9.8551	+9.0785	+8.7925	−9.2008	−9.3646	+9.7651	= +0.92	0.36
−9.8000	+8.5855	+8.7743	−9.1247	−9.5208	+9.6436	= +0.18	0.62
−9.7273	+8.2393	+8.7146	−9.0325	−9.5931	+9.5496	= +0.75	0.70
−9.4914	−7.3061	+8.3722	−8.7217	−9.6767	+9.3190	= +1.63	1.73
+9.1499	−8.3465	−8.8318	+8.7980	−9.7498	−8.6312	= +0.55	2.57
+9.9901	−8.1188	−9.4913	+9.4492	−9.7556	−9.6671	= +0.33	2.09
+0.3761	+9.1757	−9.7442	+9.5448	−9.4468	−9.8318	= +0.38	2.27
+0.4981	+9.5597	−9.4693	−9.1112	+7.5105	−8.7024	= +2.07	1.19
+0.4498	+9.6413	−8.7919	−9.6233	−8.8691	+9.5399	= +1.58	0.79
+0.3823	+9.6892	+7.8946	−9.6984	−9.2986	+9.6547	= +2.71	0.72
+0.3199	+9.7175	+8.3883	−9.5057	−9.4785	+9.6702	= +2.99	0.46
+0.2584	+9.7263	+8.4389	−9.6900	−9.5929	+9.6637	= +2.24	0.62
+0.1787	+9.7084	+8.4115	−9.6505	−9.6946	+9.6418	= +2.43	0.60
+0.0237	+9.6084	+8.3032	−9.5379	−9.8103	+9.5822	= +2.44	0.67
+9.7381	+9.2739	+8.1126	−9.2732	−9.8974	+9.4698	= +0.10	0.41

Equations from the Declinations.

							Weight.
$+0.4299\ \delta \log q$	$+9.8023\ \delta e$	$-8.9052\ \delta T$	$+9.9005\ \delta \omega$	$-8.6591\ \delta i$	$-9.2630\ \delta \Omega$	$= +4.''59$	0.29
$+0.4008$	$+9.6194$	-8.9529	$+9.8126$	-8.7826	-9.2068	$= +3.61$	0.47
$+0.4014$	$+9.3636$	-9.0475	$+9.7291$	-8.8411	-9.2140	$= +1.61$	0.60
$+0.4094$	$+9.1945$	-9.0967	$+9.6871$	-8.8276	-9.2258	$= +1.72$	0.70
$+0.4223$	$+8.9104$	-9.1314	$+9.6160$	-8.7090	-9.2238	$= +2.81$	1.71
$+0.4444$	$+8.5223$	-8.9832	$+9.4363$	$+7.0656$	-9.0928	$= +2.21$	2.57
$+0.5294$	$+7.9961$	$+9.2695$	-9.1089	$+9.0218$	$+9.0810$	$= +1.00$	2.04
$+0.6902$	-9.1747	$+9.9580$	-9.9980	$+8.5811$	$+9.7179$	$= +0.25$	2.24
$+0.6299$	-9.4551	$+9.8831$	-9.9774	$+8.1283$	-8.3321	$= -1.12$	1.15
$+0.4712$	-9.1778	$+9.5374$	-9.7775	$+9.3709$	-9.3505	$= -0.40$	0.79
$+0.3824$	-8.4943	$+9.2865$	-9.7031	$+9.5341$	-9.1944	$= +0.26$	0.72
$+0.3437$	$+8.7707$	$+9.1361$	-9.6986	$+9.5768$	-8.8537	$= +1.90$	0.44
$+0.3297$	$+9.1485$	$+9.0385$	-9.7200	$+9.5823$	$+7.6943$	$= +0.53$	0.62
$+0.3304$	$+9.3778$	$+8.9605$	-9.7595	$+9.5616$	$+8.9410$	$= +2.06$	0.60
$+0.3495$	$+9.6005$	$+8.8879$	-9.8280	$+9.4755$	$+9.3028$	$= +1.97$	0.60
$+0.3754$	$+9.7620$	$+8.8403$	-9.8951	$+9.2523$	$+9.4895$	$= +1.68$	0.39

The operations were carried through with logarithms of five decimal places, the want of breadth in the page has compelled the omission of the last figure in the above coefficients. The resulting normal equations are —

$+211.720\ \delta \log q$	$+6.2418\ \delta e$	$+9.9751\ \delta T$	$-16.7517\ \delta \omega$	$-0.8780\ \delta i$	$+1.5523\ \delta \Omega$	$-81.''633$	$=0$
$+\ 6.2418$	$+1.8262$	-0.9109	$-\ 0.0865$	-0.6452	$+0.4804$	$-\ 9.5672$	$=0$
$+\ 9.9751$	-0.9109	$+3.8401$	$-\ 4.1916$	$+1.0919$	$+2.3800$	$+\ 2.7880$	$=0$
$-\ 16.7517$	-0.0865	-4.1916	$+\ 7.2621$	-0.8845	-3.3057	$-\ 0.5563$	$=0$
$-\ 0.8780$	-0.6452	$+1.0919$	$-\ 0.8845$	$+3.5740$	$+0.0632$	$+\ 5.3321$	$=0$
$+\ 1.5523$	$+0.4804$	$+2.3800$	$-\ 3.3057$	$+0.0632$	$+3.6073$	$-\ 2.6113$	$=0$

The solution of these gives, —

$$\delta \log q = +0.''44, \quad \delta e = +2.''99, \quad \delta T = -0.''36, \quad \delta \omega = +1.''81, \quad \delta i = -0.''32, \quad \delta \Omega = +2.''04.$$

And the sum of the squares of the residuals is reduced from 87.378 to 13.547, making the probable error of a normal of the weight unity, $\pm 0''.487$; adopting this value, the elements with their probable errors are (which elements it will be remembered are the osculating of Oct. 2) : —

$$
\begin{array}{rll}
T = & 1858, \text{ Sept. } 29.970971 & \pm\ 0^{d}.0000860 \text{ Paris Mean Time.} \\
\omega = & 129^{\circ}\ 6'\ 41''.21 & \pm\ 0''.348 \\
\Omega = & 165\ 19\ 12.71 & \pm\ 0.611 \quad \text{Mean Equinox and Ecliptic 1858.0} \\
i = & 116\ 58\ 10.55 & \pm\ 0.290 \\
\varphi = & 85\ 3\ 55.22 & \pm\ 19.10 \\
\log q = & 9.7622804 & \pm\ 0.000000616 \\
\log a = & 2.19331 & \\
P = & 1949.7 \text{ years.} & \pm\ 6^{y}.25
\end{array}
$$

The normals are represented by these elements with the following residuals, (Obs. — Cal.)

	$\Delta\alpha \cos\delta$	$\Delta\delta$		$\Delta\alpha \cos\delta$	$\Delta\delta$
June 14	—0''.43	+0''.39	Oct. 19	—0''.17	—0''.11
July 13	—0.07	+0.35	Nov. 1	—0.96	+0.49
Aug. 11	—0.40	—0.89	Nov. 16	+0.11	+0.70
Aug. 23	+0.30	—0.49	Dec. 1	+0.39	+1.97
Sept. 5	+1.23	+0.93	Dec. 16	—0.32	+0.27
Sept. 17	+0.32	+0.60	Jan. 3	+0.00	+1.42
Sept. 28	+0.08	—0.44	Jan. 30	+0.41	+0.73
Oct. 8	—0.66	—0.39	Feb. 26	—1.21	—0.22

These residuals, although they appear quite small, do not indicate a completely satisfactory solution. For the probable error derived from them is much larger than that obtained from the consideration of the observations themselves. The latter quantity being $\pm 0''.27$, while the former, as stated above, is $\pm 0''.487$. The principal cause of this difference is doubtless to be sought in the small systematic errors of the observations which arise from the idiosyncrasy of the observer in selecting the proper point to be observed, influenced perhaps, in some degree by the size of the instrument he used. In Vol. III. p. 329, of the Annals of Harvard College Observatory, will be found the statement of the opinion that the observations have a tendency to place the comet too near the Sun, and the smaller the telescope the nearer the Sun. Let us see whether the observations confirm this supposition. Taking the comparisons in declination of the best observations which go to form our normal of Sept. 17, when the effect of such a tendency lies almost wholly in declination, and arranging them under the head of the different observatories and in the order of the size of the telescopes, we have the following table. The numbers beneath the names of the observatories denote the aperture of the telescope in inches.

	Ann Arbor. 12.5	Berlin. 9.6	Liverpool. 8.5	Königsberg. 6.25	Bonn. 6.0	Kremsmünster. 5.9	Pulkova. 5.8	Paris. 4.8	Geneva. 4.25
Sept. 12	+14″.33	....	+15″.63	+22″.06	+13″.93	+14″.42	+13″.41	+17″.28	+10″.28
13	+14.72	+13.93	....	+12.59	....	+15.48	....	+19.45	+15.92
14	....	....	....	+13.57	....	+13.63	....	+17.69	+12.28
15	....	+16.04	+16.06	+21.30	....	....	....	....	+ 7.63
16	+14.47	....	....	+10.53	+11.66	+15.72	+11.15	....	....
17	+16.65	....	....	+18.84	....	+16.64	+14.35	....	....
18	+17.32	....	+12.06	+10.13	....	....	+ 9.83	+25.92	....
19	+12.55	....	....	....	....	+17.81	....	+ 7.51	+13.14
20	+14.10	+13.48	....	....	....	+13.83	....	+13.09	....
21	....	+15.17	+18.13	+11.08	+14.28	....	....	....	+18.35
22	....	....	....	....	+12.90	+15.55	....	....	....
Mean,	+14.88	+14.66	+15.47	+15.01	+13.19	+15.39	+12.19	+16.82	+12.93

The existence of systematic error seems pretty well made out between the different observatories; and the Bonn, Pulkova, and Geneva observations made with small telescopes, do certainly place the comet nearer the Sun than the others. But the observatory which places the comet farthest to the north is Paris, with a very small telescope. Also Kremsmünster and Königsberg, with much smaller telescopes, put the comet farther from the Sun than Ann Arbor and Berlin. These facts militate strongly against this supposition. The quantity used in forming the normal was +14″.44, and the preceding elements give +13″.84 for the same quantity, from which it may be judged how well each of the above observations is satisfied.

Again, if this hypothesis were sufficient to account for the systematic errors, we should have almost perfect agreement in the right ascensions. Let us see whether this was the case.

	Ann Arbor.	Berlin.	Liverpool.	Königsberg.	Bonn.	Kremsmünster.	Pulkova.	Paris.	Geneva.
Sept. 12	− 7″.62	....	−11″.69	− 7″.21	−13″.13	−10″.38	−18″.72	− 4″.79	− 9″.74
13	−11.36	−18.00	....	− 9.26	....	−12.00	....	− 9.48	−14.06
14	....	....	....	− 7.87	....	− 6.96	....	− 9.41	− 9.83
15	....	−17.88	− 8.83	−11.24	....	....	....	....	−16.48
16	−14.63	....	....	−12.52	− 7.51	−10.77	−10.14	....	....
17	−13.47	....	....	−13.64	....	−15.02	−13.25	....	....
18	−13.13	....	−11.09	− 5.55	....	....	−10.07	−15.38	....
19	−17.26	....	....	....	....	−13.13	....	−18.14	−15.42
20	−16.22	− 9.98	....	....	....	−10.89	....	−20.02	....
21	....	−11.95	− 9.02	− 9.39	−14.82	....	....	....	− 8.18
22	....	....	....	....	−19.26	−12.99	....	....	....
Mean,	−13.38	−14.45	−10.16	− 9.59	−13.68	−11.52	−13.05	−12.87	−12.29

Systematic error is not quite so manifest here as in the declinations, the observations not agreeing so well among themselves, but it undoubtedly exists in considerable quantity. The quantity used for the normal of Sept. 17 was —12″.67, and the orbit found gives —13″.07.

We shall make one more trial; about Oct. 8, the effect according to the hypothesis, took place wholly in the direction of right ascension. The scheme of observations stands thus: —

	Ann Arbor.	Paris. 12.6	Berlin.	Liverpool.	Königsberg.	Bonn.	Kremsmünster.	Pulkova.	Geneva.	Greenwich. 3.75
	″	″	″	″	″	″	″	″	″	″
Oct. 4	....	....	—7.20	—2.82	....	....	....	....	—16.18	....
5	—7.94	....	+1.71	....	— 7.91	—7.65	....	....	—10.96	—9.49
6	....	....	—4.30	....	....	....	—11.01	—10.98	—11.00	....
7	....	....	—4.97	....	....	....	— 6.66	....	— 8.19	....
8	....	—2.80	....	—8.49	....	—4.85	— 2.40	....	....	....
9	—0.09	—4.29	....	....	—12.66	....	....	....	— 6.94	—0.68
10	....	....	....	....	....	....	— 6.49	....	....	....
11	....	....	....	....	....	....	— 4.78	....	....	—2.05
12	....	....	....	....	....	....	....	— 4.17	....	....
13	—1.20	....	....	+4.11	....	....	....	....	— 5.28	....
14	+5.45	....	....	....	....	....	— 3.73	....	— 4.39	....

The observations are too scattered to establish anything with certainty, but the systematic errors seem to be larger than before, and, Greenwich excepted, the observations with the small telescopes place the comet farther from the Sun than those with the large telescopes. The same thing is probably true of the observations of the rest of October, but as the northern observations here begin to fail us, we can make no comparison.

It would be very difficult, perhaps impossible, to arrive at a satisfactory explanation of these systematic errors and to assign their numerical values, consequently I shall not undertake any discussion of them. If, however, this hypothesis should be adopted, and a correction varying inversely as the size of the telescope should be applied to the observations, removing the comet from the Sun a space ranging from 1″ to 3″, the effect would be to diminish the period of revolution by about 25 or 30 years. With regard to this, the most interesting element of the orbit, we may state with confidence, I think, that it is not less than 1900 years, and cannot exceed 1975 years.

Lastly, we have settled by this discussion, that there is not the slightest indication that any other force than gravity influenced the motion of the centre of gravity of the comet. For although, on comparing our final orbit with observations made at a particular observatory, we should observe small but well-marked deviations, yet another observatory will be found, whose observations entitled to equal confidence indicate a deviation at the same time in an opposite direction.

V.

On the Secular Periodicity of the Aurora Borealis.

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Communicated October 11*th*, 1859.

It is said in Holmes's Annals of America,* that the aurora was first seen in New England on December 17, 1719. The historian refers to Dr. Trumbull's Century-Sermon, preached at New Haven, on January 1, 1801, in which occurs the following note: † — "The aurora borealis, or northern light, is a new appearance in the heavens to this country, peculiar to the eighteenth century. It had been seen in Great Britain, especially in the north of Scotland, for many centuries past; but even in that country it had not appeared for eighty or one hundred years, until March 6,‡ 1716. Its first appearance in New England was on the 17th of December, 1719. It appears to have been a great light, and began about eight o'clock in the evening. It filled the country with the greatest alarm imaginable. It was the general opinion, that it was the sign of the coming of the Son of Man in the heavens, and that the judgment of the great day was about to commence. According to the accounts given by the ancient people who were spectators of it, there was little sleep in New England that night." An anonymous account of this aurora by an eyewitness, dated December 15, 1719, has been republished in the Collections of the Massachusetts Historical Society.§ The author remarks in the first paragraph: "And I hope (though I believe I shall differ from some) I shall say nothing that shall be inconsistent either with Divinity or Philosophy." This aurora was seen from eight o'clock in the evening until an hour or two before daybreak the next morning. Its appearance at eleven o'clock "was somewhat dreadful, — sometimes it looked of a flame, sometimes a blood-red color, — and the whole northeast horizon was very light, and looked as though the moon had been near her rising." The description ends in these words: "Thus I have given you the best account I am able of this meteor, which, though very unusual here, [is] yet in

* Vol. I. p. 523. † Page 5. ‡ Old Style. § Vol. II. pp. 17-20.

Northern countries more frequent, and seems to me to be what our modern philosophers call Aurora Borealis."

I suggest, in this connection, whether the following extracts from Winthrop's History of New England * do not indicate the appearance of an aurora in New England at a much earlier date than that ascribed to it by Dr. Holmes and others. "About midnight three men, coming in a boat to Boston, saw two lights arise out of the water, near the north point of the town cove, in form like a man, and went at a small distance to the town, and to the south point, and there vanished away." "The like was seen by many a week after." In the description of the second case, it is added: "A light like the moon arose about the northeast point in Boston, and met the former at Nottle's Island, and there they closed in one, and then parted and closed and parted divers times, and so went over the hill in the island and vanished. Sometimes they shot out flames and sometimes sparkles." These luminous appearances occurred on the 11th and 18th of April, 1643.

Many of this generation, accustomed as they have been to frequent displays of the aurora, will read with surprise the statement that the aurora was observed for the first time in New England in 1719, with the possible exceptions just quoted from Winthrop's journal. For the inference is, that no aurora, or at least no conspicuous exhibition of it, had occurred here before, since the settlement of the country. The people of New England were too much inclined to exaggerate every unusual phenomenon in the heavens to have overlooked or been silent in regard to a spectacle so strange as the aurora, had they had the opportunity of beholding one. That the aurora had been equally uncommon in Old England during the century previous to 1719, appears from the fact that the great astronomer, Dr. Halley, was, as he says, dying to see one, and that he expected to die without seeing it. At last the opportunity came, on March 17, 1716, when Halley was sixty years old. In his description of it, he says: † "This was the only one I had as yet seen, and of which I began to despair, since it is certain it hath not happened to any remarkable degree in this part of England since I was born." He adds that the like is not recorded in the English annals since 1574, or for one hundred and forty years. It was then seen on two successive nights, November 25 and 26. It was also seen at London on February 10, 1560, and on October 18, 1564. Notwithstanding this infrequency of the aurora in England for a long period *prior* to 1716, John Huxham observed it at Plymouth, England, in eighty-one instances between 1728 and 1748.‡ The Philosophical Trans-

* Vol. II. pp. 152, 153.

† Phil. Trans., XXIX. 416.

‡ Amer. Journ. of Sci., XXXIII. 297.

actions of London make no mention of auroras before 1716, though they had been published for forty years; but they record two hundred observations, made in different places, between 1716 and 1750.* The Academy of Sciences, founded at Paris in 1666, is silent upon the subject for fifty years after its establishment.

E. J. Burman writes under the date of October 30, 1722, that the aurora had been seen at Upsal thirty times during the last five months. Celsius states it had been rarely seen in Sweden before 1716, and yet between 1716 and 1732 there may be found three hundred and sixteen observations on two hundred and twenty-four independent recurrences of the phenomenon. He heard from the old men then living at Upsal, that these northern lights were novel even to them in that high latitude, and he concludes, from all the evidence he could collect, that these phenomena were periodical near the arctic circle as well as on the parallels of Europe. "It is impossible to believe," says Celsius, "that the skilful observers of the last century, who passed their lives in the Observatories erected for them, particularly at Paris and Greenwich, should not have taken care to transmit to posterity their observations on this admirable phenomenon, if it had appeared in their time." Mairan adduces also the authority of Anderson, found in his Natural History of Iceland. "It has always appeared extraordinary to me," he says, "that the most ancient Icelanders, as they have assured me, should have been astonished themselves at the frequent appearance of the aurora in their island, declaring that formerly they were much less common than to-day." . . . . "But I am so much the more ready to believe them, as it is certain that in other countries of Europe this phenomenon was much more uncommon formerly than now." The first recorded appearance of an aurora in Italy belongs to the year 1727.† Zanotti and Beccari have collected fifty-two exhibitions of it in Bologna and other parts of Italy, between the years 1727 and 1751, and thirty-six doubtful cases. Zanotti, in his description of the aurora which was seen in Italy, as well as in England, on December 16, 1737, says: "The Aurora Borealis, which was formerly a rare phenomenon, and almost unknown in this our climate (Italy), is now become very frequent. A great number have been observed for some years past." ‡ The language of Halley, Leibnitz, Kirch, Fontenelle, and Miraldi, in describing the auroras of the first half of the eighteenth century as *uncommon sights*, and the silence of Cassini in respect to any such display in the latter half of the preceding century, point to one and the same explanation.

The periodicity in the occurrence of the aurora, which seems to be roughly fore-

* Bertholon in Encyc. Method.

† Musschenbroek, Nat. Phil., 1314.

‡ Phil. Trans., XLI. 593.

shadowed in what has already been said, was illustrated and confirmed by the comprehensive review of the subject which Mairan made in his *Traité Physique et Historique de l'Aurore Boréale*, published by the French Academy first in 1731, and a second edition in 1754. Mairan was incited to his great labor by the remarkable aurora of October 19, 1726. The first edition contained the record of 229 auroras, scattered over the long interval between the years 500 and 1731. These appearances were divided into 22 distinct groups, separated from each other by unequal intervals of time, themselves barren in auroral displays. But in the *Eclaircissemens* of the second edition of his work, Mairan had accumulated 2137 recorded observations, or, subtracting the duplicates, 1441 independent exhibitions of the aurora between the years 583 and 1751. With this large accession of new materials, Mairan would doubtless have modified the number and epoch of his groups, had he pursued the subject further in that direction. If the whole interval between 1541 and 1751 is divided into seven periods of thirty years each, there are, according to Mairan's catalogue, 38 appearances of the aurora in the first period, or between 1541 and 1571; 66 appearances in the second period, or between 1571 and 1601; 57 appearances in the third period, or between 1601 and 1631; 18 appearances in the fourth period, or between 1631 and 1661; 26 appearances in the fifth period, or between 1661 and 1691; 195 appearances in the sixth period, or between 1691 and 1721; and 908 in the seventh period, or between 1721 and 1751. In the year 1732, one hundred auroras were observed; and in 1730, one hundred and sixteen. The general progress of physical science and the multiplication of observers may *partly* account for the large numbers in the sixth and seventh cycles; but they fail to explain the inferiority of the fourth and fifth cycles to the second and third. Hansteen * distinguishes twenty-four periods between 502 B. C. and the present epoch, particularly the ninth, which is from 541 to 603; the twelfth, which is from 823 to 887; the twenty-second, which is from 1517 to 1588; and the twenty-fourth, which is from 1707 to 1788. He assigns the maximum of the latter period to 1752; but it is much nearer a minimum, as I shall show hereafter.

The weight of evidence which Mairan has accumulated in favor of the periodicity of the aurora has not been universally held conclusive. Bertholon † has argued against what he calls the *pretended* interruptions in the occurrence of this phenomenon. He maintains that there really exists no cessation in the auroral displays, and explains away the apparent cessation by accidental circumstances, as the lack of observatories, the scarcity of observers, their want of experience, their bad geographical position, or their inability

* Pogg. Ann., XXII. 536, 537. Kaemtz's Meteorology, p. 458.

† Encyc. Method. Phys., I. 347 – 349.

to communicate with the public by printed books, or through the transactions of academies. And he quotes from Fontenelle a remark to the effect that a man does not often see more than he already knows to exist. Considerations like these are, no doubt, entitled to great weight in comparing together the number of auroras observed in *ancient* and *modern* times. They account in part for the increasing number of observed auroras as we approach the present epoch; but they fail to explain the alternate increase and decrease in the number of auroras from one quarter of a century to another, and especially will they fail if it is found that the same fluctuations which are indicated by Mairan's historical investigations are repeated during the last century, and even under our own eyes.

I shall next consider how the observations of the last one hundred years affect this question of the periodicity of the aurora. In 1754, Mairan * commented on the fact that the aurora had begun again to diminish in frequency in France, so that he found no instance between November 3, 1740, and February 3, 1750, which would answer the purpose of determining the height of an aurora by its parallax. The St. Petersburg observations declare that Mairan was correct in his forebodings. They furnish two hundred and sixteen auroras between 1729 and 1743 inclusive, and only forty-six between 1744 and 1758 inclusive, there being thirty-five in one year, 1730, and none in 1753 and 1754. Dalton † has published two hundred and twenty-seven appearances of the aurora in Kendal and Keswick, between the years 1787 and 1793, of which twenty-nine were observed at both places, and all but ten at Kendal. In Dalton's catalogue of auroras, observed in Great Britain and Ireland between the years 1793 and 1834, fifty-five occurred before 1810, and only seventeen between 1810 and 1826 inclusive, though the latter period is longer by one year. Then, again, one hundred and thirteen auroras were observed in the last eight years of Dalton's observations, between 1827 and 1834 inclusive. In this latter period eight are designated as *grand*, and many others as *fine*. Thirty-two auroras were seen in one year, 1830. But there were none in 1807, 1809, 1810, 1811, 1812, 1813, 1815, 1822, 1823, and 1824. No wonder that Singer remarks in his Elements of Electricity, ‡ published in 1814, that the aurora was then rarely visible in England. Arago § stated in September, 1827, that no aurora had been seen before in Paris for twenty years. Böckmann ‖ observed at Carlsruhe twenty-three auroras in 1779, seventeen in 1780, fifteen in 1781, eight in 1782, ten in 1783, one in 1789, and no more for the next 12 years.

It may be interesting to inquire how the case stands in the western hemisphere, and

* Page 430. † Meteor. Observ. and Essays, p. 54. ‡ Page 253.

§ Amer. Journ. Sci., XIV. 107. ‖ Gilbert Ann., VII. 32.

whether it sustains the European history of the aurora for the last three centuries. I have mentioned the surprise excited in New England by the aurora of 1719. After such a commencement, there are scattered accounts of the aurora during the remainder of the century. Mr. Greenwood, then Hollis Professor of Mathematics and Natural Philosophy in Harvard College, described an aurora seen at Cambridge on November 2, 1730. His successor, Professor John Winthrop,* has recorded nine exhibitions of it between 1741 and 1757. Mr. Caleb Gannett † mentions an aurora, accompanied by an east and west arch, which was seen at Cambridge on March 27, 1781. Manasseh Cutler noticed the aurora repeatedly at Ipswich in 1781.‡ Auroras were seen at Salem on November 17 and 24, 1720, on January 1 and October 2, 1728, and an extraordinary one on October 22, 1730. On December 29, 1736 (probably), Dr. Holyoke witnessed an aurora of which he says: "The first aurora borealis I ever saw. The northern sky appeared suffused by a dark blood-red colored vapor, without any variety of different colored rays. I have never seen the like. Northern lights were then a great novelty, and excited great wonder and terror." On August 6, 1768, a bright streak of light extended from the west northwest to the southeast, almost as bright as a rainbow. On July 19, 1769, there was an aurora of unusual brightness.§ On April 21, 1750, the aurora was seen as far south as Charleston, S. C. One who beheld it has given the following description: — "We had a most extraordinary appearance of the aurora borealis. One half of the sky seemed like a beautiful streaked liquid flame, so terrible to many of the female inhabitants that some of them were thrown into fits." || Auroras were observed at Cambridge by Professor Williams, in co-operation with the Meteorological Society of the Palatinate, during its brief period of activity. Professor C. Dewey, then of Williams College, observed auroras on May 23 and 28, 1818; also from June 6 to June 10, on September 24 and 25, and on October 6 and 7 of the same year.

It is well known to many members of this Academy, that Dr. Holyoke, of Salem, kept a Meteorological Journal from 1754 to 1828. That part which relates to the weather has already been published in the Memoirs of the Academy. I have consulted the manuscript records of Dr. Holyoke, which he presented to the Academy, and have selected from them all the auroras he has observed and recorded. Unfortunately, the copy in possession of the Academy is not the original, until the year 1786; and, being prepared for a special purpose, it does not contain any notice of auroras, if, indeed, any were observed before 1786. But the Academy also possesses the original manuscript

* Amer. Journ. Sci., XL. 204.
† Mem. Amer. Acad., II. 136.
‡ Mem. Amer. Acad., I. 366.
§ Felt's History of Salem, II. 137.
|| Gent. Mag., XX. 418, and XXI. 39.

Journal of Meteorology kept at Cambridge by Professor John Winthrop, from 1742 to 1779; that of Professor Edward Wigglesworth, kept also at Cambridge from 1782 to 1793; and that of Dr. Enoch Hale, kept at Boston, from 1818 to 1848. In all these journals, except the last, the auroras are noted with great care. Dr. Hale, probably, recorded only the most conspicuous. This collection of manuscripts covers more than a century of time, in which only two years are unrepresented, namely, 1780 and 1781. From this rich storehouse of observations, I have been able to cull 624 recorded examples of auroras, of which only 79 are duplicates. After subtracting these, 545 independent auroras remain, which have never before appeared in print. Of the 624 observations contained in all the manuscripts, 254 were registered by Professor Winthrop, 136 by Professor Wigglesworth, 198 by Dr. Holyoke, and the balance (36) by Dr. Hale. As all these observations have been made at places only a few miles apart,* they are strictly comparable with each other, and furnish an almost uninterrupted history of the aurora in this immediate vicinity for a century. It appears from these journals, that during the thirty-three years, between 1792 and 1826 inclusive, only 48 auroras were observed; but that during the thirty-three years *next preceding* 1792, there are registered 387 independent auroras. And even during the eighteen years between 1742 and 1759 inclusive (or as far back before 1759 as the observations extend), 77 auroras are recorded. And single years may be selected in which there occurred nearly twice as many auroras as in the whole period of thirty-three years, ending with 1826. None were observed in the years 1796, 1797, 1798, 1799, 1800, 1801, 1807, 1808, 1810, 1811, 1812, 1813, 1816, and 1817. Observations on the aurora, made exclusively in the State of New York,† between the years 1826 and 1850 inclusive, exhibit 1152 independent appearances during that period of twenty-five years; and, when combined with those already described, they manifest for this part of the United States what Dalton's observations have demonstrated for England and Ireland, namely, that the displays of the aurora are in a high degree intermittent.

A study of the sequences in the succession of remarkable auroras may be even more instructive than an indiscriminate attention to all; for this reason, if for no other, that such auroras can hardly have escaped detection and description in early times. In the United States, great auroras were witnessed on December 17, 1719; on October 22

* Winthrop's Journal was kept at Andover during May, June, July, and August, at Watertown during September, October, and November, 1775; and at Concord, from December, 1775, to June 18, 1776; Cambridge being the head-quarters of the Revolutionary army.

† Results of a Series of Meteorological Observations, made in Obedience to Instructions from the Regents of the University at sundry Academies in the State of New York, from 1826 to 1850, inclusive, and compiled from the original Records and the Annual Reports of the Regents by Franklin B. Hough, M. D., p. 472.

and November 2, 1730; on December 29, 1736; on April 21, 1750; on August 6, 1768; and on July 19, 1769; descriptions of which have already been published. I have selected from the unpublished manuscripts already mentioned all those examples of auroras which are described as *red*, or *bright*, or *high*, or *great*, or *considerable*, or *brilliant*, or *unusual*, or *remarkable*. In the catalogue which follows, the *most* extraordinary exhibitions are printed in *italics*.

1743. January 12.
1746. March 1, June 1.
1747. *August* 19.
1749. October 7.*
1750. *February* 16, April 3, October 22.
1752. January 8.
1757. *November* 12.
1758. January 8, *March* 31.
1759. *September* 31.
1760. May 3.
1761. November 19.
1762. May 21, August 10, December 18.
1763. July 31, September 14, November 11.
1764. *March* 5, *June* 18.
1765. July 22, August 19.
1768. *July* 20, *December* 5.
1769. January 5, February 27, June 9, July 2, *October* 7.
1770. January 18,* *March* 23,† *April* 19, August 27.
1771. *March* 17, June 2.
1772. *July* 8.
1773. *January* 17, *January* 19, *July* 12, *July* 17.
1774. March 13, August 14.
1777. April 6, *September* 7, *November* 3, *November* 27.†
1778. February 17, March 24, April 23, *September* 24.
1779. February 10.
1786. February 28, March 13, March 19, *October* 17.
1787. June 8, July 13, July 14, October 13, November 3,* November 24.*
1788. *July* 30.*
1789. March 14, May 23, August 18, *November* 14.
1818. *March* 24.
1827. *August* 28, September 8.
1830. December 11.
1838. *February* 21, *November* 12.
1839. *January* 14, January 19.
1840. *August* 28.
1848. *April* 6.

The examination of this list of unusual auroras betrays a very partial chronological distribution. During the larger part of the 18th century, down to 1789, they were frequent, there being in all 76 examples, of which 30 are marked as particularly conspicuous. Between the years 1789 and 1826, a solitary instance is found, on March 24, 1818. From 1827 to 1848, I depend on Dr. Hale's record, which is evidently less complete in this specialty than the others. But I find even there 8 unusual auroras mentioned, of which 5 are worthy of *italics*. For more ample information in regard to remarkable displays of auroras since 1827, the catalogue of New York auroras, already quoted, may be consulted to advantage. Professor Olmsted has selected from the catalogues of Dalton and the Regents of the State of New York 12 auroras between 1827 and 1848 inclusive, which he assigns to the first rank of auroras, as he has classified them.‡ I annex a list of those which Dr. Hough has characterized as

* Red. † Very extraordinary. ‡ Smithsonian Contributions to Knowledge, VIII.

brilliant, the *very* brilliant specimens being printed *in italics*, and a few of a highly extraordinary character being signalized by an asterisk. Of brilliant auroras, 157 are registered: of these 81 are designated as very brilliant, and 4 as highly extraordinary.

1827. *August* 28.*
1828. August 16, September 8, *September* 29, October 8.
1829. *June* 1, June 7, September 26, November 9, December 28.
1830. *March* 15, *April* 19, May 2, *June* 10, *June* 11, *July* 14, July 15, *August* 19, August 20, *September* 15, *October* 28, *December* 7, *December* 10, *December* 11.
1831. January 7, *February* 6, *March* 8, March 18, *April* 1, April 18, *April* 19,* *April* 20, June 10, *June* 21, *July* 3, *July* 4, October 29, December 9.
1832. *August* 22, *August* 23, *August* 24.
1833. *March* 17, *May* 16, *May* 17, June 29, *July* 10, *September* 5, *November* 3, *December* 29.
1834. *October* 8, *November* 2.
1835. *January* 4, August 19, September 4, September 22, *November* 17, November 18.
1836. April 19, *April* 22, *April* 23, *May* 8, May 19, July 31, August 2, *August* 12.
1837. *January* 14, April 6, *July* 1, *October* 22, November 12, *November* 14.
1838. February 20, February 21, July 27, August 22, *September* 14, *September* 15, *September* 16, November 9, November 26.
1839. January 10, *January* 14, March 5, May 5, June 7, August 28, *September* 3,* October 10.
1840. *January* 3, *May* 29.
1841. *February* 23, *March* 23, *June* 15, *July* 29, *August* 4, August 6, *September* 25, October 5, *November* 18.
1842. *April* 14, June 4, July 3, *August* 5.
1843. March 7, *April* 15, *June* 3, June 22, August 3, *October* 5.
1844. *February* 4, *October* 20, November 1, *December* 15.
1845. *January* 8, January 9, February 25, April 13, *November* 3, *December* 3.
1846. March 25, May 30, *August* 6, September 23, October 2, December 9.
1847. January 17, *February* 21, *March* 19, April 6, *April* 7, August 4, *August* 29, October 24, November 1, *November* 25, December 7.
1848. January 6, *January* 16, *February* 8, February 18, *February* 21, March 23, April 1, April 5, *April* 6, July 3, July 11, July 12, July 23, August 14, August 21, *October* 23, *November* 17,* November 18, November 26, December 27.
1849. July 24.

Dr. Holyoke's Journal furnishes *positive* testimony in favor of a conclusion which had been adopted already from *negative* evidence; from the absence, that is, of any description of a remarkable aurora seen in this country during the present century *before* 1827, although the Memoirs of the American Academy of Arts and Sciences in Boston, the Transactions of the Philosophical Society of Philadelphia, and, after 1818, the American Journal of Science were in existence, and would have furnished a convenient medium for the publication of any aurora which deserved the attention of the scientific world. Therefore, as Professor Olmsted remarks, "the splendid arch and other striking accompaniments of the aurora of August, 1827, took us by surprise, and were viewed with wonder by nearly all the existing generation of the countries where it was visible." † Mr. Felt says that it caused much apprehension lest the end of all

* Red.

† Smithsonian Contributions, VIII. 6.

things had come. The arch which signalized this aurora had been seen by Dr. Holyoke, who was then ninety-nine years old, only twice before, viz. in 1755 and 1769. The sight of a magnificent aurora was so unusual, that in August, 1827, the bells were rung in Salem to call attention to it.

Accordingly, the discussion of American observations, both of ordinary and extraordinary auroras, substantiates the general conclusion first suggested by European observation; namely, that there is a *secular periodicity* to the phenomenon, twenty years or more of abundant exhibitions being separated by intervals, equally long or longer, when these displays, if not wholly wanting, are stripped of their more brilliant characteristics. Moreover, these repeated interruptions in the return of auroras are such as no failure of memory, no negligence of observers, and no deficiencies of history can adequately explain.

Another decline in the frequency and brilliancy of the aurora since 1850 is manifest. This will appear from an examination of the Regent's Reports, made since the publication of Mr. Hough's Results &c. It will also appear, by consulting the American Journal of Science, which contains probably a notice of all the conspicuous auroras in recent years. On September 29, 1851, an aurora was visible in the Southern States. On February 19, 1852, a grand aurora was seen in New Hampshire and Vermont. Splendid auroras are also recorded on June 11 and November 11, by Mr. Z. Thompson, of Burlington, Vt. On August 11, 1853, a slight aurora was visible at New Haven. On September 2, 1853, an aurora was seen at sea off Cape Race. On May 24, 1853, an aurora appeared at Perryville, in lat. 37° and long. 39°. Auroras were also seen there on April 22 and June 11, 1852. On April 5, 1853, a splendid aurora was witnessed at Burlington, Vt. On April 29, 1859, an auroral arch was seen at New Haven. But the last decade was redeemed from insignificance in respect to auroral displays by the prolonged and magnificent exhibition between August 28 and September 4, 1859, the geographical extent of which embraced the United States and Australia. Poey* states that the aurora has been registered only six times at Cuba, viz., on November 13, 1784; on November 14, 1789; in November, 1833; on November 17, 1848; and on August 28 and September 1–2, 1859. Mr. Logan says that the aurora has been seen in California eleven times in 11 years, of which four were in 1860. The best displays were on August 28, 1859, and July 4, 1860. The late Professor Olmsted, in his valuable Smithsonian Contribution to Knowledge, "On the recent secular period of the Aurora Borealis," accepted for publication January, 1855, has attempted to approx-

* American Journal of Science.

imate to a just estimate of the numerical value of this periodicity. I will state his conclusion in his own words.

"From the foregoing, and many similar inductions, I think it may be inferred, with considerable probability, that the greatest secular periods of the aurora borealis occur at intervals of about sixty-five years, reckoning from the middle of one period to the middle of another, although returns of a less remarkable character are probably interspersed among these.

"The *duration* of one of these great periods appears to be from 21 to 25 years. That which we have recently passed through commenced in 1827, and if we consider it as completed in 1848, when there was almost a cessation of the phenomenon in its higher forms for two years, its duration was 21 years. The occurrence of three exhibitions of the first class in September, 1851, and of one in February, 1852, throws some doubt on this point. Although the greatly diminished intensity since 1848 would incline me to consider the period as terminating then, yet these later exhibitions indicate a duration of 25 years. If we examine into the duration of other similar periods, we obtain corresponding results. Thus the return immediately preceding the recent one lasted from 1760 to 1783, a period of 23 years; and the next preceding that lasted from 1716 to 1740, another period of 24 years. On the whole, therefore, I conclude that the aurora borealis is subject to periodical returns, during which it is exhibited in extraordinary frequency and greatly augmented splendor and magnificence; that these periods are at intervals of about 65 years; that they last for a period not exceeding 25 years, and, consequently, that from the end of one visitation to the beginning of another is an interval of nearly 10 years, during which time the phenomenon is far less remarkable both in frequency and intensity.

"Probably similar periods occur in the polar regions, since travellers differ much in their account of the numbers and degrees of splendor of these exhibitions at different times."*

I propose to publish immediately, in a second memoir, a complete catalogue of auroras, and then to renew the discussion in regard to the periodicity of the phenomenon with more ample materials.

* Pages 38, 39.

CATALOGUE OF AURORAS.*

Compiled from Professor Winthrop's Journal.

1742.

1 Dec. 29. A small northern light this evening.
2 " 30. (11^h, P. M.) A faint appearance of the northern light.

1743.

3 Jan. 23. (11^h, P. M.) A considerable northern light.
4 Nov. 5. A northern light in the evening.

1744 – 5, None.

1746.

5 Mar. 12. Evening. A considerable northern light.
6 " 13. ($11\frac{3}{4}^h$, P. M.) Northern light.
7 " 29. About midnight a northern light.
8 June 12. Evening. A considerable Aurora Borealis.
9 " 15. Evening. Aurora Borealis.
10 July 4. Aurora Borealis from $11\frac{1}{2}^h$ to $6\frac{1}{2}^h$.
11 Dec. 14. Evening. Small Aurora Borealis.

1747.

12 Jan. 9. Evening. Small Aurora Borealis.
13 Feb. 6. Evening. Aurora Borealis.
14 " 7. Evening. Aurora Borealis.
15 May 29. Evening. Small Aurora.
16 July 5. Morning. An Aurora Borealis.
17 Aug. 29. Evening. A small Aurora Borealis.
18 " 30. Morning. Great Aurora Borealis.
19 Sept. 5. Evening. Aurora Borealis.
20 " 27. Evening. Aurora Borealis.
21 Dec. 26. (10^h, P. M.) A small Aurora Borealis.

1748.

22 Feb. 3. ($10\frac{1}{4}^h$, P. M.) Aurora Borealis.
23 April 7. Evening. Aurora Borealis.
24 Aug. 30. Evening. Aurora Borealis.
25 Nov. 1. Evening. Aurora Borealis.
26 Dec. 15. Evening. Aurora Borealis.
27 " 20. Evening. Aurora Borealis.

1749.

28 Feb. 16. (10^h, P. M.) Small Aurora Borealis.
29 March 9. Evening. Aurora Borealis.
30 April 5. Evening. Aurora Borealis.
31 " 18. Evening. Small Aurora Borealis.
32 May 11. Evening. Aurora Borealis.
33 June 7. ($10\frac{1}{2}^h$, P. M.) Aurora Borealis.
34 Sept. 13. Evening. Small Aurora Borealis.
35 Oct. 18. At 9, P. M., a crimson Aurora Borealis.
36 Nov. 6. Evening. Aurora Borealis.
37 " 9. Evening. Small Aurora Borealis.

1750.

38 Jan. 5. Evening. Small Aurora Borealis.
39 " 6. Aurora Borealis.
40 Feb. 4. ($10\frac{3}{4}^h$, P. M.) Small Aurora Borealis.
41 " 27. Evening. Great Aurora Borealis.
42 Mar. 4. Aurora Borealis.
43 " 11. Evening. Small Aurora Borealis.
44 April 8. Evening. Small Aurora Borealis.
45 " 9. Evening. Aurora Borealis.
46 " 14. ($7\frac{3}{4}^h$, P. M.) Considerable Aurora Borealis.
47 " 30. (After $8\frac{1}{2}^h$, P. M.) Aurora Borealis.
48 May 1. Evening. Aurora Borealis.
49 July 26. Evening. Aurora Borealis.
50 " 28. Evening. Aurora Borealis.
51 Aug. 26. Evening. Aurora Borealis.
52 Sept. 4. Evening. Aurora Borealis.
53 Nov. 2. Evening. Bright Aurora Borealis.
54 " 4. ($6\frac{1}{4}^h$, P. M.) Aurora Borealis.

1751.

55 Feb. 12. (8^h, P. M.) Aurora Borealis.
56 " 19. Evening. Aurora Borealis.
57 " 24. Evening. Aurora Borealis.
58 April 2. Evening. Aurora Borealis.
59 June 27. ($11\frac{3}{4}^h$, P. M.) Aurora Borealis.

1752.

60 Jan. 19. Evening. Bright Aurora Borealis.
61 Oct. 12. ($8\frac{1}{4}^h$, P. M.) Aurora Borealis.

1753.

62 April 30. ($10\frac{1}{4}^h$, P. M.) Aurora Borealis.

1754 – 6, None.

1757.

63 Aug. 13. Evening. Aurora Borealis.
64 Sept. 7. Evening. Small Aurora Borealis.
65 " 13. Evening. Small Aurora Borealis.

* All the dates are reduced to *new style*.

66 Sept. 14. Evening. Aurora Borealis.
67 Nov. 12. Evening. Great Aurora Borealis.
68 " 13. Evening. Small Aurora Borealis.

1758.

69 Jan. 8. Evening. Bright Aurora Borealis.
70 Mar. 12. Evening. Aurora Borealis.
71 " 31. Evening. Very bright Aurora Borealis.
72 April 1. Evening. Small Aurora Borealis.

1759.

73 Mar. 8. Evening. Bright Aurora Borealis.
74 Sept. 9. Evening. Aurora Borealis.
75 " 11. Evening. Aurora Borealis.
76 " 18. (7^h, P. M.) Small Aurora Borealis.
77 Oct. 31. Evening. Very bright Aurora Borealis.

1760.

78 Mar. 14. (10^h, P. M.) Aurora Borealis.
79 May 3. ($10\frac{1}{4}^h$, P. M.) Bright Aurora Borealis.
80 Sept. 10. Evening. Aurora Borealis.
81 Oct. 4. Evening. Aurora Borealis.
82 " 25. Evening. Aurora Borealis.
83 Nov. 12. (After $6\frac{3}{4}^h$, P. M.) Aurora Borealis.

1761.

84 Mar. 10. Evening. Aurora Borealis.
85 " 11. Evening. Aurora Borealis.
86 Sept. 26. (After $7\frac{1}{2}^h$, P. M.) Aurora Borealis.
87 Nov. 6. Evening. Aurora Borealis.
88 " 19. Evening. Large Aurora Borealis.

1762.

89 Feb. 4. Evening. Aurora Borealis.
90 May 21. Evening. Bright Aurora Borealis.
91 Aug. 10. Evening. Bright Aurora Borealis.
92 " 11. Evening. Aurora Borealis.
93 " 13. (After $7\frac{3}{4}^h$, P. M.) Aurora Borealis.
94 Sept. 15. (After $7\frac{1}{2}^h$, P. M.) Aurora Borealis.
95 Dec. 18. Evening. Bright Aurora Borealis.

1763.

96 April 24. ($10\frac{1}{2}^h$, P. M.) Small Aurora Borealis.
97 May 2. Evening. Aurora Borealis.
98 July 31. Bright Aurora Borealis.
99 Aug. 4. Evening. Aurora Borealis.
100 Sept. 14. Evening. Bright Aurora Borealis.
101 Nov. 11. Evening. Bright Aurora Borealis.

1764.

102 Feb. 16. ($10\frac{1}{2}^h$, P. M.) Aurora Borealis.
103 Mar. 5. ($10\frac{1}{4}^h$, P. M.) Unusual Aurora Borealis.
104 May 16. Evening. Aurora Borealis.
105 June 3. Evening. Aurora Borealis.
106 " 18. ($10\frac{1}{4}^h$, P. M.) Remarkable Aurora Borealis.
107 " 19. ($10\frac{1}{4}^h$, P. M.) Faint Aurora Borealis.
108 July 27. Evening. Aurora Borealis.
109 Aug. 25. Evening. Aurora Borealis.
110 Sept. 20. At 8, P. M., an Aurora Borealis.
111 " 28. Evening. Aurora Borealis.
112 " 29. (After $6\frac{3}{4}^h$, P. M.) Aurora Borealis.

1765.

113 Feb. 19. Evening. Aurora Borealis.
114 July 21. Evening. Little Aurora Borealis.
115 " 22. Evening. Bright Aurora Borealis.
116 Aug. 17. Evening. Little Aurora Borealis.
117 " 19. Evening. Bright Aurora Borealis.
118 " 21. Evening. Little Aurora Borealis.
119 Oct. 12. Evening. Aurora Borealis.

1766, None.

1767.

120 July 18. Evening. Aurora Borealis.
121 " 28. Evening. Aurora Borealis.
122 Aug. 18. Evening. Aurora Borealis.

1768.

123 Feb. 18. Evening. Aurora Borealis.
124 July 20. Evening. Remarkable Aurora Borealis.
125 Aug. 6. Remarkable Aurora Borealis, like that of July 20.
126 Sept. 14. Evening. Small Aurora Borealis.
127 Oct. 2. Evening. Aurora Borealis.
128 Dec. 5. Evening. Very bright Aurora Borealis.

1769.

129 Jan. 5. Evening. Considerable Aurora Borealis.
130 Feb. 27. Evening. Bright Aurora Borealis.
131 June 9. Evening. Bright Aurora Borealis.
132 " 28. Evening. Small Aurora Borealis.
133 " 29. Evening. Small Aurora Borealis.
134 July 2. Evening. Bright Aurora Borealis.
135 " 7. Evening. Aurora Borealis.
136 " 8. Evening. Aurora Borealis.
137 Aug. 5. Evening. Aurora Borealis.
138 Sept. 26. Evening. Small Aurora Borealis.
139 Oct. 7. Evening. Very bright Aurora Borealis.
140 " 23. Evening. Aurora Borealis.

141 Oct. 25. Evening. Small Aurora Borealis.
142 " 26. Evening. Small Aurora Borealis.
143 " 27. Evening. Aurora Borealis.
144 Dec. 6. Evening. Aurora Borealis.

1770.

145 Jan. 17. Evening. Aurora Borealis.
146 " 18. Evening. Aurora radiating in the East.
147 Feb. 28. Evening. Aurora Borealis.
148 Mar. 23. Evening. Aurora Borealis, and a glorious arch from East to West.
149 April 19. Evening. Great Aurora Borealis.
150 May 16. Evening. Small Aurora Borealis.
151 July 14. Evening. Aurora Borealis.
152 " 16. Evening. Aurora Borealis.
153 " 27. Evening. Small Aurora Borealis.
154 " 31. Evening. Aurora Borealis.
155 Aug. 27. Evening. Bright Aurora Borealis.
156 " 31. Evening. Aurora Borealis.
157 Sept. 9. Evening. Aurora Borealis.
158 " 10. Evening. Aurora Borealis.

1771.

159 Feb. 19. ($10\frac{1}{4}^h$, P. M.) Aurora Borealis.
160 Mar. 17. Evening. Very bright Aurora Borealis.
161 " 18. Evening. Small Aurora Borealis.
162 June 2. Evening. Bright Aurora Borealis behind the clouds.
163 " 12. Evening. Little Aurora Borealis.
164 Aug. 16. Evening. Small Aurora Borealis.
165 " 17. Evening. Small Aurora Borealis.
166 Sept. 13. Evening. Small Aurora Borealis.
167 " 17. Evening. Aurora Borealis.
168 Oct. 4. Evening. Small Aurora.
169 " 18. Evening. Aurora Borealis.
170 Nov. 1. Evening. Aurora Borealis.
171 " 5. Evening. Aurora Borealis.
172 " 12. Evening. Small Aurora Borealis.
173 Dec. 4. Evening. Little Aurora Borealis.

1772.

174 Feb. 27. Evening. Aurora Borealis.
175 April 12. (10^h, P. M,) Small Aurora Borealis.
176 July 5. Evening. Aurora Borealis.
177 " 7. Evening. Aurora Borealis.
178 " 8. Evening. Great Aurora Borealis.
179 " 25. Evening. Remarkable Aurora Borealis.
180 Oct. 2. Evening. Aurora Borealis.

1773.

181 Jan. 17. Evening. Great Aurora Borealis.
182 " 19. Evening. Great streaming Aurora Borealis.
183 Mar. 15. Evening. Aurora Borealis.
184 " 23. Evening. Aurora Borealis.
185 May 19. Evening. Aurora Borealis.
186 June 6. Evening. Aurora Borealis.
187 " 10. Evening. Small Aurora Borealis.
188 " 13. Evening. Small Aurora Borealis.
189 " 14. Evening. Small Aurora Borealis.
190 " 24. Evening. Aurora Borealis.
191 July 12. Evening. Uncommon Aurora Borealis.
192 " 17. Evening. Great Aurora Borealis.
193 " 18. Evening. Small Aurora Borealis.
194 " 25. Evening. Aurora Borealis.
195 Aug. 15. Evening. Aurora Borealis.
196 " 25. Evening. Aurora Borealis in Northwest.

1774.

197 Feb. 28. Evening. Aurora Borealis.
198 Mar. 13. Evening. Great Aurora Borealis.
199 April 3. Evening. Aurora Borealis.
200 June 4. Evening. Little Aurora Borealis.
201 July 9. Evening. Aurora Borealis.
202 " 12. Evening. Aurora Borealis.
203 " 13. Evening. Small Aurora Borealis.
204 " 28. Evening. Little Aurora Borealis.
205 " 29. Evening. Little Aurora Borealis.
206 Aug. 14. Evening. Great Aurora Borealis.
207 " 26. Evening. Aurora Borealis.
208 Sept. 1. Evening. Aurora Borealis.
209 " 10. Evening. Aurora Borealis.
210 Nov. 3. Evening. Aurora Borealis.
211 " 5. Evening. Aurora Borealis.

1775.

212 Jan. 29. Evening. Aurora Borealis.
213 Feb. 2. ($11\frac{1}{4}^h$, P. M.) Little Aurora Borealis.
214 " 21. Evening. Aurora Borealis.
215 Mar. 25. ($10\frac{1}{4}^h$, P. M.) Small Aurora Borealis.
216 July 18. Evening. Aurora Borealis.

1776.

217 April 18. Evening. Aurora Borealis.
218 Sept. 4. Evening. Aurora Borealis.
219 " 5. Evening. Aurora Borealis.
220 " 16. Evening. Aurora Borealis.

1777.

221 Jan. 28. Evening. Aurora Borealis.

222 Mar. 29. Evening. Aurora Borealis.
223 April 6. Evening. Considerable Aurora Borealis.
224 " 11. Evening. Aurora Borealis.
225 June 28. Evening. Aurora Borealis.
226 July 27. Evening. Little Aurora Borealis.
227 Sept. 7. Evening. Great Aurora Borealis.
228 " 24. Evening. Aurora Borealis.
229 Oct. 8. Evening. Aurora Borealis.
230 Nov. 3. Evening. Great Aurora Borealis.
231 " 27. Evening. Extraordinary Aurora Borealis.
232 " 28. ($12\frac{1}{4}^h$, A. M.) Evening. Aurora Borealis continues.
233 Dec. 2. Evening. Aurora Borealis.
234 " 21. Evening. Aurora Borealis.

1778.

235 Jan. 18. Evening. Aurora Borealis.
236 Feb. 15. Evening. Aurora Borealis.
237 " 17. Evening. Bright Aurora Borealis.
238 Mar. 17. Evening. Small Aurora Borealis.
239 " 22. Evening. Aurora Borealis.
240 " 24. Evening. Bright Aurora Borealis.
241 April 20. Evening. Small Aurora Borealis.
242 " 23. Evening. Bright Aurora Borealis.
243 " 26. Evening. Aurora Borealis.
244 May 13. Evening. Aurora Borealis.
245 June 14. Evening. Aurora Borealis.
246 " 28. Evening. Aurora Borealis.
247 July 7. Evening. Uncommon Aurora Borealis.
248 " 31. Evening. Little Aurora Borealis.
249 Sept. 22. Evening. Aurora Borealis.
250 " 24. Evening. Great Aurora Borealis.

1779.

251 Feb. 10. Evening. Bright Aurora.
252 " 11. Evening. Aurora Borealis.
253 " 13. Evening. Aurora Borealis.
254 April 5. Evening. Aurora Borealis.

CATALOGUE OF AURORAS.

Compiled from Professor Wigglesworth's Journal.

1780, defective.

1781.

255 Mar. 27. Aurora Borealis.

1782.

256 Sept. 12. Aurora Borealis.
257 " 13. Aurora Borealis.
258 " 30. Aurora Borealis.
259 Oct. 2. Aurora Borealis.
260 " 3. Aurora Borealis.
261 " 10. Aurora Borealis.
262 Nov. 26. Aurora Borealis.

1783.

263 Jan. 31. Aurora Borealis.
264 Feb. 2. Aurora Borealis.
265 " 27. Aurora Borealis.
266 Mar. 2. Aurora Borealis.
267 " 4. Aurora Borealis.
268 " 26. Aurora Borealis.
269 " 29. Aurora Borealis.
270 " 30. Aurora Borealis.
271 April 2. Aurora Borealis.
272 " 7. Aurora Borealis.
273 " 27. Aurora Borealis.
274 May 2. Aurora Borealis.
275 " 13. Aurora Borealis.
276 " 29. Aurora Borealis.

1784.

277 June 11. Aurora Borealis.
278 Sept. 15. Aurora Borealis.
279 Nov. 15. Aurora Borealis.
280 Dec. 11. Aurora Borealis.

1785.

281 May 9. Aurora Borealis.
282 Sept. 8. Aurora Borealis.
283 Oct. 6. Aurora Borealis.
284 Nov. 29. Aurora Borealis.
285 " 30. Aurora Borealis.
286 Dec. 19. Aurora Borealis.

1786.

287 Feb. 28. White Aurora Borealis.

288	Mar. 13.	(9^h, P. M.) White Aurora Borealis.
289	" 19.	Bright Aurora Borealis at 8 o'clock.
290	" 22.	Aurora Borealis.
291	" 25.	Small Aurora Borealis.
292	" 27.	Aurora Borealis.
293	" 30.	Aurora Borealis.
294	" 31.	Aurora Borealis.
295	April 26.	Small Aurora Borealis.
296	" 28.	Small Aurora Borealis.
297	" 29.	Aurora Borealis.
298	May 31.	Aurora Borealis.
299	June 30.	Aurora Borealis.
300	July 1.	White Aurora Borealis.
301	" 2.	Aurora Borealis.
302	" 5.	Aurora Borealis.
303	" 15.	Aurora Borealis.
304	" 19.	Aurora Borealis.
305	" 27.	Aurora Borealis.
306	Aug. 22.	Aurora Borealis.
307	" 25.	Aurora Borealis.
308	Sept. 28.	Aurora Borealis.
309	" 29.	Aurora Borealis.
310	Oct. 13.	Aurora Borealis.
311	" 17.	Bright and red Aurora Borealis.
312	" 25.	Small Aurora Borealis.
313	Nov. 16.	Aurora Borealis.
314	" 21.	Aurora Borealis.

1787.

315	Jan. 17.	Aurora Borealis.
316	Feb. 15.	Aurora Borealis.
317	Mar. 20.	Aurora Borealis.
318	" 21.	Aurora Borealis.
319	April 14.	Aurora Borealis.
320	" 18.	Aurora Borealis.
321	May 15.	Aurora Borealis.
322	" 16.	Aurora Borealis.
323	June 9.	Aurora Borealis.
324	" 10.	Aurora Borealis.
325	" 14.	Aurora Borealis.
326	July 13.	Bright Aurora Borealis.
327	" 14.	Bright Aurora Borealis.
328	Aug. 1.	Aurora Borealis.
329	" 10.	Aurora Borealis.
330	" 19.	Aurora Borealis.
331	Sept. 6.	Aurora Borealis.
332	" 18.	Aurora Borealis.
333	Oct. 4.	Aurora Borealis.
334	" 5.	Aurora Borealis.
335	" 13.	Aurora Borealis.
336	Oct. 31.	Aurora Borealis.
337	Nov. 3.	Red Aurora Borealis.
338	" 4.	Aurora Borealis.
339	" 24.	Red Aurora Borealis.
340	Dec. 1.	Aurora Borealis.
341	" 9.	Aurora Borealis.

1788.

342	Jan. 14.	Aurora Borealis.
343	Feb. 5.	Aurora Borealis.
344	Mar. 27.	Aurora Borealis.
345	" 28.	Aurora Borealis.
346	April 1.	Aurora Borealis.
347	" 29.	Aurora Borealis.
348	May 29.	Aurora Borealis.
349	July 25.	Aurora Borealis.
350	" 30.	Red Aurora Borealis.
351	" 31.	Aurora Borealis.
352	Aug. 1.	Aurora Borealis.
353	" 23.	Aurora Borealis.
354	" 26.	Aurora Borealis.
355	" 27.	Aurora Borealis.
356	" 28.	Aurora Borealis.
357	Sept. 2.	Aurora Borealis.
358	" 3.	Aurora Borealis.
359	" 4.	Aurora Borealis.
360	" 10.	Aurora Borealis.
361	" 24.	Aurora Borealis.
362	" 25.	Aurora Borealis.
363	" 26.	Aurora Borealis.
364	Oct. 4.	Aurora Borealis.
365	" 5.	Aurora Borealis.
366	" 22.	Aurora Borealis.
367	" 30.	Aurora Borealis.

1789.

368	Feb. 15.	Aurora Borealis.
369	" 26.	Aurora Borealis.
370	Mar. 14.	Aurora Borealis.
371	" 26.	Aurora Borealis.
372	" 27.	Aurora Borealis.
373	April 17.	Aurora Borealis.
374	" 26.	Aurora Borealis.
375	" 27.	Aurora Borealis.
376	May 21.	Aurora Borealis.
377	" 22.	Aurora Borealis.
378	" 23.	Vivid Aurora Borealis.
379	" 24.	Aurora Borealis.
380	" 28.	Aurora Borealis.
381	July 25.	Aurora Borealis.

382 July 27. Aurora Borealis.
383 Aug. 18. Aurora Borealis.
384 " 24. Aurora Borealis.
385 Oct. 19. Aurora Borealis.
386 " 23. Aurora Borealis.
387 Nov. 14. Vivid and red Aurora Borealis.
388 Dec. 18. Aurora Borealis.

1790 - 1792, no observations were made.

1793.

389 Mar. 13. Aurora Borealis.
390 Sept. 4. Aurora Borealis.

CATALOGUE OF AURORAS.

Compiled from Dr. Holyoke's Journal.

1786.

391 Feb. 16. Aurora Borealis.
392 " 28. Aurora Borealis.
393 Mar. 19. Aurora Borealis.
394 " 21 or 22. Aurora Borealis.
395 " 25. Aurora Borealis.
396 " 27. Aurora Borealis.
397 " 30. Aurora Borealis.
398 " 31. Aurora Borealis.
399 April 2. Aurora Borealis.
400 " 6. Aurora Borealis.
401 " 11. Aurora Borealis.
402 " 28. Aurora Borealis.
403 " 29. Aurora Borealis.
404 May 23. Aurora Borealis.
405 " 31. Aurora Borealis.
406 June 1. Aurora Borealis.
407 " 19. Aurora Borealis.
408 " 30. Aurora Borealis.
409 July 1. Aurora Borealis.
410 " 2. Aurora Borealis.
411 " 5. Aurora Borealis.
412 " 15. Aurora Borealis.
413 " 17. Aurora Borealis.
414 " 20. Aurora Borealis.
415 " 27. Small Aurora Borealis.
416 Aug. 3. Aurora Borealis.
417 " 22. Aurora Borealis.
418 " 25. Aurora Borealis.
419 " 31. Aurora Borealis.
420 Sept. 3. Aurora Borealis.
421 " 28. Aurora Borealis.
422 " 29. Aurora Borealis.
423 Oct. 13. Aurora Borealis.
424 " 17. Aurora Borealis.
425 Oct. 25. Small Aurora Borealis.
426 " 27. Aurora Borealis.
427 Nov. 15. Small Aurora Borealis.
428 " 18. Aurora Borealis.
429 " 21. Aurora Borealis.
430 Dec. 5. Aurora Borealis.
431 " 15. Aurora Borealis.

1787.

432 Jan. 16. Aurora Borealis.
433 Feb. 6. Aurora Borealis, low.
434 " 9. Aurora Borealis, small and low.
435 " 15. Aurora Borealis.
436 " 20. Aurora Borealis.
437 " 22. Small Aurora Borealis.
438 Mar. 20. Aurora Borealis.
439 " 23. Aurora Borealis.
440 April 14. Aurora Borealis.
441 " 18. Aurora Borealis.
442 " 19. Aurora Borealis.
443 May 16. Aurora Borealis.
444 " 17. Very small Aurora.
445 June 8. Large Aurora Borealis behind clouds.
446 " 9. Aurora Borealis.
447 July 4. Small Aurora Borealis.
448 " 13. Aurora Borealis.
449 " 14. Aurora Borealis: streamers.
450 " 18. Aurora Borealis.
451 " 23. Small Aurora Borealis.
452 Aug. 1. Aurora Borealis.
453 " 10. Aurora Borealis.
454 " 13. Small Aurora Borealis.
455 " 19. Aurora Borealis.
456 Sept. 6. Small Aurora Borealis.
457 " 18. Aurora Borealis.

458 Oct. 5. Aurora Borealis.
459 " 7. Small Aurora Borealis.
460 " 13. Bright Aurora Borealis.
461 Nov. 3. Aurora Borealis.
462 " 4. Red Aurora Borealis.
463 Dec. 8. Small Aurora.
464 " 9. Small Aurora.

1788.

465 Jan. 10. Small Aurora Borealis.
466 " 14. Small Aurora Borealis.
467 Feb. 5. Aurora Borealis.
468 July 2. Very small Aurora Borealis.
469 " 25. Small Aurora Borealis.
470 " 30. Notable Aurora Borealis.
471 " 31. Small Aurora Borealis.
472 Aug. 1. Aurora Borealis.
473 " 23. Aurora Borealis.
474 " 26. Aurora Borealis.
475 " 27. Aurora Borealis.
476 " 28. Aurora Borealis.
477 Sept. 2. Aurora Borealis.
478 " 3. Aurora Borealis.
479 " 4. Aurora Borealis.
480 " 5. Aurora Borealis.
481 " 25. Aurora Borealis.
482 " 26. Aurora Borealis.
483 " 28. Small Aurora Borealis.
484 Oct. 5. Small Aurora Borealis.
485 " 6. Small Aurora Borealis.
486 " 22. Aurora Borealis.
487 " 30. Small Aurora Borealis.

1789.

488 Feb. 16. Aurora Borealis.
489 " 26. Aurora Borealis.
490 Mar. 14. Bright Aurora Borealis.
491 " 26. Small Aurora Borealis.
492 " 27. Small Aurora Borealis.
493 " 29. Small Aurora Borealis.
494 April 26. Aurora Borealis.
495 " 27. Aurora Borealis.
496 May 20. Aurora Borealis.
497 " 21. Aurora Borealis.
498 " 22. Aurora Borealis.
499 " 23. Aurora Borealis.
500 " 24. Aurora Borealis.
501 " 28. Aurora Borealis.
502 June 15. Aurora Borealis.
503 " 17. Aurora Borealis.
504 " 19. Aurora Borealis.
505 June 20. Aurora Borealis.
506 " 23. Aurora Borealis.
507 " 29. Aurora Borealis.
508 July 11. Aurora Borealis.
509 " 20. Aurora Borealis.
510 " 26. Aurora Borealis.
511 " 28. Aurora Borealis.
512 " 29. Aurora Borealis.
513 Aug. 18. Bright Aurora Borealis.
514 " 24. Aurora Borealis.
515 Sept. 9. Aurora Borealis.
516 " 21. Aurora Borealis.
517 " 22. Aurora Borealis.
518 Oct. 16. Aurora Borealis.
519 " 19. Aurora Borealis.
520 " 21. Aurora Borealis.
521 " 23. Aurora Borealis.
522 Nov. 14. Aurora Borealis, beautifully variegated, and reaching to the zenith.
523 Dec. 13. Aurora Borealis.
524 " 14. Aurora Borealis.

1790.

525 Feb. 11. Aurora Borealis.
526 " 13. Aurora Borealis.
527 May 16. Aurora Borealis.
528 " 17. Aurora Borealis.
529 June 3. Aurora Borealis.
530 " 30. Aurora Borealis.
531 July 1. Aurora Borealis.
532 " 3. Aurora Borealis.
533 Aug. 16. Aurora Borealis.
534 Oct. 31. Aurora Borealis.
535 Nov. 10. Aurora Borealis.

1791.

536 Jan. 6. Aurora Borealis.
537 " 12. Aurora Borealis.
538 Mar. 22. Aurora Borealis.
539 " 26. Aurora Borealis.
540 April 1. Small Aurora Borealis.
541 " 5. Aurora Borealis.
542 July 22. Aurora Borealis.
543 " 23. Aurora Borealis.
544 " 24. Aurora Borealis.
545 " 28. Aurora Borealis.
546 Dec. 27. Aurora Borealis.

1792.

547 July 7. Aurora Borealis.
548 " 11. Aurora Borealis.

549 July 18. Aurora Borealis.
550 Oct. 13. Aurora Borealis.
551 " 14. Aurora Borealis.
552 Dec. 15. Aurora Borealis.

1793.

553 Jan. 12. Aurora Borealis.
554 " 13. Aurora Borealis.
555 Mar. 13. Aurora Borealis.
556 Aug. 6. Aurora Borealis.
557 " 28. Aurora Borealis.
558 Sept. 2. Aurora Borealis.

1794.

559 April 30. Aurora Borealis.

1795.

560 May 24. Aurora Borealis.
561 Oct. 14. Aurora Borealis.

1796 – 1801, none.

1802.

562 June 16. Aurora Borealis.

1803.

563 Mar. 19. Aurora Borealis.
564 Aug. 23. Evening. Aurora Borealis.
565 Sept. 17. Evening. Aurora Borealis.
566 " 19. Evening. Aurora Borealis.

1804.

567 April 1. Evening. Aurora Borealis.
568 May 2. Evening. Aurora Borealis.
569 " 12. Evening. Aurora Borealis.
570 Nov. 5. Evening. Aurora Borealis.

1805.

571 Jan. 4. (10^h, P. M.) Aurora Borealis.
572 Sept. 15. (10^h, P. M.) Aurora Borealis.
573 Nov. 18. (10^h, P. M.) Aurora Borealis.

1806.

574 April 13. (10^h, P. M.) Aurora Borealis.
575 Oct. 5. (10^h, P. M.) Aurora Borealis.

1807, 1808, none.

1809.

576 Jan. 31. (10^h, P. M.) Aurora Borealis.
577 June 13. (10^h, P. M.) Aurora Borealis.

1810 – 1813, none.

1814.

578 Feb. 28. Aurora Borealis.
579 April 17. (10^h, P. M.) Aurora Borealis.
580 Sept. 11. (11^h, P. M.) Aurora Borealis.

1815.

581 Sept. 26. Aurora Borealis.

1816, 1817, none.

1818.

582 Sept. 27. (10^h, P. M.) Aurora Borealis.

1819.

583 Feb. 19. (10^h, P. M.) Aurora Borealis.
584 Mar. 25. (10^h, P. M.) Aurora Borealis.
585 " 26. (10^h, P. M.) Aurora Borealis.
586 Nov. 13. (10^h, P. M.) Aurora Borealis.
587 " 14. (10^h, P. M.) Aurora Borealis.

1820.

588 April 3. (10^h, P. M.) Aurora Borealis.

CATALOGUE OF AURORAS.

Compiled from Dr. Hale's Journal.

1818.

589 Sept. 26. (12^h, P. M.) Aurora Borealis.

1819.

590 Feb. 19. Aurora Borealis.
591 Mar. 25. Brilliant Aurora Borealis.
592 Oct. 12. Aurora Borealis in evening.
593 Nov. 13. (10^h, P. M.) Aurora Borealis.

1820.

594 April 3. Aurora Borealis, night of 3 – 4.

1821, none.

1822.

595 Oct. 22. Aurora Borealis.

1823, 1824, none.

1825.

596 April 14. Aurora Borealis.
597 Dec. 7. Aurora Borealis.

1826, none.

458	Oct.	5.	Aurora Borealis.
459	"	7.	Small Aurora Borealis.
460	"	13.	Bright Aurora Borealis.
461	Nov.	3.	Aurora Borealis.
462	"	4.	Red Aurora Borealis.
463	Dec.	8.	Small Aurora.
464	"	9.	Small Aurora.
		1788.	
465	Jan.	10.	Small Aurora Borealis.
466	"	14.	Small Aurora Borealis.
467	Feb.	5.	Aurora Borealis.
468	July	2.	Very small Aurora Borealis.
469	"	25.	Small Aurora Borealis.
470	"	30.	Notable Aurora Borealis.
471	"	31.	Small Aurora Borealis.
472	Aug.	1.	Aurora Borealis.
473	"	23.	Aurora Borealis.
474	"	26.	Aurora Borealis.
475	"	27.	Aurora Borealis.
476	"	28.	Aurora Borealis.
477	Sept.	2.	Aurora Borealis.
478	"	3.	Aurora Borealis.
479	"	4.	Aurora Borealis.
480	"	5.	Aurora Borealis.
481	"	25.	Aurora Borealis.
482	"	26.	Aurora Borealis.
483	"	28.	Small Aurora Borealis.
484	Oct.	5.	Small Aurora Borealis.
485	"	6.	Small Aurora Borealis.
486	"	22.	Aurora Borealis.
487	"	30.	Small Aurora Borealis.
		1789.	
488	Feb.	16.	Aurora Borealis.
489	"	26.	Aurora Borealis.
490	Mar.	14.	Bright Aurora Borealis.
491	"	26.	Small Aurora Borealis.
492	"	27.	Small Aurora Borealis.
493	"	29.	Small Aurora Borealis.
494	April	26.	Aurora Borealis.
495	"	27.	Aurora Borealis.
496	May	20.	Aurora Borealis.
497	"	21.	Aurora Borealis.
498	"	22.	Aurora Borealis.
499	"	23.	Aurora Borealis.
500	"	24.	Aurora Borealis.
501	"	28.	Aurora Borealis.
502	June	15.	Aurora Borealis.
503	"	17.	Aurora Borealis.
504	"	19.	Aurora Borealis.
505	June	20.	Aurora Borealis.
506	"	23.	Aurora Borealis.
507	"	29.	Aurora Borealis.
508	July	11.	Aurora Borealis.
509	"	20.	Aurora Borealis.
510	"	26.	Aurora Borealis.
511	"	28.	Aurora Borealis.
512	"	29.	Aurora Borealis.
513	Aug.	18.	Bright Aurora Borealis.
514	"	24.	Aurora Borealis.
515	Sept.	9.	Aurora Borealis.
516	"	21.	Aurora Borealis.
517	"	22.	Aurora Borealis.
518	Oct.	16.	Aurora Borealis.
519	"	19.	Aurora Borealis.
520	"	21.	Aurora Borealis.
521	"	23.	Aurora Borealis.
522	Nov.	14.	Aurora Borealis, beautifully variegated, and reaching to the zenith.
523	Dec.	13.	Aurora Borealis.
524	"	14.	Aurora Borealis.
		1790.	
525	Feb.	11.	Aurora Borealis.
526	"	13.	Aurora Borealis.
527	May	16.	Aurora Borealis.
528	"	17.	Aurora Borealis.
529	June	3.	Aurora Borealis.
530	"	30.	Aurora Borealis.
531	July	1.	Aurora Borealis.
532	"	3.	Aurora Borealis.
533	Aug.	16.	Aurora Borealis.
534	Oct.	31.	Aurora Borealis.
535	Nov.	10.	Aurora Borealis.
		1791.	
536	Jan.	6.	Aurora Borealis.
537	"	12.	Aurora Borealis.
538	Mar.	22.	Aurora Borealis.
539	"	26.	Aurora Borealis.
540	April	1.	Small Aurora Borealis.
541	"	5.	Aurora Borealis.
542	July	22.	Aurora Borealis.
543	"	23.	Aurora Borealis.
544	"	24.	Aurora Borealis.
545	"	28.	Aurora Borealis.
546	Dec.	27.	Aurora Borealis.
		1792.	
547	July	7.	Aurora Borealis.
548	"	11.	Aurora Borealis.

549 July 18. Aurora Borealis.
550 Oct. 13. Aurora Borealis.
551 " 14. Aurora Borealis.
552 Dec. 15. Aurora Borealis.

1793.

553 Jan. 12. Aurora Borealis.
554 " 13. Aurora Borealis.
555 Mar. 13. Aurora Borealis.
556 Aug. 6. Aurora Borealis.
557 " 28. Aurora Borealis.
558 Sept. 2. Aurora Borealis.

1794.

559 April 30. Aurora Borealis.

1795.

560 May 24. Aurora Borealis.
561 Oct. 14. Aurora Borealis.

1796 – 1801, none.

1802.

562 June 16. Aurora Borealis.

1803.

563 Mar. 19. Aurora Borealis.
564 Aug. 23. Evening. Aurora Borealis.
565 Sept. 17. Evening. Aurora Borealis.
566 " 19. Evening. Aurora Borealis.

1804.

567 April 1. Evening. Aurora Borealis.
568 May 2. Evening. Aurora Borealis.
569 " 12. Evening. Aurora Borealis.
570 Nov. 5. Evening. Aurora Borealis.

1805.

571 Jan. 4. (10^h, P. M.) Aurora Borealis.
572 Sept. 15. (10^h, P. M.) Aurora Borealis.
573 Nov. 18. (10^h, P. M.) Aurora Borealis.

1806.

574 April 13. (10^h, P. M.) Aurora Borealis.
575 Oct. 5. (10^h, P. M.) Aurora Borealis.

1807, 1808, none.

1809.

576 Jan. 31. (10^h, P. M.) Aurora Borealis.
577 June 13. (10^h, P. M.) Aurora Borealis.

1810 – 1813, none.

1814.

578 Feb. 28. Aurora Borealis.
579 April 17. (10^h, P. M.) Aurora Borealis.
580 Sept. 11. (11^h, P. M.) Aurora Borealis.

1815.

581 Sept. 26. Aurora Borealis.

1816, 1817, none.

1818.

582 Sept. 27. (10^h, P. M.) Aurora Borealis.

1819.

583 Feb. 19. (10^h, P. M.) Aurora Borealis.
584 Mar. 25. (10^h, P. M.) Aurora Borealis.
585 " 26. (10^h, P. M.) Aurora Borealis.
586 Nov. 13. (10^h, P. M.) Aurora Borealis.
587 " 14. (10^h, P. M.) Aurora Borealis.

1820.

588 April 3. (10^h, P. M.) Aurora Borealis.

CATALOGUE OF AURORAS.

Compiled from Dr. Hale's Journal.

1818.

589 Sept. 26. (12^h, P. M.) Aurora Borealis.

1819.

590 Feb. 19. Aurora Borealis.
591 Mar. 25. Brilliant Aurora Borealis.
592 Oct. 12. Aurora Borealis in evening.
593 Nov. 13. (10^h, P. M.) Aurora Borealis.

1820.

594 April 3. Aurora Borealis, night of 3 – 4.

1821, none.

1822.

595 Oct. 22. Aurora Borealis.

1823, 1824, none.

1825.

596 April 14. Aurora Borealis.
597 Dec. 7. Aurora Borealis.

1826, none.

1827.

598 Sept. 8. Brilliant Aurora Borealis.
599 " 25. Aurora Borealis.

1828, 1829, none.

1830.

600 June 10. Aurora Borealis.
601 " 11. Aurora Borealis.
602 Dec. 11. Brilliant Aurora Borealis.

1831 – 1834, none.

1835.

603 Nov. 17. Aurora Borealis.
604 " 18. Aurora Borealis.

1836.

605 April 22. Aurora Borealis.

1837.

606 July 1. Aurora Borealis.
607 Dec. 1. Aurora Borealis.
608 " 4. Aurora Borealis in evening.

1838.

609 Feb. 21. Brilliant Aurora Borealis. At 7, P. M., of a beautiful color; at 10, P. M., of a pale color.
610 July 15. Aurora Borealis.
611 Sept. 13. Aurora Borealis in evening.
612 Sept. 16. Aurora Borealis in evening.
613 Nov. 12. (10^h, P. M.) Aurora Borealis. From 11 to 12 very bright.
614 " 14. (9^h, P. M.) Aurora Borealis.
615 " 25. Aurora Borealis.

1839.

616 June 14. (7^h, P. M.) Aurora Borealis. At 10, brilliant.
617 " 19. Evening. Aurora Borealis. From 9 to 10 bright.
618 Sept. 3. Aurora Borealis.

1840.

619 May 29. Aurora Borealis.
620 Aug. 28. (9^h to 12^h, P. M.) Aurora Borealis. At 11, very bright.

1841.

621 Feb. 22. (8^h to 9^h, P. M.) Aurora Borealis.
622 " 23. (11^h, P. M.) Aurora Borealis.

1847.

623 April 7. Aurora Borealis, from 10^h, P. M., to 3^h, A. M., next morning.

1848.

624 April 6. (10^h to 11^h, P. M.) Brilliant Aurora Borealis.

VI.

On a Process of Fractional Condensation; applicable to the Separation of Bodies having small Differences between their Boiling-Points.

By C. M. WARREN.

Communicated May 10th, 1864.

It is well known that the process in general use for the proximate analysis of mixtures of volatile liquids, — viz., that of simple fractional distillation, either from a tubulated retort or from a flask with bulbs, as proposed by Wurtz,* — affords but very imperfect and unsatisfactory results, and not unfrequently leads to gross errors and misconceptions, except in those cases in which the boiling-points of the constituents are widely different, or in which some auxiliary method can be advantageously employed.

The want of a more efficient process for effecting such separations has long been recognized. There are numerous natural and artificial products, of the highest scientific interest, — such as petroleums, essential oils, tars, and other mixtures of oils obtained by the distillation, under varied circumstances, of bituminous, vegetable, and animal substances, — of which it may at least be said that we have but very imperfect knowledge, — I might almost say no knowledge, except such as could be derived from the study of very impure materials, — still mixtures of different bodies, — with which, instead of the pure substances sought for, chemists have felt compelled to content themselves, as the best results which they were able to obtain by the means at their command.

In repeated instances, apparently after persevering and protracted efforts, investigators have been forced to assert either the impossibility, or their inability, to obtain, from such mixtures, bodies of constant boiling-point, — a property which is generally received as a test of purity for liquid bodies.

I may here specify a few recent instances of this kind.

* Annales de Chimie et de Physique, 3e Série, XLII. 132.

1. Warren de la Rue and Hugo Müller,* in their paper entitled "Chemical Examination of Burmese Naphtha or Rangoon Tar," after detailing the preliminary treatment by distillation in a current of steam, add that "A further separation of the various products was effected by repeated fractional distillations; but no absolutely constant boiling-points could be obtained, notwithstanding the great number of distillations and the large quantity of material at command. It is true that considerable portions of distillates could be collected between certain ranges of temperature, tending to indicate a constant boiling-point; nevertheless it soon became evident that distillation alone could not effect the separations of the various constituents, and that recourse must be had to other processes." The other processes resorted to were, treatment with sulphuric and nitric acids, either separately or mixed; but still with very imperfect results. This acid treatment, which was first proposed by De la Rue, and subsequently employed by C. Greville Williams,† Schorlemmer, and others, will be further noticed below.

2. Frankland,‡ in speaking of a mixture of the hydrocarbons of the formulæ $C_n H_n$ and $C_n H_{n+1}$ (now generally considered as $C_n H_{n+2}$), which have a difference of 6° to 7° C. between their boiling-points, says, "The separation of two such bodies by distillation alone is impossible"; and suggests that the employment of anhydrous sulphuric acid may accomplish the object by dissolving out the body of the formula $C_n H_n$.

3. And so recently as 1862, Schorlemmer,§ in his first paper "On the Hydrides of the Alcohol-Radicals existing in the Products of the Destructive Distillation of Cannel Coal," remarks that "it was, however, found impossible to obtain a product of constant boiling-point by repeated fractional distillations"; and he also had recourse to the acid-treatment above referred to.

4. Pebal,|| after an elaborate research on the petroleum from Galicia, in which Wurtz's bulbs were employed, and also Eisenstuck,¶ who made an extended investiga-

* Proceedings of the Royal Society, VIII. 221.

† Philosophical Transactions, 1857, 447.

‡ Quarterly Journal of the Chemical Society, 1851, 3, 43.

§ Journal of the Chemical Society, XV. 419.

|| Annalen der Chemie und Pharmacie, CXV. 20, asserts the "Unmöglichkeit, das Gemenge durch fractionirte Destillationen zu entwirren."

¶ Annalen der Chemie und Pharmacie, CXIII. 169, says as follows: — "Mit den 5° zu 5° aufgesammelten Destillaten wurde die fractionirte Destillation wieder von Neuem vorgenommen, aber nachdem diese Operation sieben Wochen mit etwas 50 Pfund Steinöl fortgesetzt worden war, erhielt ich doch kein Product von irgend constantem Siedepunkt. Nach diesen Versuchen halte ich es für Unmöglich, das Steinöl durch fractionirte Destillationen allein in Producte mit constantem Siedepunkt, zu scheiden."

tion of the petroleum from Sehnde, near Hannover, also with the use of Wurtz's bulbs, both assert in the most positive manner the impossibility of separating from petroleum, by fractional distillation, products of constant boiling-point.

Such is the general character of the results obtained in the attempts which have been made to separate the constituents of such mixtures by fractional distillation.

The treatment with strong acids, etc., as an auxiliary to the common method of fractional distillation, which is claimed to have given good results in some cases, is open to serious objections in its application to mixtures of unknown substances, as must be readily apparent. The further consideration of this subject is reserved for another occasion, when I shall submit the results which I have obtained by my process in the study of mixtures almost identical with some of those in the investigation of which the acid process has been employed. I shall then be able to show that the results obtained by that process are, to a considerable extent, inaccurate and by no means exhaustive; and that it is still of the highest importance to have a process which shall be generally applicable in all such cases, without resort to any harsh and uncertain treatment.

With regard to the value of constancy of boiling-point above referred to, as a test of purity of a liquid substance, I may here say that, without scarcely lessening the importance of obtaining constancy of boiling-point, before resorting to harsher treatment, in the study of mixtures of unknown substances, I think I shall be able to show, on another occasion, that this property is not necessarily indicative of so high a degree of purity as has generally been supposed; and that a body may have a constant boiling-point, and yet contain enough of a foreign substance to appreciably — and, in delicate cases, seriously — affect the determination of its constitution and of some of its other properties. But in no such case have I yet found that the removal of the impurity by chemical means has essentially changed the boiling-point, — *i. e.*, never to the extent of 1° C. of temperature. I propose, at a future time, to study this question synthetically, operating with pure liquid substances, with the view to determine, in a few cases, how much of a foreign substance may be present, — which would probably be variable in different cases, — without sensibly affecting the boiling-point. A solution of this question would, I think, be of considerable practical value in some instances.*

* Since this was prepared for the press I notice that late experiments by Berthelot go to show the correctness of my conception of the value of constancy of boiling-point, as above stated.

Of the New Process.

The chief distinctive feature of my process, as compared with the common one, consists in this, — that the operator has complete and easy control of the temperature of the vapors given off in distillation; and consequently can readily cool these vapors to the lowest limit of temperature which the most volatile portion, under the circumstances, is able to bear and retain its vaporous condition. It will be seen at a glance that, under these conditions, the operator has it in his power to secure in any case the very largest possible amount of condensation of the heavier from the lighter vapors. The liquids resulting from the condensation of the less volatile portions of course fall back into the retort, while the vapors of the more volatile parts continue to go forward to a cold condenser, descending in the opposite direction, from which the condensed product falls into a special receiver. In this manner he is able to obtain, in each successive operation, a series of products which shall contain the minimum quantity of the less volatile constituents, which a single distillation is capable of affording.

Of the common process, on the contrary, nearly the reverse of all this is true: the operator having no control whatever; being forced to receive the vapors at the temperature which they naturally acquire in passing from the retort, and laden with such proportion of the less volatile bodies as may be carried forward with them.*

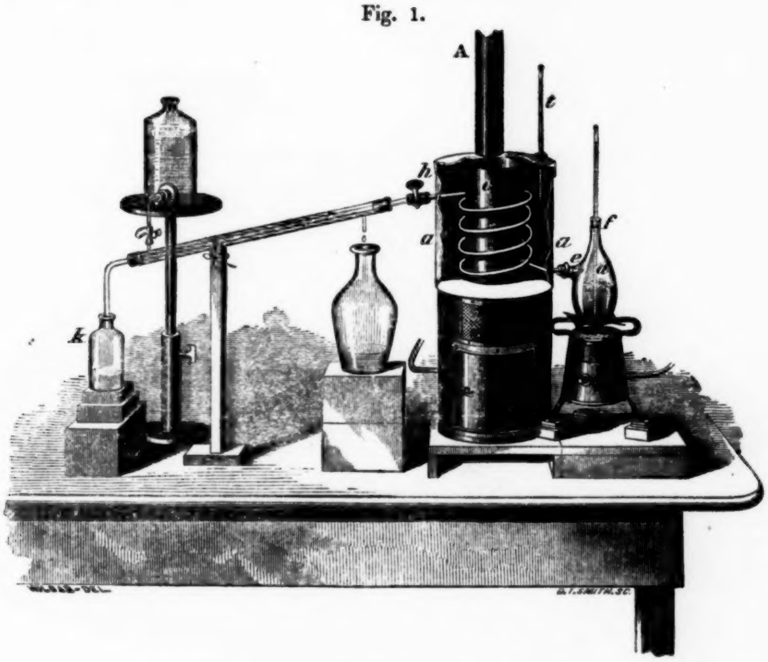
* The only apparatus, of which I have any knowledge, which can be regarded as bearing any analogy to my own, is that employed in the rectification of alcoholic spirits, on a manufacturing scale. In one of the older forms of this apparatus, that of Solimani, to which my attention was first called by a friend, after my process had been in use more than a twelvemonth, the temperature of a dephlegmator is kept within such limits as to give alcohol of any required strength more readily than by the common methods. The mode of construction of this apparatus is, however, only adapted to manufacturing purposes, and it could not be utilized in the more exact experiments required in scientific research. Either on account of its complication, or some other cause, the apparatus of Solimani has, I believe, long since been abandoned.

Mansfield (Quarterly Journal of the Chemical Society, 1849, I. 264), observing that "the boiling-point of benzole is the same as that of alcohol of sp. gr. 0.825," remarks that "any of the summary processes of rectification which are practised by distillers in the manufacture of alcoholic spirits, are applicable to the separation of benzole from the less volatile fluids of naphtha"; and, appended to his scientific treatise on coal-tar, under the title "*Of a Practical Mode of Preparing Benzole*," goes on to describe a process for that purpose, which I believe, he had previously patented. It appears that Mansfield did not employ this process in his research, but obtained his benzole, as well as the other less volatile hydrocarbons, in the usual manner, — by simple distillation.

In the belief that no process of fractioning at all analogous to mine has ever been employed in scientific research, and that I am not in any way directly indebted to any of the devices of my predecessors, I have taken no special pains to consider these devices in much detail. I may say, however, that I have found no

In the new process, perfect control of the temperature of the vapors is secured by simply conducting these vapors upward through a worm contained in a bath, *aa*, Figs. 1 and 2, the temperature of which is regulated by means of a separate lamp, *b*, Fig. 2, or by a safety-furnace, *p*, as shown in Fig. 1. The bath may be of oil or water, or of metal for very high temperatures, as the case may require, and is furnished with a thermometer, *t*.

Fig. 1.

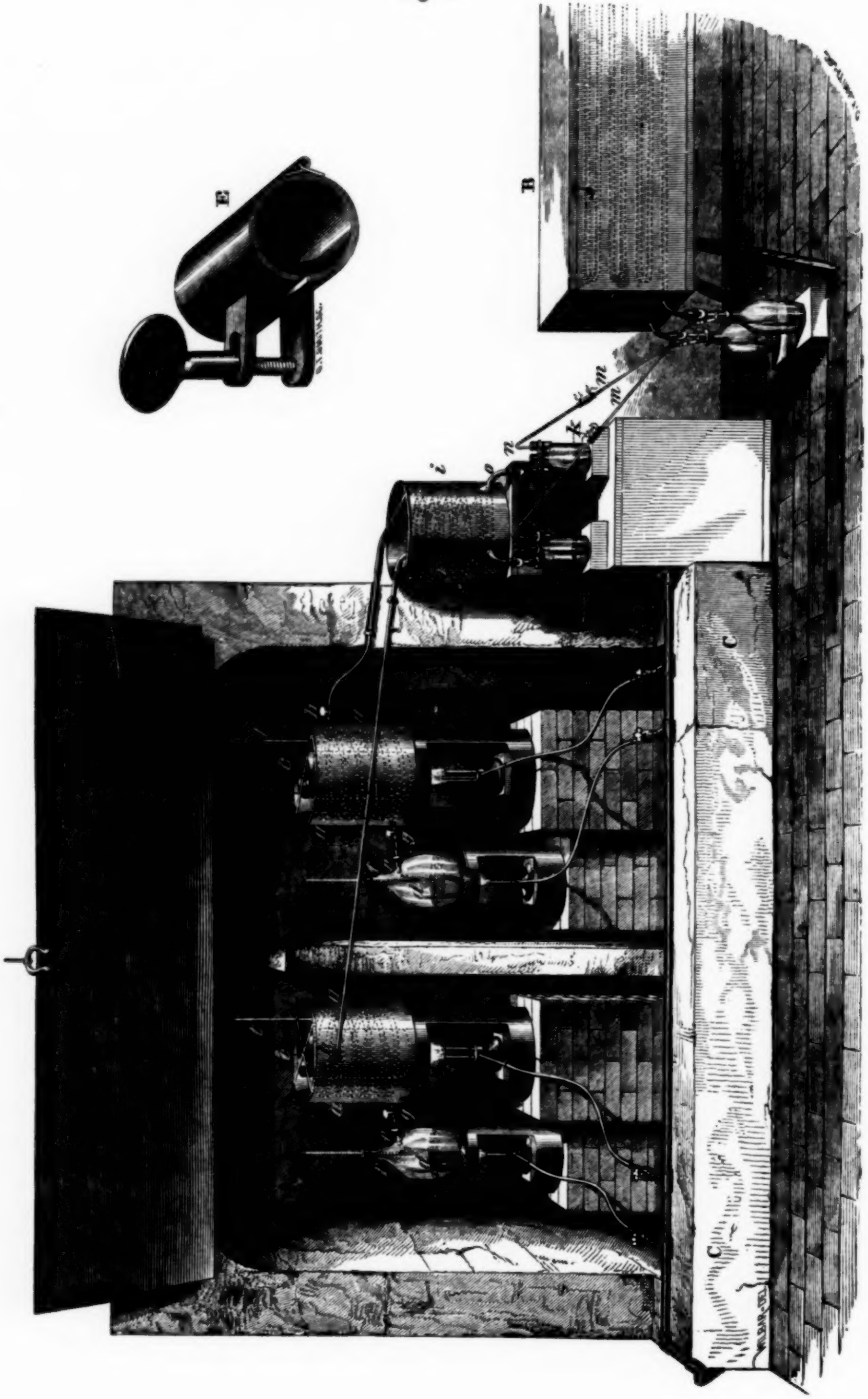


That this bath may be equally adapted for the separation of liquids boiling below the common temperature, an empty vessel, *c*, Fig. 1 and 2, is permanently secured in the interior of the bath by means of straps of metal across the top, to serve as a convenient receptacle for ice or iced water, by means of which a low temperature may be steadily maintained. This interior vessel also serves a good purpose in economizing

record of any one's ever having employed the oil bath and a separate fire to regulate a heated condenser, this being the essential feature on which the superiority of my process is based; adapting it at once to both high and low temperatures, and for the most delicate work.

The employment of bulbs, above referred to, as proposed by Wurtz, is simply a modification of the old process. The bulb apparatus furnishes the same, or, at most, but slightly better results than a simple retort; being no more than equivalent to increasing the height of the sides of the retort itself, without introducing any control over the accuracy of the results; the only advantage gained being, that these results are obtained somewhat more quickly.

Fig. 2.



time, and fuel in heating the bath, as it diminishes the quantity of oil required to cover the worm. It is made to extend to within about three inches of the bottom of the bath, and large enough to fill the greater part of the space in the centre of the coil. The bath and interior vessel are both made of sheet-copper, with joints brazed so that they will bear a high temperature. I generally use, also, copper worms, especially in the earlier distillations, the quantities then operated upon being larger, as such worms are conveniently procured, and not liable to break. In the larger-sized apparatus, the tube of which the worm is made measures ten feet in length and half an inch in diameter. I have tried several lengths of worm and several diameters of tube, but not, as yet, with any special view of determining the precise proportions, in relation to the size of the retort, which would be best adapted to the purpose. There appears, however, to be nothing gained by increasing the length of the worm beyond what is required to reduce the temperature of the vapors to that of the bath. I have in use three sizes of apparatus: the largest has a copper worm 10 feet long and $\frac{1}{2}$ inch bore; the medium size, a worm 5 feet long and $\frac{3}{8}$ inch bore; and the smallest size, for very small quantities, a worm 1 foot 6 inches long and $\frac{1}{4}$ inch bore. Each of these has been found to answer a good purpose. The distillation may be conducted in a glass flask, or more conveniently in a glass retort of the form shown at *d*, Fig. 1 and 2. The body of this retort, as appears in the figure, is of the form of the corresponding part of the common retort; but which, in place of a long neck, has only a short tubulure, *e*, in the side, for escape of the vapors, and another tubulure, *f*, in the top, which contains the thermometer, and through which the retort is charged.

In the larger apparatus the retort is connected with the lower end of the elevated worm by means of a glass tube of about the same diameter as the end of the worm. One end of this tube enters the retort at the lateral tubulure through a perforated cork, and the other end is joined to the end of the worm either by being firmly bound with a strip of cloth thickly covered with vulcanized caoutchouc, — such as is found in commerce, — or by means of a perforated cork, which is made to fit the ends of both tubes as snugly as possible, and then tightly pressed together upon the joint by means of an iron clamp, as shown at *g*, Fig. 2. This clamp is figured on a larger scale at E. As it is highly important that all joints in the apparatus should be perfectly tight, inasmuch as the least leakage, when continued a long time, would cause, in the aggregate, a serious loss of material, I would call special attention to the clamp joint as the best which I have tried. Before falling upon this device I had used exclusively the vulcanized caoutchouc joints, which were found to answer a good purpose, in most cases,

except that they required too frequent renewal. I have found the cloth covered with vulcanized caoutchouc preferable to the common caoutchouc tubing. In the smaller sizes of apparatus I have the end of the worm itself project far enough from the bath to connect directly with the retort by means of a perforated cork, without the use of an additional connecting tube.

The upper end, *h*, of the elevated worm is brought out through the side of the bath at a point about three inches below the top; so that, when working with a low temperature of the bath, the worm may still be completely covered with oil, and also give sufficient space above the worm for the expansion of the oil when higher temperatures are employed. To avoid contaminating the atmosphere of the laboratory with the disagreeable fumes which are given off, in large quantity, from such a mass of heated oil, the top of the bath is tightly closed with a sheet-iron cover, from which a small funnel, A, Fig. 1, conducts these fumes to a chimney.

In the larger apparatus, the vapors which succeed in passing through the heated worm are conducted downward into a cooled worm contained in a bath of water, *ii*, Fig. 2, and the liquid product is collected in the receiver, *k*. The cold bath, *ii*, contains two condensing worms, — one for each apparatus, — and is large enough to condense for both without the necessity of renewing the water. I have represented two apparatuses combined, as it will be found more economical of time to operate with two at once. In the smaller apparatus, for the table, a Liebig condenser may be conveniently substituted for the cold worm, as shown in Fig. 1.

For collecting liquids which boil below the common temperature, when such are present, I attach a refrigerator, B, Fig. 2, which is provided with two block-tin condensing-tubes, — one for each apparatus. These are bent in a zigzag form, and attached to the inner sides of the refrigerator. The lower ends of the tubes extend through the end of the refrigerator far enough to form a convenient connection with the second receiver, *l*, Fig. 2, which communicates with the first receiver, *k*, by means of the glass tube, *m*.

In order to successfully collect and condense the vapors of such extremely volatile liquids as are now under consideration, it is of course indispensable that the apparatus should be constructed with very tight joints; and for greater convenience, but more especially to prevent breakage, such of the joints as require to be frequently taken apart should be made flexible. A very convenient and perfectly tight joint of this kind may be made as follows: — the short stationary tube, *n*, in the cork of the receiver, *k*, Fig. 2, is made with the opening somewhat divergent upward; the end, *o*, of the worm is enough smaller than the inside diameter of the upper end of the tube,

n, to leave room for a piece of caoutchouc tube to be drawn over it, and still admit of its being inserted in the end of the tube, *n*; the flexible tube is drawn on far enough to prevent the drops which form on the end of the worm from coming in contact with the caoutchouc; a perfectly tight and convenient flexible joint is now made by pressing the tube, *n*, over the caoutchouc covering of the end of the worm, *o*. The joints of the receivers, *ll*, are made in the same manner.

The vapors which escape condensation in *ii* pass through the receivers, *kk* and *ll*, to the refrigerator B, which contains ice, or a mixture of ice and salt, are there condensed and fall back into the receivers, *ll*; which should stand in a wooden vessel also containing ice or a freezing mixture. The refrigerator, B, is made with double bottom and sides, with an inch space between, which is filled with pulverized charcoal. Being tightly covered, a charge of ice and salt will serve for a long day's operations without renewal. In this manner I have been able to collect, in considerable quantity, bodies boiling nearly at 0° C., and this from mixtures in which such bodies had been quite overlooked by previous investigators.

It will be observed, on reference to Fig. 2, that the larger distilling apparatus is represented as standing in a brick fire-place, with brick-work, CC, a few inches high, built up in front; and a sheet-iron apron, DD, folded above. This is for security against fire in case of accident, either to the retort or hot bath of oil. As arranged, the contents of either or both of these could run out and burn without danger to the operator or the premises, as the brick-work in front would prevent the liquid from spreading beyond the fire-place, and the dropping of the sheet-iron apron would cause an additional draft, and thus insure the passage of the flames into the chimney. Instead of placing the apparatus in a fire-place, where that is not convenient, equal security against accidents may be attained by the use of my safety heating-lamp,* *q*, Fig. 1, to heat the retort, and safety-furnace, *p*, containing a Bunsen's burner, for heating the bath. The bottom of this furnace, and also a large part of the sides, is formed of wire gauze, such as described for the safety-lamp.† The gauze upon the bottom need not be permanently attached to the furnace, but may be simply laid over an opening cut in the stool or board on which the furnace is to be placed; if the furnace be then set upon it, taking care that the joint shall be tight around the edge, nothing more will be required. A strip of vulcanized caoutchouc, about an eighth of an inch in thickness, is riveted around the edge of the opening for the door; against this the door tightly closes, so that no ignition can take place through the cracks which would otherwise remain under the edges of the door.

* American Journal of Science, 1862 (2), XXXIII. 275.

† *Loc. cit.*

For an apparatus to stand upon the table, the safety-lamp and furnace are especially desirable. I have also used them for the larger apparatus, placed upon the floor of the laboratory. As a practical test of the security which they afford, I may relate an incident which happened to myself. I had left the laboratory for a short time, with such an apparatus in full operation; the retort containing nearly a quart of light petroleum boiling below 100° C. Having been detained longer than I expected, on returning I found the laboratory filled with the vapors of hydrocarbons; and, on approaching the retort, found that the caoutchouc joint, connecting the retort with the elevated worm, had failed, and that the larger portion of the liquid had distilled into the room, having been mainly condensed in the upper worm, and conducted thence down the outside of the retort into the safety-lamp. This process was still going on, the lamp being highly-heated from the excess of fuel thus added to it, but no ignition took place outside the lamp. Although this experiment was rather injudicious, it furnishes a valuable test of the efficiency of the safety-lamp and furnace.

Having described the apparatus, I now proceed to give such details of the method of conducting the separations as have been found, in my experience, most efficient and economical of time. In commencing with a crude mixture of unknown liquids, I deem it advisable to operate at once on a tolerably large quantity of material, especially if the constituents are supposed to be numerous, and to omit chemical treatment till after the separations have so far progressed as to indicate the number and species of bodies present, and, approximately, their several boiling-points.

Notwithstanding the precautions taken to avoid loss from evaporation and leakage, I have at times been surprised at the large waste of material which has been made apparent after a long series of operations. When it is considered, however, that the time required to make a complete separation of a very complex mixture of liquids must necessarily be very protracted, during which more or less of evaporation is constantly taking place, it will be a matter of no surprise that the loss is so considerable. The quantity of material required must depend also on the proportions in which the various constituents are contained in the crude mixture, and upon their degree of volatility; but as these cannot be known *a priori*, it may suffice to make a single preliminary distillation of a portion of the mixture, from a tubulated retort, to ascertain the range of temperature within which it distills, noting at the same time the proportions which come over between certain temperatures; as, for example, below 50° C.; between 50° and 100°, etc.; from these data one may judge pretty nearly of the quantity which it will be advisable to take. It is evident that, when very volatile

bodies are present, even in considerable proportion, a much larger quantity would be required than if the material were but slightly volatile; as the waste in the former case, from evaporation, would be much greater.

But in many cases it will be found that highly volatile bodies are present only in very small proportion, — e. g. in viscid petroleums like Rangoon tar, and in the products of distillation of some species of asphalt. In such cases, the requisite quantity to be operated upon, to obtain the most volatile constituents in sufficient quantity for anything like a complete study of their chemical relations, would be extremely large, — too large to be conducted in the laboratory, — and one would have to resort to the manufactory for the first distillation. I have dwelt at some length on this point, having experienced the disappointment which one feels, after months of labor, on finding the products insufficient for his requirements, when the expenditure of a little more time, comparatively, might have given double the quantities obtained.

In the first series of fractioning I generally operate on successive portions, of about one gallon each, of the crude material, and take off a fraction for every 20° C. rise of temperature of the retort. These fractions are preserved in well-stoppered bottles, and each carefully labelled with the temperatures between which it was obtained. The fractions for each fresh portion of the crude material, being collected between the same limits of temperature, are added to the corresponding products from the preceding operations, till enough of the crude material has been taken to insure, ultimately, a sufficiency of the pure products.

In the commencement, not only of this but of all subsequent fractionings, when the temperature to which the bath should be raised is unknown, I first bring the liquid in the retort into full ebullition, so that a steady stream of liquid shall flow back from the end of the worm into the retort. I then carefully raise the temperature of the bath until the vapors from the retort pass through the heated worm so freely that the liquid, in condensing from them, shall drop with tolerable rapidity into the cold receiver. In order that this dropping may be continuous, it is necessary that the temperature of the bath should rise *very gradually* as the more volatile constituents of the mixture are taken off; this is easily effected by carefully regulating the flame under the bath.

It is advisable to boil the retort as rapidly as possible without choking the lower end of the heated worm with the returning liquid. As this choking would give rise to additional pressure in the retort, and consequently occasion abnormal elevation of the temperature, and possibly a rush of liquid into the receiver, and thus introduce irregularities in the work, excessive heat under the retort should be avoided. The

first indication of choking of the worm is a partial or entire stoppage of the stream of liquid which normally flows steadily from the end of the worm into the retort. Any interruption or unsteadiness of this flow would indicate too rapid ebullition.

As a rule, other things being equal, the greater the difference between the temperature of the bath and that of the retort, the slower the products will come off, and the more effectual will be the separation. I think it possible, however, that the earlier fractionings may be conducted so slowly that the loss of time would more than counterbalance what might be gained by more thorough separation, and that equally good results may be more economically obtained by more frequent operations, somewhat more rapidly conducted.

A striking illustration of the advantage to be gained by this process is presented by the fact that, during the first fractioning of a crude mixture, such as American petroleum or coal-tar naphtha, for example, the difference between the temperature of the bath and that of the retort may sometimes be as much as 35° C., or even more. While, as the products become purer, this difference between the temperatures of the bath and retort proportionally decreases, till finally, in operating on a pure product, the temperature of the bath must be brought to within a few degrees of that of the retort, in order to bring the vapors through. But the amount of this difference is variable for different bodies of equal purity.

These first fractionings must necessarily be quite arbitrary; for, as a general rule, when operating on such mixtures as those just mentioned, neither the thermometer nor the quantities obtained for any given range of temperature will indicate any decided preponderance of any one substance. On the contrary, the temperature rises uniformly, and about the same quantity is generally obtained for the same number of degrees of temperature throughout the operation. In other mixtures, in which certain bodies may seem to be present in much larger proportion than others, or in which there may be a greater difference between the boiling-points of the constituents than in the cases referred to, — facts which would be indicated by the thermometer of the retort, and by the relative quantities of the products obtained, — there might be something gained by exercising discretion in taking off fractions according to these indications.

In the second series of fractioning, the first or lowest fraction of the preceding series, which is large enough to operate upon by itself, is transferred to the retort, and brought into ebullition. The temperature of the bath is then adjusted as above described, and the distillation continued, the fractions obtained being placed in their appropriate bottles until the temperature of the retort shall have risen to, or somewhat above, the point at which the second or next succeeding fraction of the first series may be sup-

posed, or has been found by experiment, to boil. This fraction is then added to the residue in the retort, and the distillation is continued as before. In the same manner, I proceed with the remaining fractions of the first series.

All subsequent fractionings are similarly conducted. As the work progresses, however, the fractions are taken for a gradually decreasing number of degrees of temperature, until finally it becomes necessary, for the attainment of absolute constancy of boiling-point, to take off a fraction for every degree, centigrade; and to continue thus to operate on these fractions, each representing one degree of temperature, until the desired end is attained.

The operator will observe that, in each series of fractions, in which each fraction has been taken for the same range of temperature, the difference between the boiling-points of any two contiguous fractions is nearly the same as the difference between any other two contiguous fractions, — in other words, that the difference referred to approximates to a common difference throughout the same series. Once ascertained, this difference serves as a valuable guide in determining with sufficient accuracy when to add the next fraction to the retort. By observing this systematic course, irregularities, from the improper mixture of products, may be avoided, and time thus economized.

After a few series of fractionings, — sometimes after two or three, variable in number, according to the nature or complication of the mixture, — it will be found that some of the fractions are considerably larger than others for the same range of temperature, indicating approximately the boiling-points of the several constituents. But fractions of constant boiling-point, or those, the boiling-points of which cannot be sensibly changed by further fractional condensation, are not obtained, as already mentioned, till after repeated careful fractioning for every degree of temperature. When fractioning for every degree, it is important to use every precaution to protect the thermometer from external influences, and to carefully apply the corrections for variations in the atmospheric pressure. This may even be desirable earlier; but it is of so much importance in the case specified, that, if omitted, the operator would be liable one day to mix products which he had separated the day previous.

In this way, certain larger fractions are obtained, which are not susceptible of further alteration in their boiling-points; but there are yet considerable quantities of liquid in the intermediate fractions, which still continue to change more or less in each succeeding operation. When the fractions of constant boiling-point have once been obtained, if it were not important to test for other bodies in the intermediate fractions, the operation might here be suspended, provided the pure products already obtained should be large enough for the purposes required.

But in my investigations, I have undertaken to prove the negative as well as the positive. I have attempted to carry the process of separation so far, that I might assert the absence of other bodies, as well as the presence of those obtained; and this clearing up of the intermediate fractions has generally been the most tedious part of the work. I have continued to operate upon these by themselves, until they also have become distributed in regular course — no new bodies appearing — among the fractions of constant boiling-point, or to such an extent that the intermediate quantities have become too small to admit of further continuance of the process.

This process has been in constant use in my laboratory during the last three years. In this time it has been applied in the study of petroleums, coal oils, the more volatile parts of coal- and wood-tars, the essential oil of cumin, commercial fusel oil, from corn whiskey, and even to mixtures more complex than either of these. As the result of this long experience, I can say that as regards bodies not decomposed by heat in distillation, I have not yet found a mixture so complex that it may not be resolved by this process into its proximate constituents so completely, that these shall have almost absolutely constant boiling-points. In repeated instances, even from petroleums, I have obtained these constituents so pure, that the contents of an ordinary tubulated retort charged with one of them has been completely distilled off without any essential change of temperature; i. e., not to the amount of $\frac{1}{2}^{\circ}$ C., the thermometer frequently remaining absolutely constant for more than half an hour, a constancy of boiling-point not exceeded by that of distilled water. This state of purity, I think I may safely assert, has never before been attained from such mixtures by any system of fractional distillation.

As I shall soon be prepared to present to the Academy detailed results of the investigations above referred to, I may omit further allusion to them on this occasion.

I would remark, in conclusion, that it seems to me not improbable that this process may ultimately prove to be of great value in the arts. It is not too much to anticipate that, whenever the various constituents of the mixtures referred to shall have been separately and thoroughly studied in a pure state, some of them may be found to possess properties which will give to them great commercial value, sufficient to justify the expenditure necessary to separate them in large quantities.

VII.

Researches on the Volatile Hydrocarbons.

By C. M. WARREN.

Communicated October 11th, 1864.

Introductory Remarks. — While engaged, a few years since, in attempting to separate some of the constituents of coal-tar naphtha by the common process of fractional distillation, I was forced to the conviction that that process could not be safely relied upon for anything like a complete and accurate analysis of such a complex mixture of liquids; and that, at best, the products thus obtained could not be regarded as anything better than remote approximations to pure substances; leaving reason to fear that there might still be other bodies present, in lesser quantities perhaps, which had escaped detection.

An examination of the results of previous researches on tars, petroleums, etc., served in general to confirm the impressions induced by my own less extended experiments; and to increase, rather than lessen, the doubts already existing in my mind as to the trustworthiness of the results which had hitherto been published concerning the neutral constituents of such mixtures. Influenced by these considerations, and by the belief that, if I could succeed in finding a process capable of effecting a more complete separation of the constituents of such mixtures, it might probably lead to the discovery of new bodies, lying between those which had already been described, — I was led to undertake the researches, the results of which I am about to record. Even if this chief purpose should fail, I was convinced that the expenditure of labor in isolating those bodies in a state of greater purity, would be amply compensated by the much needed confirmation, or perhaps correction, of the results previously published, in addition to the valuable incidental evidence of the absence of other bodies which would thus be furnished. The results which I have obtained in the pursuit of this object are abundantly sufficient to show that I did not undervalue the work of my predecessors, nor over-estimate the importance of the work before me.

The success which attended my efforts in search of a better process of separation

has already been described in detail, in a memoir "On Fractional Condensation," etc. (Memoirs of the American Academy, 1864.)

This new process was first applied, more especially for the purpose of testing its efficiency, in the separation of benzole from coal-tar naphtha. This mixture was selected for the test on account of the property which benzole possesses, in contradistinction from its associates, of being crystallizable at a low temperature, thus affording an additional test of the purity of the product which might be obtained by the process of fractioning. Somewhat to my surprise I found that, after only the fifth series of fractionings, I had obtained benzole so nearly pure that the whole of it would distil from a tubulated retort between 80° and 81° C.; and that when congealed, which was effected by placing the containing bottle in pounded ice, not a drop of liquid could be poured from the mass of crystals. From this result, — which, at the least, indicated a near approximation to purity, — taken in connection with other favorable indications, I felt confident that I had accomplished my first object, and had found a process that could, in all probability, be successfully applied in the study of the petroleums, which up to that time (1861) had baffled every attempt to resolve them into their proximate constituents.

Being naturally anxious to apply the new process in this seemingly more promising field of inquiry, I at once suspended, for the time being, my operations on coal-tar naphtha, and commenced simultaneously the investigation of Pennsylvanian petroleum, and of the oils distilled from Albert coal (from Hillsboro, New Brunswick) in the process of manufacturing illuminating oil. These two substances, neither of which had ever been made the subject of special scientific investigation, were selected as being fair representative types, on the one hand of the native liquid petroleums, and on the other of the artificial coal oils. The comparative study of these two substances seemed to promise additional interest on account of the close analogies which they present, especially when this circumstance is considered in connection with the fact of their great diversity of origin. This is the limit which at that time was assigned for these researches; my intention being, so soon as the separations and analyses should be completed, and the boiling-points and some of the other more important physical characteristics determined, briefly to publish the results, together with the process of fractioning, — preliminary to a complete memoir at a more advanced stage of the work. Before this work had been accomplished, however, it became evident that the bodies contained in these mixtures could not be studied so satisfactorily by themselves as in comparison with other series of hydrocarbons, especially with reference to certain important questions of more general interest; for example, the question in regard

to the increment of boiling-point corresponding to the addition of $C_2 H_2$ in homologous series. It was therefore deemed advisable to extend the inquiry so as to include the naphthas from coal- and wood-tars, and the oil of cumin. And there are still other mixtures of hydrocarbons, that have been made the subjects of previous research, which must yet be brought into the work, in order to clear up, in a satisfactory manner, the confusion and obscurity that seem to exist in our publications regarding some questions relating to the different series of this class of bodies.

This digression from my original plan, having caused much additional labor, has necessarily delayed publication longer than was desirable, until now the results of more than three years of work have accumulated. In this connection I may remark, so far as regards petroleum, that I had nearly completed the fractional separations — except of the bodies of high boiling-point — so long ago as June, 1862, having been for a long time occupied with this work before the appearance, in that month, of the first memoir of Pelouze and Cahours on the same subject. At that time my work was considerably in advance of theirs, and their results differed widely from mine in some important particulars; yet after the publication of their memoir I felt reconciled to a continuance of the delay which had been caused by the change of plan above mentioned, considering it due to these chemists that they should have time to complete the publication of the results of their investigations before I should publish mine. Similar remarks might be made respecting the first publication of Schorlemmer, which appeared soon after, on the products of distillation of cannel coal; this substance being so closely analogous to the Albert coal (upon the products of which I had at that time been long engaged) as to induce the belief that, under the same circumstances, either would afford the same products.

I.

On the Volatile Hydrocarbons from Coal-tar Naphtha, Oil of Cumin, and Cuminic Acid.

PART I.

HYDROCARBONS FROM COAL-TAR NAPHTHA.

In presenting the results of a re-examination of a series of substances upon which so much labor had already been bestowed, and upon the nature and properties of which so little doubt has seemed to exist, it may confer an interest on the subject to state briefly some of the more important results and conclusions that previous investigators have arrived at in the study of these substances.

The discovery by Faraday,* in 1825, of benzole ("bicarburetted hydrogen") in the oil compressed from oil-gas, rendered it highly probable, and indeed led this distinguished philosopher to suspect, that this substance might be found in coal-tar naphtha. His search for it, however, proved unsuccessful, it having been first detected by Hofmann in 1845.† This chemist, however, did not attempt to isolate this body, and the bare fact of its presence appears to be all that was definitely known of the composition of coal-tar naphtha prior to 1849, in which year Mansfield‡ published his elaborate and valuable research, being the first effort at a proximate analysis of this mixture which appears to have been attended with any considerable measure of success. Although a fatal accident, while engaged in his experiments, prevented Mansfield from completing the investigation which he had so well begun, yet the work that he had already published in an unfinished state must always be regarded as having contributed much towards a clear and definite knowledge of the nature of the neutral pyrogenous oils contained in coal-tar naphtha. Indeed, it may be said that little has since been added to our knowledge on this subject. Notwithstanding the incompleteness of his separations of the hydrocarbons, the extent to which he had carried them, with the limited means employed, is truly remarkable, and could not have been accomplished without an expenditure of labor, and a degree of patient endurance, which only those who have experienced the tediousness of such operations can appreciate.

Mansfield claimed to show that the light coal-tar naphtha is composed of a mixture of four distinct hydrocarbons, boiling within the range of 80° to 175°, C.; and probably having the general formula $C_n H_{n-6}$. The first of these, which he found to boil constant at 80°, was proved to be identical with benzole, $C_{12} H_6$. The second, boiling at about 113°, was determined, from certain reactions, to be identical with toluole, $C_{14} H_8$. The special study of this body was deferred, however, with the remark that it had not yet been isolated in a state of sufficient purity to claim an analysis. The third body, boiling at about 140° to 145°, was said to present all the characteristics of cumole, $C_{18} H_{12}$; but this view was not founded on a careful study and comparison of the chemical or physical properties of these bodies, but was merely an expression of opinion in advance of anticipated results. Of the fourth body, boiling at about 170° to 175°, Mansfield remarks that it bears so strong a resemblance, in odor and other properties, to cymole, $C_{20} H_{14}$, as to induce the belief that this substance is

* Philosophical Transactions of the Royal Society, 1825, CXV. 465.

† Annalen der Chemie und Pharmacie, 1845, LV. 200.

‡ Quarterly Journal of the Chemical Society, 1849, I. 244.

identical with the hydrocarbon existing in oil of cumin. It thus appears that of the four bodies which Mansfield detected in coal-tar naphtha, benzole is the only one which he had studied in any detail. Indeed he distinctly states that the others had not yet been isolated in such a state of purity as to entitle them to analysis. And yet his conjectures as to the identity of these bodies, thrown out by way of preliminary notice of results which were acknowledged to be incomplete and inaccurate, have nevertheless been extensively quoted, and generally received as established facts. In addition to the bodies already mentioned, Mansfield also detected the presence of a body more volatile than benzole, having an alliaceous odor, which he found to boil between 60° and 70°. Ritthausen* made a re-examination of the light coal-tar naphtha, in order to obtain the hydrocarbons in a state of greater purity, and to prove the correctness of Mansfield's view of the composition of this naphtha. In regard to the results which he obtained, he says they fully confirm those of Mansfield. Of the body which Mansfield designated as probably identical with cymole, and of the oil more volatile than benzole, Ritthausen obtained quantities too small to admit of investigation. In regard to the latter, however, he remarks,† that to Mansfield's account he can add, that "its nitro-product quite resembles that of benzole, and hence that at all events it belongs to the series $C_n H_{n-6}$, and perhaps has the formula $C_{10} H_4$."‡ It is to be regretted that Ritthausen also omitted to analyze and determine the vapor density of any one of these substances, he having added, therefore, nothing more than a confirmation of the results of Mansfield. He gives the boiling-point of benzole at 80°, of toluole at 109°, and of the so-called cumole at 139–140°, which will be found to agree very nearly with my own determinations. Church,§ in the following year, published a paper on the "Determination of Boiling-points" in the "Ben-

* Journal für praktische Chemie, 1854, LXI. 74.

† "Ich kann den Angaben von Mansfield über das letztere nur das hinzufügen, das seine Nitroproducte denen des Benzols, etc. ganz ähnlich sind, daher es jedenfalls der Reihe $C_n H_{n-6}$ angehört und vielleicht die Formel $C_{10} H_4$ besitzt."

‡ On a future occasion I shall show that Ritthausen was in error in placing this body in the benzole series, and indeed in considering it as a hydrocarbon at all. He was evidently deceived by operating on a mixture containing benzole. Furthermore, as Mansfield suggested might be possible, that part of the naphtha more volatile than benzole is by no means composed of a single substance. Having had a large quantity of this volatile material at my command, I have been able to obtain the separate constituents apparently in a state of great purity. Of the two bodies separated, one of them boils constant at about 40°, and the other near 0°. Both are compounds containing sulphur, and therefore will more properly form the subject of a separate paper.

§ Philosophical Magazine, 1855, 4th Series, IX. 256.

zole Series." I cannot better present his results than by quoting the following table: —

	Formula.	Boiling-point.	Difference.
Benzole,	$C_{12} H_6 = C_6\ 3\ (C_2 H_2)$	80°.8	
			22°.9
Toluole,	$C_{14} H_8 = C_6\ 4\ (C_2 H_2)$	103°.7	
			22°.5
Xylole,	$C_{16} H_{10} = C_6\ 5\ (C_2 H_2)$	126°.2	
			22°.2
Cumole,	$C_{18} H_{12} = C_6\ 6\ (C_2 H_2)$	148°.4	
			22°.3
Cymole,	$C_{20} H_{14} = C_6\ 7\ (C_2 H_2)$	170°.7	

Church states that he obtained all of these bodies from coal-naphtha, and also that he obtained benzole from benzoic acid, toluole from toluylic acid, xylole from wood-spirit, cumole from cuminic acid, and cymole from oil of cumin; and that he has found the corresponding bodies from these different sources to be identical. It will be observed that Church claims to have found in coal-tar a body boiling at 126°.2, which he calls *xylole,* thus supplying from this source a fifth member of the benzole series; whereas Mansfield and Ritthausen found only four bodies within the range of temperature indicated by the table. It will also be observed that his determination of the boiling-point of toluole is much lower, and that of cumole much higher, than the corresponding determinations of Mansfield and Ritthausen; thus giving room for a middle member between them, and preserving a remarkable uniformity of difference — viz. 22° and a fraction — between the boiling-points of any two contiguous members of the series, for the addition of $C_2 H_2$.

That the earlier investigators had found in coal-tar naphtha only the two lower members ($C_{12} H_6$ and $C_{14} H_8$) and the two upper members ($C_{18} H_{12}$ and $C_{20} H_{14}$), indicating the absence of the middle member ($C_{16} H_{10}$) of the benzole series, was always to me an anomaly which I could not reconcile with any plausible theory in regard to the formation of these bodies; and I was led, therefore, to question whether this body had not been overlooked in making the separations. The alleged discovery of this body in coal-naphtha by Church, together with the beautiful uniformity of the boiling-point difference throughout the series which he presented, and the apparent care with which the whole research had been conducted, led me to regard his results as being more reliable than those which had previously been published. I remained under this conviction until I had discovered the boiling-point difference of 30° in other series of hydrocarbons,* which led me to doubt the accuracy of Church's determinations of boiling-points, and to consider those of Mansfield and Ritthausen as probably more correct.

In the first paragraph of his paper, Church remarks that, "although doubts still

* See the following Memoir on this subject.

remain as to the relations of these bodies to one another, yet their composition has been ascertained with certainty." It does not appear, however, that an analysis or vapor density of any one of the members of this series, as obtained from coal-tar, except benzole, had ever been published. As already indicated by the title of his paper, it appears to have been the design of Church to treat only of the boiling-points of these bodies; yet finding that his preparations of toluole — prepared both from coal-naphtha and toluylic acid — gave a boiling-point differing considerably from observations previously published, he took occasion to make analyses of his preparations of this substance, which he regards as "perfectly satisfactory"; and adds that "the details and numerical results of these analyses, and of many others which the present inquiry necessitated, the limits and special object of the present paper do not admit of my giving here." As he undertook to correct the work of his predecessors, to do which fairly would seem to require the publication of these "details and numerical results," their omission is to be regretted, the more since he found space and purpose for matter apparently less relevant to his special subject. I am prompted to these remarks from having been led to undertake the tedious task of making a re-examination of coal-tar naphtha mainly on account of the disagreement between Church's determinations, which I have found to be mostly incorrect, and those which had been previously published.

In addition to the bodies mentioned in the foregoing table, Church alludes to the discovery of two other bodies, boiling respectively at 97° and 112°. Subsequently, in a "Note on Parabenzole, a new Hydrocarbon from Coal-Naphtha,"* he publishes the details of an investigation of the former of these two bodies, which he finally found to boil "*perfectly constant at* 97°.5," and to be isomeric with benzole.

I think I shall be able to show in the following pages, —

1. That coal-tar naphtha contains only four hydrocarbons within the range of 80° to 170°, as taught by Mansfield, and confirmed by Ritthausen.

2. That the benzole series within that range of temperature is limited to four members, and therefore does not contain five, as has been generally supposed.

3. That these four members have the boiling-points 80°, 110°, 140°, and 170° respectively; and consequently that the boiling-point difference in this series, for an elementary difference of C_2H_2, is 30°, instead of 22° and a fraction, as alleged by Church.

4. That the body obtained from coal-tar naphtha, boiling at 140°, is not identical with cumole from cuminic acid, as assumed by Mansfield, nor even isomeric with it;

* Philosophical Magazine, 1857, 4th Series, XIII. 415.

but that it has the formula which has been assigned to xylole, containing C_2H_2 less than that of cumole.

5. That the body obtained from coal-tar naphtha, boiling at 170°, is quite a different body from cymole obtained from oil of cumin, — with which it has been considered identical, as assumed by Mansfield, — these bodies differing from each other by C_2H_2.

6. That cumole from cuminic acid, and cymole from oil of cumin, do not even belong to the benzole series.

7. That the Parabenzole of Church was in all probability only a mixture of benzole and toluole.

Of the Quality of Naphtha employed in this Investigation. — As I have taken occasion to question the existence in coal-tar naphtha of two of the substances which it has been said to contain, — viz. cymole, $C_{20}H_{14}$, and parabenzole, $C_{12}H_6$, — it is a matter of some importance that I should clearly state the kind or quality of the naphtha employed. The tar from which this naphtha was obtained was a mixture of the tar furnished by the following companies, viz. the New York and the Manhattan Gas-Light Companies, of New York; Brooklyn Gas-Light Company, of Brooklyn, N. Y.; Albany Gas-Light Company, of Albany, N. Y.; and the Gas-Light Companies of Newark and Jersey City, in New Jersey. It was mostly made from Cannel and Newcastle caking coals, which were imported from Liverpool, and mixed in the proportions of one third to five eighths Cannel, to two thirds to three eighths Newcastle. In some of the works a portion of the caking coal was from mines in Pennsylvania. The tar from these different gas-works, as regularly received at the naphtha manufactory, was poured into a large tank provided for this purpose. The stills were uniformly charged with tar directly from this tank; so that there can be no doubt that the naphtha employed was made from a mixture of the tar supplied by the six different companies above enumerated. Most of the gas-works referred to are large, the annual production of tar amounting in the aggregate to upwards of 50,000 barrels. It does not appear, therefore, that the absence of the bodies in question from the naphtha which I have employed, can be attributed to any peculiarity of the tar. The naphtha was prepared in a manufactory in New York over which I had at that time personal control, and was purified under my own direction. The process of purification did not differ essentially from that in common use in England, the reagents employed being oil of vitriol and alkali. One hundred barrels of the purified naphtha were subjected, under my personal superintendence, to repeated fractional distillation from an iron still. The chief object in operating on so large a quantity, was to insure the detec-

tion of any constituent which might be present in small proportion. The process of fractioning was continued on this large scale until the separations had so far progressed, that at certain temperatures a full barrel of distillate would come off from the ten-barrel still employed, without a variation of more than one or two degrees of the thermometer. Finally a sample gallon was taken from each of the barrels composing the last series of products, and these samples were set aside for this investigation, which was afterwards conducted in the laboratory.

Of the Results of Fractional Condensation. — Such of the samples above mentioned as promised to yield the different constituents of the naphtha in the largest proportion, were subjected to repeated series of fractionings by my process of "Fractional Condensation."* As full details of this process have already been given in the memoir referred to, it will be needless to repeat them here. It will suffice to say that the fractioning in this case was conducted in all respects as there described, and continued until the whole of the naphtha taken, boiling between 80° and 170°, had accumulated at the four points already indicated, viz. at 80°, 110°, 140°, and 170°; or so nearly the whole that the intermediate quantities had become too small to admit of being further operated upon. Having therefore so thoroughly exhausted the intermediate fractions, I can have no hesitation in asserting that no other body than those alluded to was present in the naphtha, — at least, in appreciable quantity, — hence, that the parabenzole of Church was probably only a mixture of benzole and toluole. I may here remark that each of the sample-gallons employed, when subjected to my process of fractioning, was found to contain, in variable proportion, all of the constituents of the naphtha.

Of some of the Properties of the Bodies obtained by Fractioning.

1. Benzole.

Specific gravity, 0.8957 at 0°, and 0.882 at 15°.5.†

Determination of Boiling-point. — This experiment was conducted in a tubulated retort, operating on 150–200 *c. c.* of the benzole, containing some pieces of sodium. The benzole employed had previously been repeatedly boiled with sodium, until the latter

* Memoirs of the American Academy, 1864.

† It would appear that the specific gravities of liquids are usually determined at the temperature of the air. The result of this is that the determinations made by different observers are not comparable with one another. That these specific gravities are not uniformly taken at 0° C. — the temperature which, on account of greater convenience, etc. is generally acknowledged to be preferable — is probably due to the fact that the more

ceased to have any action. The thermometer bulb extended into the liquid * nearly to the bottom of the retort. A second thermometer was attached, by means of flexible bands, to the side of the one in the retort; the bulb being placed, during ebullition, at a point midway between the centre of the cork (— 5°) and the upper end of the mercurial column, viz. at 35°. A paper screen, closely fitting the thermometer spindle, was placed across at the top of the cork. With the retort neck slightly inclined upward, and cooled to prevent the escape of vapor, ebullition was continued for considerable time, until the mercury in the thermometer ceased to rise. The lamp being removed for the moment, the neck of the retort was then turned downward, and quickly inserted in a Liebig's condenser. On replacing the lamp, distillation commenced almost immediately at 79°.

Observations. —

Temperature.		Time.		Temper. by Side Thermom.
°		h. m.		
79.0	at	2.40	5 minutes.	
79.2	"	2.45	15 "	22°.
79.4	"	3.00	12 "	24°.
79.5	"	3.12	20 "	25°.
79.6	"	3.32	18 "	26°.
79.6	"	3.50		26°.

common specific gravity bottle is not suited to this purpose. Indeed, with a volatile body that bottle cannot serve for an accurate determination at any temperature. A reform in this regard being highly desirable, I would call attention to a specific gravity bottle which I obtained a few years ago from Fastrè, in Paris, which is admirably adapted for taking specific gravities, even of volatile liquids, at a low temperature. The accompanying figure represents this bottle of one half its natural size. Who was the author of this particular form I am not informed, although it may have been already noticed in some publication. A bottle analogous to this is figured by Schiel ("Einleitung in das Studium der organischen Chemie," page 76); but his bottle has an oval bottom, which makes it less convenient. The particular advantage of this bottle over the more common one, which advantage Schiel omits to notice, consists in this: that the space or chamber above the line on the capillary neck is large enough to allow for the expansion of the liquid consequent upon the elevation of temperature from 0° to that of the surrounding air; and that the ground stopper fits so closely that no perceptible loss from evaporation can take place during the time occupied by an experiment.

In order to furnish determinations of the specific gravities of the bodies to be treated of in these researches, which shall be comparable with corresponding determinations by other observers, I shall generally record one or more special determinations made for this purpose.

* For critical remarks on the question of propriety of placing the thermometer bulb in the liquid, etc.; and for further details of the method of taking boiling-points, especially at low temperatures, see the accompanying Memoir, "On the Influence of $C_2 H_2$ on the Boiling-points in Homologous Series of Hydrocarbons," etc.

Distillation therefore occupied one hour and ten minutes, during which time the thermometer rose only 0°.6, being fifty minutes in rising 0°.2 from 79°.4 to 79°.6, at which temperature it had distilled nearly to dryness. Height of the barometer during the experiment reduced to $0° = 761.9^{mm.}$ Taking 79°.4, this being the average of the last five observations, and applying the corrections for the upper column of mercury, and for atmospheric pressure, according to the directions given by Kopp,* we find the corrected boiling-point of benzole to be 80°.1.

Analysis. — 0.2339 gramme of benzole gave, by my process† of combustion in a stream of oxygen gas, 0.7903 of carbonic acid, and 0.1683 of water.

		Calculated.		Found.
Carbon,	C_{12}	72	92.31	92.15
Hydrogen,	H_6	6	7.69	7.99
		78	100.00	100.14

Determination of Vapor Density. —

Temperature of balance,	15°
Temperature of oil bath,	171°
Height of barometer,	$764.1^{mm.}$ at 9°
Increment of balloon,	0.2447
Capacity of balloon,	265 *c. c.*

Density of vapor found,	2.688
Theory $C_{12} H_6 = 4$ volumes,	2.698

2. Toluole.

Specific gravity, 0.8824 at 0°, and 0.872 at 15°.

Determination of Boiling-point. — The preparation employed for this determination had also been repeatedly boiled with sodium until the latter ceased to have any action upon it. Operating in this case also upon a pretty large quantity, the distillation occupied about an hour. The experiment was conducted as detailed under the head of Benzole. Distillation commenced at 110°.6; two minutes later the temperature had fallen to 110.4, at which point it remained absolutely constant during the lapse of forty-eight minutes. Five minutes later the temperature had risen again to 110°.6;

* Poggendorff's Annalen, 1847, LXXII. 38.

† Proceedings of the American Academy, 1864, p. 251.

and five minutes later to 110°.8, at which point, having distilled nearly to dryness, the operation was suspended. The corrections for pressure (— 0°.16) and for the upper column of mercury — which, with the thermometer used in this experiment, was only 7° in length, — gives 110°.3 as the corrected boiling-point of toluole. Church* remarks that toluole, when distilled in the ordinary manner, is liable to become oxidized, and its boiling-point thereby raised, in consequence of the upper part of the retort becoming heated above the boiling-point of toluole. He found that toluole which, by ordinary distillation, had come over between 108° and 109°, would distil eight tenths between 103° and 104°, after repeated purification with sodium. I would therefore state that my preparation of toluole was never subjected to a temperature above its boiling-point; and that I have never noticed any reduction of the boiling-point of this body by purification with sodium.

Analysis. — 0.1628 gramme of toluole gave, by combustion in a stream of oxygen gas, 0.5447 of carbonic acid, and 0.1315 of water.

		Calculated.		Found.
Carbon,	C_{14}	84	91.3	91.20
Hydrogen,	H_8	8	8.7	8.97
		92	100.0	100.17

Determination of Vapor Density. —

Temperature of balance,	17°
Temperature of oil bath,	209°
Height of barometer,	760.1$^{mm.}$ at 15°
Increment of balloon,	0.287
Capacity of balloon,	249.5 *c. c.*
Density of vapor found,	3.2196
Theory $C_{14}H_8$ = 4 volumes,	3.1822

3. Xylole. (*Cumole of Mansfield and Ritthausen.*)

Specific gravity, 0.878 at 0°, and 0.866 at 15°.5.

Determination of Boiling-point. — This determination was made in all respects like that of benzole, the xylole employed having been also subjected to the same treatment. The quantity operated upon was, however, smaller, and the experiment con-

* Philosophical Magazine, 1855 (4), IX. 256.

ducted more rapidly. Distillation began at 138°.6, and terminated at 139°, having distilled almost to dryness. The time occupied was seventeen minutes. Taking the average of these observations, viz. 138°.4, and applying the customary corrections, we find 139°.8 to be the corrected boiling-point of xylole.

Analysis. — 0.1333 gramme of xylole gave, by combustion in a stream of oxygen gas, 0.4413 of carbonic acid, and 0.1185 of water.

		Calculated.		Found.
Carbon,	C_{16}	96	90.57	90.29
Hydrogen,	H_{10}	10	9.43	9.87
		106	100.00	100.16

Determination of Vapor Density. —

Temperature of balance,	16°.5
Temperature of oil bath,	207°.5
Height of barometer,	760$^{mm.}$ at 14°
Increment of balloon,	0.3528
Capacity of balloon,	228 *c. c.*

Density of vapor found,	3.7517
Theory $C_{16} H_{10}$,	3.6665

These results show clearly that this body has the formula $C_{16} H_{10}$, and that it is doubtless the third member of the benzole series.* Although xylole, first discovered by Cahours in the oil separated from wood-spirit, has had a much lower boiling-point assigned to it, I have retained that name for this body, since the results which I have obtained in the study of the light oil from wood-tar indicate that when the corresponding body from this source is in a state of equal purity, its boiling-point will agree with the above determination. I may here mention that in my researches on the light oil from wood-tar, I have obtained a body at about 140°, but nothing between that and 110° (these temperatures are not corrected), although special pains were taken to work up the intermediate fractions. So that I am in a position to justify

* As this memoir is passing through the press, the receipt of my journals for September calls attention to late publications of Hugo Müller, Béchamp, and Naquet concerning this hydrocarbon. Müller concludes that it is xylole, a result which agrees with my own. (Annalen der Chemie und Pharmacie, 1864, CXXXI. 321.) Béchamp, on the contrary, erroneously regards it as being a *new* hydrocarbon, not belonging to the benzole series. (Bulletin de la Société Chimique, Paris, 1864, 204.) Naquet also calls it a *new* hydrocarbon, and gives it the formula $C_{18} H_{12}$. (Bulletin de la Société Chimique, Paris, 1864, 205.)

the assertion that no other body was present in appreciable quantity between the temperatures mentioned.

That this body from coal-tar naphtha, boiling at 140°, is not identical with cumole from cuminic acid, will be made apparent on comparison of the results above stated, with those which will be given when treating of cumole.

4. Isocumole. (*Cymole of Mansfield.*)

Specific gravity, 0.8643 at 0°, and 0.853 at 15°.

Determination of Boiling-point. — This was conducted with the usual precautions, and under conditions similar to those detailed above. The distillation, as in the foregoing determinations, was continued nearly to dryness, and occupied twenty-five minutes. Before distillation was commenced, the temperature of the boiling liquid was found to be 166°.5, and at the close of distillation 167°. Applying the customary corrections to the average of these observations, viz. 166°.75, we obtain for the corrected boiling-point 169°.8.

Analysis. — 0.1944 gramme of the substance gave, by combustion in a stream of oxygen, 0.6366 of carbonic acid, and 0.1896 of water.

		Calculated.		Found.
Carbon,	C_{18}	108	90.00	89.31
Hydrogen,	H_{12}	12	10.00	10.84
		120	100.00	100.15

Determination of Vapor Density. —

Temperature of balance,	13°.5
Temperature of oil bath,	241°.0
Height of barometer,	769.5$^{mm.}$ at 9°
Increment of balloon,	0.4206
Capacity of balloon,	239 *c. c.*

Density of vapor found,	4.3019
Theory $C_{18}H_{12}$,	4.151

Hence it appears that the calculated density on the formula $C_{18}H_{12}$ is 0.151 *less* than that found by experiment. The calculated density on the formula $C_{20}H_{14}$, which has previously been assigned to this body, — although, as above stated, without an analysis or determination of vapor density, — is 4.645; which is 0.302 *greater* than that found by experiment. It will be observed that the difference between the den-

sity found and that calculated on the formula $C_{20} H_{14}$ is not only more than twice as large as the corresponding difference calculated on the formula $C_{18} H_{12}$, but that the error is reversed; being with $C_{20} H_{14}$ a *deficiency*, while with $C_{18} H_{12}$ it is an *excess*. This circumstance has to my mind a good deal of significance, as it goes strongly to show that the lower formula is the true one. For of the many vapor densities of hydrocarbons which I have determined, I have but rarely met with an instance in which the density found was not greater than the theoretical density. And I have usually observed that the excess of the experimental over the theoretical density is larger in proportion as the boiling-point of the body is higher, a fact which needs explanation. Wurtz* observed a similar difference between the determined and calculated vapor densities of bodies of the formulæ $C_n H_n$ and $C_n H_{n+2}$, which he accounted for on the ground that his preparations contained an admixture of bodies less volatile, the vapors of which would remain in the balloon, and increase the density. But I cannot accept this explanation for the substances here treated of, since they invariably distil without residue within a range of one degree of temperature. I would rather rely upon the supposition that the high temperature employed causes partial decomposition of the substance, which would be the more liable to occur the higher the boiling-point of the body. I do not, however, offer this as an explanation, but merely make the suggestion.

PART II.

HYDROCARBONS FROM OIL OF CUMIN AND CUMINIC ACID.

THE oil of cumin employed in this research was furnished by Messrs. Reed and Cutler, wholesale importers of drugs, etc. of Boston. The package bore the label of Eduard Büttner, manufacturer, of Leipzig, and purported to be a genuine preparation, answering in all of its obvious physical properties — odor, color, etc. — the description given of this oil by Gerhardt and Cahours† in their original memoir on this substance, who, it appears, also employed a commercial preparation. Its behavior in distillation left no doubt of its being a genuine article; and this was afterwards confirmed by treatment of the cuminole with fused potash, for the production of cuminic acid, its comportment with this reagent being in all respects identical with that described by Gerhardt and Cahours. Subjected to repeated series of fractionings by my process of

* Bulletin de la Société Chimique de Paris, 1863, 309.

† Annales de Chimie et de Physique, 1841, 3e Série, I. 60.

fractional condensation already referred to, it gave, in addition to cymole and the residue of cuminole, a body boiling at about 155°, which so closely resembles oil of turpentine in odor, etc. as to be hardly distinguishable from the latter substance. The presence of this body may account for the very low boiling-point which Gerhardt and Cahours assigned to cymole, viz. 165°. The boiling-point of cymole was subsequently found by Gerhardt* to be 175°, but my own determination places it still lower by about 5°. It is evident, therefore, from a comparison of their own determinations, that the oil of cumin which they originally operated upon contained an oil boiling below cymole; and hence the finding of such an oil in that which I employed need not raise a doubt as to its being genuine. This lighter body is present in so small quantity as hardly to admit of its being detected, or at least identified, by the old process of fractioning; and its detection and isolation by the new process is but another illustration of the superior excellence of this method.

1. Of the Body resembling Oil of Turpentine.

Specific gravity, 0.8772 at 0°, and 0.8657 at 15°.

Determination of Boiling-point. — The quantity of material at command was too small to admit of attaining so high a degree of purity for this body as was desirable. The product obtained, however, distilled almost to dryness between 153°.4 and 155°.5. Taking the average of these observations, and applying the usual corrections, we obtain 155°.8 for the boiling-point of this body.

Analysis. — 0.2575 gramme of the substance gave, by combustion with oxide of copper, 0.8283 of carbonic acid, and 0.2766 of water.

		Calculated.		Found.
Carbon,	C_{20}	120	88.24	87.73
Hydrogen,	H_{16}	16	11.76	11.94
		136	100.00	99.67

Determination of Vapor Density. —

Temperature of balance,	16°
Temperature of oil bath,	211°
Height of barometer,	758°.4$^{mm.}$ at 14°
Increment of balloon,	0.4939
Capacity of balloon,	221 *c. c.*

* Annales de Chimie et de Physique, 1845, 3e Série, IV. p. 111.

Density of vapor found,	4.7281
Theory $C_{20} H_{16} =$ 4 volumes,	4.7028
Excess found,	.0253

The calculated density on the formula $C_{20} H_{14}$ is 4.635; which, compared with the density found, would increase the excess to 0.093. Although the determination agrees more nearly, indeed almost exactly, with the calculated density on the formula $C_{20} H_{16}$, the calculation on the formula $C_{20} H_{14}$ does not show a greater variation from the density found, than we have observed to be quite frequent with hydrocarbons of so high boiling-point; so that it may be questionable which of these formulæ is the true one. I cannot regard the determination of a vapor density as reliable for fixing the formula nearer than to within two equivalents of hydrogen. In the absence of opposing evidence, it will be wiser, however, to take the formula which agrees best with the results of experiment; at least until it shall be shown that the discrepancy between the calculated and observed vapor densities of bodies of high boiling-point, which appears to be so frequent, is nearly constant, or variable by some fixed law by which the amount of the error, in any given case, may be pretty nearly estimated. I shall therefore regard this body as having the formula $C_{20} H_{16}$, which is also better supported by the results of analysis. On account of its source, and close resemblance to oil of turpentine, I think of no better appellation for this body than cumo-oil of turpentine; thus adding another to the long list of isomers of the former substance, the chemical relations of which stand in so much need of being further studied.

2. Cumole.

This body was first obtained by Gerhardt and Cahours,* by the dry distillation of a mixture of six parts of crystallized cuminic acid, and twenty-four parts of caustic baryta. Abel † obtained the same result by substituting caustic lime for the baryta. His product, however, was found to boil 4° above that of Gerhardt and Cahours. My preparation was also made by the use of lime. Although the results of my experiments confirm the conclusions arrived at by Gerhardt and Cahours as to the composition of this body, yet the numerical results differ considerably from theirs. I have also observed some new facts regarding the formation of this body. They have described the reaction between the baryta and cuminic acid as being much more simple than my experiments seem to indicate. On this point they re-

* Annales de Chimie et de Physique, 1845, 3e Série, IV. 87.

† Annalen der Chemie und Pharmacie, 1847, LXIV. 312.

mark: "The formation of cumene is easily explained. In effect, the cuminic acid being represented by $C_{40}H_{24}O_4$, it appears that C_4O_4, that is to say, 2 equivalents of carbonic acid, are retained by the baryta, while $C_{36}H_{24}$ are set free." *

$$C_{20}H_{12}O_2 = C_2O_2 + C_{18}H_{12}.$$

In another place (p. 88) they remark, that "by suitably managing the heat, and employing no more than 6 gr. of cuminic acid at a time, no other products are ever obtained than those which we mention." † My experiments show that this reaction is by no means so simple as thus described. The crude product obtained from the mixture of lime and cuminic acid, when subjected to a simple distillation from a tubulated retort, was found to distil between 155° and 250°, leaving a residue at the latter temperature which became semi-fluid on cooling. The distillate thus obtained gave, by my process of fractional condensation, an oil boiling at 151°.1, and a residue at 170°. It is not improbable that the latter may prove to be mostly cymole, $C_{20}H_{14}$; but the quantity was too small to admit of pursuing this inquiry with the probability of deciding the question. There is evidence, however, that the product obtained by Gerhardt and Cahours was not simply pure cumole, as they described it, but a mixture of different bodies, which would necessitate a more complicated reaction than that which they assigned. Gerhardt and Cahours found the boiling-point of their cumole to be constant at 144°. Four years later, Gerhardt,‡ having occasion to make a very accurate determination of the boiling-point of this body, in connection with his research to find a law governing the boiling-points of the hydrocarbons, found its boiling-point to be 9° higher, viz. 153°, which is but 2° higher than my own determination. The disagreement between their determinations, it being so considerable, may be more reasonably accounted for on the supposition that they operated, in the first instance, upon a mixture of different bodies; and yet I cannot see how they could have obtained the product boiling below 150°. Additional evidence on this point will be found in the discrepancy which appears between their determination of the vapor density, and that calculated upon theory.

The specific gravity of my preparation of cumole was found to be 0.8792 at 0°, and 0.8675 at 15°.

* "La formation du cumène s'explique aisément. En effet, l'acide cuminique etant représenté par $C_{40}H_{24}O_4$, on voit que C_4O_4, c'est-a-dire 2 equivalents d'acide carbonique sont retenus par la baryte, tandis que $C_{36}H_{24}$ sont dégage." — Annales de Chimie et de Physique, 1841, 3e Série, I. 89.

† "En dirigeant la chaleur convenablement, et en n'employant pas plus de 6gr. d'acide cuminique à la fois, on n'obtient jamais d'autre produits que ceux que nous venons de nommer."

‡ Annales de Chimie et de Physique, 1845 (3), XIV. 107.

Determination of Boiling-point. — The quantity of material being quite small, this determination was made in a large test tube, with the usual precautions. It had not a perfectly constant boiling-point, the distillation ranging from 148°.4 to 151°.6. Applying the proper corrections to the mean of these observations, gives, for the boiling-point of cumole, 151°.1, which is doubtless a little too high from the impracticability of making a complete separation with the small quantity of material employed. If the boiling-point difference between cumole and cymole, for the difference of $C_2 H_2$ in their elementary formulæ, is 30°, as there is every reason to believe, then the boiling-point of cumole should be 150°, as I have found the boiling-point of cymole to be but a fraction under 180°.

Analysis. — 0.1700 gramme of cumole gave, by combustion with oxide of copper, 0.563 of carbonic acid, and 0.1557 of water.

		Calculated.		Found.
Carbon,	C_{18}	108	90.00	90.35
Hydrogen,	H_{12}	12	10.00	10.18
		120	100.00	100.53

Determination of Vapor Density. —

Temperature of balance,	17°
Temperature of oil bath,	203°
Height of barometer,	760.1$^{mm.}$ at 15°
Increment of balloon,	0.4428
Capacity of balloon,	232 *c. c.*

Density of vapor found,	4.2003
Theory $C_{18} H_{12} = 4$ volumes,	4.151

This determination, as well as the results of analysis, confirms, therefore, the formula which Gerhardt and Cahours had assigned to this body. I had anticipated a different result from this, inasmuch as the hydrocarbon from coal-tar naphtha, which I have called *iso-cumole*, boiling at 170°, or nearly 20° higher than cumole from cuminic acid, — had been found, as I have shown above, to have the formula $C_{18} H_{12}$. I am forced to the conclusion, therefore, that these two bodies are isomeric, and belong to different series. A preliminary examination of their behavior with reagents indicates that their chemical properties are also different. These will be treated of on a future occasion, in Part III.

3. Cymole.

Notwithstanding that this body is so much more volatile than the cuminole with which it is associated in the oil of cumin, — there being a difference of 40° between their boiling-points, — Gerhardt and Cahours found it necessary to resort to chemical means, viz. treatment with fused potash, in order to isolate it. Being desirous of testing the efficiency of my process in effecting the separation, the preparation employed in this investigation was obtained by fractional condensation, this process having been found as effective in this as in other cases. This will appear by a comparison of the results obtained in the study of this body before and after treatment with concentrated sulphuric acid, which is also effective to remove cuminole.

Specific Gravity. —

At 0°, before	treatment	with	$HO\ SO_3$,		0.8697
At 0°, after	"	"	"		0.8724
At 14°, before	"	"	"		0.8592

Determination of Boiling-point before Treatment with Sulphuric Acid. — The preparation was found to distil to dryness between 175°.8 and 177°. The temperature remained absolutely constant at 176° during the lapse of ten minutes, and occupied fifteen minutes in rising from 176° to 176°.5. Taking the mean of the former numbers, viz. 176°.4, and applying the proper corrections for pressure, etc., we obtain 179°.5 for the boiling-point of cymole.

After Treatment with Sulphuric Acid. — The preparation distilled to dryness between 176° and 177°, the temperature remaining thirteen minutes constant at 176°.3, indicating that no essential change in the boiling-point had been produced by the acid treatment. It was nevertheless evident that some impurity was being removed by the acid, as the first portions of the latter became dark-colored and thickened on being agitated with the oil. Successive portions of acid were therefore employed, until it ceased to produce any marked effect.

Analysis before Treatment with Sulphuric Acid. — 0.1589 gramme of cymole gave, by combustion in a stream of oxygen gas, 0.5200 of carbonic acid, and 0.1532 of water.

		Calculated.		Found.
Carbon,	C_{20}	120	89.55	89.25
Hydrogen,	H_{14}	14	10.45	10.71
		134	100.00	99.96

After Treatment with Sulphuric Acid, and Distillation in Vacuo. — 0.1623 gramme of cymole, by combustion in a stream of oxygen gas, gave 0.5324 of carbonic acid, and 0.1561 of water.

		Calculated.		Found.
Carbon,	C_{20}	120	89.55	89.46
Hydrogen,	H_{14}	14	10.45	10.68
		134	100.00	100.14

The removal of impurity by treatment with sulphuric acid had therefore hardly a sensible effect on the results of analysis.

Determination of Vapor Density before Treatment with Sulphuric Acid. —

Temperature of balance,	11°
Temperature of oil bath,	259°
Height of barometer,	$740.6^{mm.}$ at 5°
Increment of balloon,	0.4446
Capacity of balloon,	239 *c. c.*

Density of vapor found,	4.742
Theory $C_{20}H_{14} = 4$ volumes,	4.6351

After Treatment with Sulphuric Acid. —

Temperature of balance,	25°.5
Temperature of oil bath,	255°
Height of barometer,	$760^{mm.}$ at 26°
Increment of balloon,	0.4647
Capacity of balloon,	232 *c. c.*

Density of vapor found,	4.7536
Ditto before treatment with $HO\,SO_3$,	4.742
Difference,	.0116

The results of the two determinations are therefore almost identical.

A comparison of the above results with those obtained in the study of isocumole, the body from coal-tar naphtha boiling at 170°, will show that the two bodies are far from being the same substance, as Mansfield assumed, and that they have a constitutional difference of C_2H_2, and therefore doubtless belong to different series.

NOTE. — I had hoped to be able to present on this occasion the results of the study of some of the more important reactions of the hydrocarbons treated of in the preceding pages, — at least, of those in regard to which I have differed from my predecessors; but as this work is yet incomplete, and as I am at present occupied with the study of other substances of more immediate interest, I will defer this branch of the subject for future consideration, in Part III. I may here remark, however, that the behavior of these bodies with reagents is such as to strengthen the conclusions already expressed in regard to them.

II.

On the Influence of C_2H_2 upon the Boiling-points in Homologous Series of Hydrocarbons, and in some Series of their Derivatives; with Critical Observations on Methods of taking Boiling-points.

It is well known that we are indebted to H. Kopp* for the discovery of certain definite relations existing between the chemical constitution and some of the physical properties of homologous liquid bodies. Of these, one of the most important is that of a uniform difference between the boiling-points of the contiguous members of an homologous series, corresponding to the uniform difference in their elementary constitution. Kopp has shown by numerous examples, that, as a general rule, in those series which are characterized by a common elementary difference of C_2H_2 between the members, in the order of the series, the corresponding difference of boiling-point is about 19° C.; hence, that the difference between the boiling-points of any two members of such a series is $x . 19°$ for a difference of $x\, C_2H_2$ in the elementary formulæ. In the earlier observations on this subject, this relation between the boiling-points and formulæ was found so nearly constant in the different series examined, that any deviations from this apparent general law were referred, not unreasonably, to assumed inaccuracies in the determination of the boiling-points of the bodies compared. But the more recent and extended generalizations of Kopp† have led him to point out several exceptional series, in which the boiling-point difference is greater, and others in which it is less, than 19° for an elementary difference of C_2H_2. That there are such exceptional series is confirmed in a very decisive manner by my own observations, as I shall proceed to show. My determinations make the boiling-point differences in some cases so much larger than those of other observers as to leave no room for doubt on this point; especially if the comparative value of these determinations be duly estimated with reference to the more reliable character to which the preparations are entitled, on account of the more efficient means which I have employed for separating the liquids. Since Kopp first called the attention of chemists to this subject, different theories have from time to time been advanced by Schröder, Löwig, Gerhardt, and others, and supported by laborious research and observation. It will be interesting to examine some of these theories in the light

* Annalen der Chemie und Pharmacie, 1842, XLI. 79, 169; 1845, LV. 177, etc.

† Annalen der Chemie und Pharmacie, 1855, XCVI. 2.

of the new facts which I am about to present. Schröder,* not satisfied with Kopp's explanation of the discrepancies between the observed and theoretical boiling-points, on the ground of errors of determination of the former, argues that the influence of $C_2 H_2$ on boiling-points is variable in different series according to the peculiar nature of the $C_2 H_2$ in each case. He regards organic compounds for the most part made up of radicals, which he calls "components," of which he makes seven. Three of these are composed of carbon and hydrogen, viz.: —

Formyl = $(C_4 H_2)$ — ("$C_4 H_4$") — which is supposed to raise the boiling-point of a substance 52° C.

Methylen = $(C_2 H_2)^m$ — "$(C_2 H_4)^m$" — which is supposed to raise the boiling-point of a body 21°.

Elayl = $(C_2 H_2)^e$ — "$(C_2 H_4)^e$" — which is supposed to raise the boiling-point 17°. Subsequently (Pogg. Ann., 64, 101) the latter number was changed by Schröder to 16°.

A fourth component was made up of a double atom of hydrogen, (H_2)—"(H_4)"—, which was supposed to lower the boiling-point 3°; but this also was afterwards changed to 10° (Pogg. Ann., 64, 372). (The other three components, having no direct bearing on the hydrocarbons, are omitted.) By means of these components Schröder (Pogg. Ann., 62, 188) proposed to calculate the boiling-points of different substances in the following manner. Having estimated the sum of the influence of the different components of a body, the number 70 was in all cases to be deducted. Subsequently Schröder† was led to substitute, in these calculations, the influence of the separate elements for that of the components. Each double atom of carbon (C_2) was estimated to raise the boiling-point of a compound 31°; and each double atom of hydrogen (H_2) to lower it 10°. As in the former case, the number 70 was to be deducted from the sum of the influences of the different elements contained in the compound, to give the true boiling-point. Example: calculation of the boiling-point of benzole, $C_{12} H_6$; $C_{12} = 6\, C_2$; $31° \times 6 = 186°$; $H_6 = 3\, H_2$; $-10° \times 3 = -30°$; $186° - 30° - 70° = 86°$ = the calculated boiling-point of benzole by this method; which agrees exactly with the latest determination at the date of Schröder's memoir.

Löwig ‡ estimates the influence of the elementary atoms on the boiling-point differently from Schröder; and obtains numbers such that, to find the boiling-point of a compound it is only required to add together the numbers corresponding to the

* Poggendorff's Annalen, 1844, LXII. 184, 337.

† Poggendorff's Annalen, 1845, LXIV. 367; 1846, LXVII. 45.

‡ Poggendorff's Annalen, 1845, LXIV. 250.

elementary atoms which it contains, without deducting from this sum a constant number, as by Schröder's method. According to Löwig's theory, one atom of carbon (C) raises the boiling-point 38°.4, and one atom of hydrogen (H) lowers it 29°.2; these numbers being for carbon nearly two and one half times, and for hydrogen nearly three times as great, as those of Schröder. Gerhardt,* in a special paper "On the Boiling-point of the Hydrocarbons," observes that "The boiling-point of the hydrocarbons appears to obey a very simple law, according to which it is raised or depressed a certain number of degrees, corresponding to the number of equivalents of carbon or hydrogen contained in its equivalent." † From a comparison of the boiling-points and formulæ of several well-known hydrocarbons, the determinations of which were repeated with special care for this purpose, Gerhardt finds that the addition of C_2 to the molecule of an hydrocarbon raises its boiling-point 35°.5, and that the addition of H_2 lowers it 15°. The boiling-point of a body is calculated from these numbers by comparing its formula with oil of turpentine, $C_{20}\,H_{16}$, as a standard, the boiling-point of which is taken at 160° C. Example: cumole (from cuminic acid) has the formula $C_{18}\,H_{12}$; hence it contains C_2 less than oil of turpentine; therefore 35°.5 must be deducted from 160° (the boiling-point of oil of turpentine), which leaves 124°.5; but as the cumole contains 2 H_2 less than oil of turpentine, $15° \times 2 = 30°$ is to be added to the above remainder; thus $124°.5 + 30° = 154°.5$, the calculated boiling-point of cumole. Gerhardt's direct determination was 153°, which very nearly coincides with his theory.

It would be foreign from my purpose on the present occasion to consider these different hypotheses, or even the empirical law of Kopp, beyond their special relation to the boiling-points of the hydrocarbons, and such other series, derivatives from the hydrocarbons, as have been made the subjects of my own experiments. Anything more than this would be merely speculative. The want of more accurate determinations of boiling-points as essential to safe and reliable deductions and generalizations on this question, has frequently been observed. The need of this will be made strikingly apparent by comparison of my results with those of previous observers. Indeed, if my determinations may be taken as a criterion, — which, considering the nature of the materials operated upon, might not be quite fair, — the inaccuracies of the boiling-points which have hitherto been published are probably so numerous, and

* Annales de Chimie et de Physique, 1845, 3e Série, XIV. 107.

† "Il parait que le point d'ébullition des hydrogène carbonés est soumis à une loi fort simple, d'après laquelle il s'élèverait ou s'abaisserait d'un certain nombre de degrés suivant le nombre des équivalents de carbone ou d'hydrogène renfermés dans leur équivalent."

in many cases so considerable, as to make it appear almost useless to attempt further generalizations upon those unreliable data. It may be hoped, however, that the superior means which my process furnishes for separating mixtures of liquids, will lead to the accumulation of reliable facts of sufficient number and variety for a profitable review of this question in its different bearings, which, from its importance, it clearly merits.

The frequent inaccuracy of the determinations of boiling-points, upon which Kopp has justly laid so much stress, may, I think, be more reasonably attributed, at least in a majority of cases, to a want of purity in the substances themselves, rather than to a neglect of the precautions and corrections which he recommends to be observed in such determinations; although errors as great as those mentioned by Kopp* may doubtless occur, and in the particular instances which he had in mind may have occurred from the cause which he assigned for them. It should be borne in mind, however, that these errors, in the case of an impure substance, may be compensating errors; or, on the other hand, they may go to increase that which would arise from impurity.

That the conditions under which my results have been obtained may be clearly understood, and hence the value of these results fairly estimated, in comparison with those of others, I shall endeavor, as I proceed with these researches, to specify, in sufficient detail, the processes which I have employed. Having, in the memoir previously referred to, described the process by which the hydrocarbons were separated, the special object of this paper only requires, in this regard, that I should add a description of the method employed in determining the boiling-points of these bodies, which has already been partially given in the foregoing memoir, when treating of the boiling-point of benzole.

Of the Method of determining Boiling-points. — I use for this purpose a small tubulated glass retort, and usually operate on about 150 *c. c.* of the liquid. The thermometer extends into the liquid, even nearly to the bottom of the retort, taking care that the bulb shall not come in contact with the glass, but remain free in the liquid. To prevent abnormal elevation of temperature from adhesion to the glass, — which I have observed in some instances, when operating on impure hydrocarbons, to amount to several degrees, — I introduce pieces of sodium, instead of platinum, as it seems to serve at least as well for this purpose, and at the same time tends to preserve the purity of the material. Sodium has also this advantage over platinum for hydrocar-

* "Bestimmung des Siedepunkts." Poggendorff's Annalen, 1847, LXXII. 38.

bons, viz. that it does not lose its virtue by use, so long as any of it remains; platinum, on the contrary, being liable, especially if the liquid is not quite pure, to become after a while slightly coated, and its efficiency thus impaired.*

Except for low temperatures, the retort rests on a piece of wire gauze laid over the ring of an iron lamp-stand, and is heated with a small gas-flame. When operating on liquids of low boiling-point, I have observed the liability of the thermometer to be considerably affected by the ascending current of hot air striking the sides of the retort above the level of the liquid, thus causing an elevation of several degrees of temperature. To prevent this, I proceed as follows. For low temperatures, and yet above the common temperature, I place upon the gauze on which the retort is to stand, a screen of felt or thick woollen paper, which has been provided with a hole in the centre about two inches in diameter. This screen extends several inches from the sides of the retort, and has been found effectual for the purpose.

For temperatures below the common temperature, the retort is set in a water-bath containing ice-water, the temperature of the bath being gradually raised by means of a small gas-flame.

As is customary, in order to ascertain the temperature by which to calculate the correction for the upper column of mercury, a thermometer is attached, by means of elastic bands, to the side of the thermometer in the retort; the bulb being placed, during ebullition, midway between the centre of the cork and the upper extremity of the mercurial column. And, as usual, a paper screen, closely fitting the thermometer, is placed across at the top of the cork to shield the upper column of mercury from the direct influence of the ascending heat.

I have observed that it often requires considerable time — variable according to its length and the thickness of the glass spindle — for that part of the thermometer above the retort to acquire the highest temperature which the boiling liquid can communicate to it. During this time the thermometer evidently is not in a fit state for an observation. While this gradual change in the condition of the thermometer is taking place, it is desirable, for obvious reasons, that no vapors should escape from the retort. I therefore proceed as follows. The retort, the neck of which has previously been wrapped with a wet cloth, is placed in such a position that the neck

* For common use in fractioning, when not desirable to use sodium, I have found pieces of coke to be more effectual and much more durable than platinum. Not unlikely it would be found equally preferable to platinum for general use in taking the boiling-points of liquids in which sodium could not be employed. It is certain that nothing could operate better than coke for the nitro-compounds and alkaloids derived from benzole and its homologues.

shall slightly incline towards the body of the retort. If necessary, some pieces of ice, which will adhere firmly to the cloth, may be laid along the neck to insure complete condensation of the vapors during ebullition. While the retort is in this position, ebullition is continued for considerable time, until it ceases to have any effect on the height of the mercury in the thermometer. The lamp being now removed for the moment, the neck of the retort is turned down, and quickly connected with a Liebig condenser. The lamp being now replaced, the distillation is commenced. So soon as the mercury in the thermometer shall have become constant, which will now occupy but a few seconds, the temperatures by the retort thermometer and the side thermometer are carefully noted, and also the time at which these observations are made. During the distillation, which is continued nearly to dryness, the readings of the thermometers and of the watch are noted at regular intervals, or so often as any appreciable variation of the retort thermometer shall have taken place. The average of the several observations, or of those corresponding to the longer intervals of time, applying the corrections for atmospheric pressure and for the upper column of mercury, according to Kopp,* is taken for the true boiling-point. I have generally obtained the hydrocarbons so pure that the whole quantity operated upon would distil within the range of 1° of temperature, and not unfrequently within 0°.5. In a few cases, however, when the quantity of material at command would not permit of the attainment of so high a degree of purity, the distillation would range over two or three degrees; in such cases I have generally taken the average of the temperatures corresponding to the longest interval of time, as probably representing more nearly the true boiling-point of the body. In stating my results, however, I shall give the limits of temperature within which the distillation was effected. The thermometers employed in the determinations were the best that I could obtain from Fastrè of Paris; for the temperatures below 100° the instrument used was calibrated, and the scale divided into fifths of a degree. The determinations above 100° were all made with one thermometer.

The method just described differs in some respects from that of Kopp. He objects to the practice of taking boiling-points with the thermometer bulb immersed in the liquid,† on the ground that the thermometer in this condition hardly ever indicates a

* Poggendorff's Annalen, 1847, LXXII. 38.

† "Lässt man die Kugel des Thermometers in die siedende Flüssigkeit tauchen, so zeigt fast nie das Instrument eine constante Temperatur, sondern das Ende des Quecksilberfadens ist in stetter hüpfender oder zittender Bewegung; der auf diese Art gefundene Siedepunkt kann nur mehrere Grade höher liegen, als der, welcher gefunden wird, wenn sich die Kugel des Thermometers in dem Dampf der siedenden Flüssigkeit befindet."

constant temperature, the end of the mercurial column being in a state of motion. He states that a boiling-point taken in this manner may lie several degrees above that found with the thermometer bulb in the vapor. As bearing on this point, I propose, a little farther on, to give the results of a few experiments and observations, which, with others of similar character, have induced me to depart from the now more common custom of taking boiling-points with the thermometer bulb in the vapor.

Under normal conditions, the temperature of the boiling liquid and that of the vapor evolved should be the same. The only disturbing influence which appears to have been specially dwelt upon as likely to alter these conditions in the taking of boiling-points, is the liability of some liquids to adhere to the surface of the glass in such a manner as to produce abnormal elevation of temperature, generally attended with irregular ebullition, and consequent fluctuation of the thermometer. To remedy this it is usual to introduce pieces of platinum; iron filings, coal, etc., have also been employed. As above remarked, pieces of coke — or, when admissible, sodium — are found to be more surely effectual with hydrocarbons than platinum. Indeed, during more than three years of experience and careful observation upon a large number of hydrocarbons, I have not yet met with a single instance in which irregular ebullition and its consequent disturbing influence upon the boiling-point might not be completely prevented by these means. Although I cannot, of course, go so far as to say that equally satisfactory results would be obtained with other liquids by the use of coke, it is nevertheless my belief that in a majority of instances such would be the case.

I have dwelt upon this point for the reason that the objections to the custom of taking boiling-points with the bulb in the vapor, appear to be even greater than those which Kopp has raised against the opposite course of placing the bulb in the liquid, as I shall proceed to show. It therefore becomes a matter of some importance that the objections to one or the other custom should be removed; and I think it will be found easier to overcome the objections to placing the bulb in the liquid, as I have done in the case of many hydrocarbons, even if coke shall not be found equally efficient with most other liquids.

My experience has shown that, when irregular ebullition is effectually prevented, the temperature of the vapor from a boiling liquid is more liable to lead to an erroneous determination of the boiling-point, than the temperature of the liquid itself. The reasons for this are, first, that the vapor is liable to become superheated by the hot air from the flame coming in contact with the sides of the retort above the surface of the liquid; second, that, with the bulb in the vapor, the thermometer is more liable to sudden depression from currents of cool air passing over the retort.

If the bulb be in the vapor, the occurrence of either of these disturbing influences would then affect the principal mass of the mercury in the thermometer; while, on the contrary, if the bulb were in the liquid, only the small quantity of mercury in the stem of the thermometer would be subjected to these influences; the liquid then serving as a regulator, and reducing the error from these sources to a minimum. Fluctuations from currents of cold air are comparatively slight, and more easily prevented than those from overheating the vapor. The latter is the more likely to occur the lower the boiling-point of the liquid, or when the quantity of liquid in the retort is small. I have, however, observed from this cause an elevation of 3°–4° in distilling a body boiling as high as 98° C., without an unnecessarily large flame. But the liquid in this instance was pretty low in the retort.

In the case of liquids boiling below the common temperature, it seems indispensable that the bulb of the thermometer should be placed in the liquid. As evidence of this I will here state the results of observations made while occupied in fractioning some exceedingly volatile products from American petroleum.

Experiment 1. — The liquid operated upon boiled at so low a temperature that the distillation was effected by the heat of the surrounding atmosphere. The distillation was conducted in a flask, and the bulb of the thermometer placed in the vapor. The flask was attached to my condensing apparatus, including the "refrigerator, B, Fig. 2."* The temperature of the condensing-worm contained in the "elevated bath, *aa*, Fig. 2,"* and also that of the "first receiver, *k*, Fig. 2,"* was 11°.5. The temperature of the "cold bath, *ii*, Fig. 2,"* was 11°. The condenser in "the refrigerator, B," and the "second receiver," were cooled in a mixture of ice and salt. With the liquid boiling steadily from several points on the bottom of the flask, and the condensed product from the distillation running well from the refrigerator into the "second receiver," not a drop was condensed in any of the apparatus intervening between the flask and the "second receiver," although this part of the apparatus was cooled, as already stated, to about 11°. The temperature of the vapor in the flask at this time was 18°.5, or only 2°.5 below the temperature of the laboratory. These observations show that the liquid was boiling at a temperature considerably below that indicated by the thermometer in the vapor. Additional evidence of this was furnished by the fact that, during the distillation, the exterior of the flask, from the bottom to about one quarter of an inch above the surface of the liquid, was thickly covered with water condensed from the atmosphere, resembling heavy dew; while above, the sides of the

* See Memoir "On Process of Fractional Condensation," Memoirs of the American Academy, 1864.

flask were perfectly dry. It was these observations which first directed my attention to the fact that the temperature of the vapor could not in all cases be depended upon for the true boiling-point of a liquid, and naturally led me to make other experiments with special reference to this question.

Experiment 2. — The conditions of this experiment were somewhat different from those of the first. The liquid operated upon was the extremely volatile product collected in the "second receiver" of Experiment 1. The flask employed was smaller, and provided with two thermometers; the bulb of one of these was placed in the liquid, and that of the other in the vapor. The flask stood in a water-bath containing ice-water; this bath was also provided with a thermometer. The temperature of the ice-water bath was very gradually raised by means of a small flame from a Bunsen's burner. Temperature of the laboratory, 20° C. Observations during the distillation: —

	1.	Temperature of the water-bath,	10°
		" " boiling liquid,	8°
		" " vapor,	18°.5
	2.	Temperature of the water-bath,	12°
		" " boiling liquid,	9°
		" " vapor,	18°
15 minutes later.	3.	Temperature of the water-bath,	18°
		" " boiling liquid,	10°
		" " vapor,	14°
10 minutes later.	4.	Temperature of the water-bath,	20°
		" " boiling liquid,	12°
		" " vapor,	19°
20 minutes later.	5.	Temperature of the water-bath,	23°
		" " boiling liquid,	15°
		" " vapor,	19°

Experiment 3. — The subject of this experiment was a liquid which I had separated from the most volatile product of the re-distillation, on a manufacturing scale, of the crude benzole obtained in the distillation of coal-tar. The apparatus employed was essentially the same as that used in Experiment 1, with the addition of the extra thermometers, as in Experiment 2. The condensing-worm in the "elevated bath," and that in the "cold bath," and also the "first receiver," were all cooled in pounded ice. The condenser in the "refrigerator," and also the "second receiver," were both

cooled in a mixture of ice and salt. The retort, which stood in a small copper bath containing pounded ice, was charged with about 250 *c. c.* of the liquid, which had been previously cooled in a mixture of ice and salt. Temperature of the laboratory, 16° C. Observations during the distillation: —

	1.	Temperature of the retort-bath, . .	0°
		" " boiling liquid, . . .	0°.6
		" " vapor, . . .	13°.5
45 minutes later.*	2.	Temperature of the retort-bath, . . .	0°
		" " boiling liquid, . .	1°.3
		" " vapor, ...	12°.2
15 minutes later.	3.	Temperature of the retort-bath, . .	6°
		" " boiling liquid, . . .	1°.8
		" " vapor, . . .	12°.6
30 minutes later.	4.	Temperature of the retort-bath, . . .	11°
		" " boiling liquid, . .	3°.8
		" " vapor, ...	12°.4
30 minutes later.	5.	Temperature of the retort-bath, . . .	14°.5
		" " boiling liquid, . . .	7°.3
		" " vapor, . . .	13°.8

The apparent inconsistency that the temperature of the boiling liquid should be above that of the heating medium, — viz. an ice-bath — which continued during the first forty-five minutes of the experiment, is to be explained by the fact that there was a long column of mercury, above the surface of the liquid, which was subjected to the heating influence of the vapor. I would further remark that the gradual elevation of the boiling-point, as indicated by the thermometer in the liquid, is also only apparent, and is due to the gradual uncovering of the bulb as the liquid was distilled off. At the close of the experiment only about one fifth of the bulb, which unfortunately was a long one, was under the surface of the liquid. That this is the true explanation is evinced by the fact that during the experiment not a drop of liquid was observed to fall back into the retort from the "elevated condenser," although this was a tube ten feet in length, and cooled to the temperature of 0°.

I will now proceed to give my determinations of the boiling-points of various hydrocarbons, and of some of their derivatives, and then pass directly to consider the bearing of these results on the question concerning the increment of boiling-point for

* From this point the temperature of the retort-bath was gradually raised by means of a small gas-flame.

the addition of $C_2 H_2$ in homologous series.* The data for these considerations may be more conveniently arranged in tabular form, exhibiting at once, in serial order, the formulæ, boiling-points, elementary difference, and the corresponding difference of boiling-point.

1. *Of the Hydrocarbons obtained from Pennsylvania Petroleum.*

1st Series.

Formula.	Boiling-point.	Elementary Difference.	Difference of boiling-point found.	Range of Temperature within which the substance would all distil.†
$C_8 H_{10}$	$0^\circ.0$ (?)		°	°
$C_{10} H_{12}$	30.2	$C_2 H_2$	30.2	1.5
$C_{12} H_{14}$	61.3	$C_2 H_2$	31.1	0.8
$C_{14} H_{16}$	90.4	$C_2 H_2$	29.1	1.0
$C_{16} H_{18}$	119.5	$C_2 H_2$	29.1	1.0
$C_{18} H_{20}$	150.8	$C_2 H_2$	31.3	0.8
			150.8 ÷ 5 = 30°.16	

Average increment of boiling-point for the addition of $C_2 H_2 = 30^\circ.16$.

* In considering this question I shall include the boiling-points of the substances which I have separated from Pennsylvania petroleum, and the oil distilled from Albert coal; reserving for a subsequent memoir all other facts which have been derived from the study of these bodies.

† The ranges of temperature given in this and in the corresponding columns of the following tables, are for the purpose of showing the impossibility of there having been any essential error in the determinations of the boiling-points; as is evinced by the fact, in each case, that the whole product was found to distil without residue within such narrow limits. With so small a range of temperature, it is evident that it would make no practical difference whether either extreme or the mean of the observations be taken for the boiling-point.

The fact that these substances distil without residue within so short a range of temperature, is also of much value as proof of the existence of the two parallel series in petroleum and in coal-oil, with boiling-points so near together; [as shown by comparison of the boiling-points of the first with the second series from petroleum; and also of the two corresponding series from Albert coal-oil]; especially if this is considered in connection with the fact, so far as my experience goes, that the quantities of material in one series are generally about equal to those in the other.

That no erroneous conception may be formed as to the degree of purity of the substances treated of in this and in the following tables, from a mere inspection of the *ranges* of temperature here given; and in order that the almost absolute constancy of the boiling-points, in most cases, may not be overlooked, I would refer to the preceding memoir for further details concerning the boiling-points of such of these bodies as are therein treated of. For example, it will be found under the head "Determination of boiling-point" of benzole, that in the distillation it required 50 minutes for the temperature to rise 0°.2; while in one of the following tables it will be seen that the *range* of temperature within which the benzole distilled to dryness was found to be 0°.8. Likewise, by reference to the "Determination of boiling-point" of toluole it will be observed that it was found to boil absolutely constant 48 minutes; while the *range* of temperature given in the table referred to is 0°.7. In such cases as these, the slight rise of temperature which takes place just before going to dryness, is doubtless to be attributed to superheating of the vapor, in consequence of there being so small

2D SERIES.*

Formula. (?)	Boiling-point.	Elementary difference.	Difference of boiling-point found.	Range of Temperature within which the substance would all distil.
	°		°	°
$C_8\ H_{10}$	8 – 9			
$C_{10}\ H_{12}$	37.0	$C_2\ H_2$	29.0	0.4
$C_{12}\ H_{14}$	68.5	$C_2\ H_2$	31.5	0.6
$C_{14}\ H_{16}$	98.1	$C_2\ H_2$	29.6	1.2
$C_{16}\ H_{18}$	127.6	$C_2\ H_2$	29.5	1.5
			119.6 ÷ 4 = 29°.9	

Average increment of boiling-point for the addition of $C_2\ H_2$ = 29°.9.

3D SERIES. (*Not completed.*)

Formula.	Boiling-point.	Elementary difference.	Difference of boiling-point found.	Range of Temperature within which the substance would all distil.
	°		°	°
$C_{20}\ H_{20}$	174.9			1.7
$C_{22}\ H_{22}$	195.8	$C_2\ H_2$	20.9	1.5
$C_{24}\ H_{24}$	216.2	$C_2\ H_2$	20.3	2.2
			41.2 ÷ 2 = 20°.6	

Average increment of boiling-point for the addition of $C_2\ H_2$ = 20°.6.

2. *Of the Hydrocarbons obtained from Albert Coal.*

1ST SERIES. (*Not completed.*)

Formula.	Boiling-point.	Elementary difference.	Difference of boiling-point found.	Range of Temperature within which the substance would all distil.
	°		°	°
$C_{10}\ H_{12}$				
$C_{12}\ H_{14}$	59.9	$C_2\ H_2$		1.5
$C_{14}\ H_{16}$	90.6	$C_2\ H_2$	30.7	0.5
$C_{16}\ H_{18}$	119.7	$C_2\ H_2$	29.1	0.5
			59.8 ÷ 2 = 29°.9	

The average boiling-point difference, in this series, for the addition of $C_2\ H_2$, is, therefore, 29°.9.

a quantity of liquid in the retort. Similar instances of absolute constancy of boiling-point as those just cited, might be given from among the products in either series from petroleum and Albert coal; which the *ranges* of temperature given in these tables do not indicate.

* I am somewhat in doubt whether the bodies composing this series and the 2d Series from Albert coal have the formula $C_n\ H_{n+2}$ as here represented, there being some indication that they contain less of hydrogen. For the purpose for which they are now presented, it is immaterial which formula is employed, as the common elementary difference and the boiling-point differences would remain the same; the solution of this question is therefore deferred for a subsequent memoir.

2D SERIES. (*Not completed.*) *

Formula. (?)	Boiling-point.	Elementary difference.	Difference of boiling-point found.	Range of Temperature within which the substance would all distil.
$C_{10}H_{12}$	°		°	°
$C_{12}H_{14}$	68.0	C_2H_2		1.0
$C_{14}H_{16}$	98.5	C_2H_2	30.5	0.6
$C_{16}H_{18}$	125.1	C_2H_2	26.6	
			$57.1 \div 2 = 28°.6$	

Average boiling-point difference = 28°.6.

3. *Of Hydrocarbons obtained from Coal-tar Naphtha.*

Name of Substance.	Formula.	Boiling-point.	Elementary difference.	Difference of boiling-point found.	Range of Temperature within which the substance would all distil.
Benzole,	$C_{12}H_6$	80.0°		°	0.8°
Toluole,	$C_{14}H_8$	110.3	C_2H_2	30.3	0.7
Xylole,	$C_{16}H_{10}$	139.8	C_2H_2	29.5	0.4
Isocumole,	$C_{18}H_{12}$	169.9	C_2H_2	30.1	1.0
				89.9	

Average increment of boiling-point for the addition of $C_2H_2 = 89.9 \div 3 = 29°.97$.

4. *Of Cumole from Cuminic Acid, and Cymole from Oil of Cumin.*

Name of Substance.	Formula.	Boiling point.	Elementary difference.	Difference of boiling-point found.	Range of Temperature within which the substance would all distil.
Cumole,	$C_{18}H_{12}$	151.1°		°	3.6°
Cymole,	$C_{20}H_{14}$	179.6	C_2H_2	28.5	1.2

With only a single exception, the results presented in the above tables point clearly to 30° as the common increment for the addition of C_2H_2 in homologous series of hydrocarbons. Indeed, leaving out of the calculation the third series from petroleum (having the general formula C_nH_n), — which must remain anomalous, — and also the products from oil of cumin, the average of all the other boiling-point differences is 29°.75. The few individual variations from the number 30°, rarely exceeding a single degree, may reasonably be attributed to errors of the thermometer (especially in case of temperatures above 100°), or in some instances to a want of purity of one of the compared substances; which latter cause I doubt not is the case with the body from petroleum boiling at 37°, as upon this body I had bestowed less labor in fractioning than upon most of the others, on account of the extreme volatility and consequent

* See foot-note on the preceding page.

loss of the substance, by which the quantity had become so much reduced that I could ill afford further loss. In the case, also, of cymole from oil of cumin, and cumole from cuminic acid, in which the boiling-point difference varies only 1°.5 from the common difference of 30°, the want of perfect agreement may be fairly accounted for by the fact that the quantity of cumole at command was too small to admit of continuing the process of fractioning far enough to obtain perfect constancy of boiling-point. In consequence, also, of the quantity being so small, the *determination* of the boiling-point of cumole is less reliable, as this had to be conducted in a test-tube. It came into full ebullition at 148°.4, the temperature rising gradually to 151°.6 (observed temperatures), at which latter temperature it had distilled nearly to dryness. The distillation occupied thirteen minutes in passing over the range of three degrees. The average of the extremes, with the usual corrections for pressure, &c., was taken for the boiling-point. Abel,* who probably operated on a larger quantity, found the boiling-point of cumole to be 148°. It does not appear that he applied the corrections for pressure and the upper column of mercury. I do not doubt that the true boiling-point of this body will be found to be 150°, which would establish the difference of 30° between it and cymole.

I would here remark that this difference of 30° for the addition of $C_2 H_2$ was first observed while engaged in fractioning Pennsylvania petroleum, and the oil from Albert coal, — substances the most difficult to separate, on account of the presence in each of two parallel series of constituents, whose boiling-points lie so near together.

As no one had preceded me in the investigation of these substances, my mind was as far as possible unbiased as to the boiling-points of the constituents of these mixtures. I was, however, aware of the beautiful relation between elementary constitution and boiling-point which Kopp had discovered, and familiar with the fact that the more recent investigations had shown the boiling-point difference among homologous hydrocarbons to be about 22°.5. If there was any one thing which more than another tended to bias me, it was the recent work of Church† on the boiling-points in the benzole series, in which he made the boiling-point difference invariably 22° and a fraction, a number varying but 3° from the theory of Kopp. Soon after the publication of Church's results, however, Kopp‡ accepted the number 22°.5 as about the boiling-point difference in this series, therefore regarding it as one of the exceptional series in which the boiling-point difference is greater than 19°. The work of Church had cer-

* Annalen der Chemie und Pharmacie, 1847, LXIII. 308.

† Philosophical Magazine, 1855, (4.) IX. 256.

‡ Annalen der Chemie und Pharmacie, 1855, XCVI. 29.

tainly the appearance of having been performed with great care, conducting to a beautiful harmony of results. My confidence in his determinations of boiling-points was increased not a little by his alleged discovery in coal-naphtha of xylole, boiling at 126°.2, indicating a more thorough analysis of this naphtha than those which had been previously published. This body, the supposed middle member of the benzole series, had up to that time been regarded as wanting in coal-tar naphtha, although all of the other members, above and below it, were found to be present, — an anomaly not easily reconciled with any plausible theory in regard to the formation of these bodies. In view of these circumstances, therefore, I was naturally led, from analogy, to anticipate that the boiling-point difference among the hydrocarbons from petroleum and Albert coal would not vary much from 20°. Not being able, however, to reconcile with previous facts and theories on this subject, the indications which were being gradually unfolded by my seemingly unerring process of separation, I was compelled to lay aside all bias, and to regard these indications as pointing unmistakably to a much greater difference of boiling-point for the addition of C_2H_2 than had previously been supposed to exist in this class of substances.

Having finally established beyond question the common difference of 30° for the addition of C_2H_2 among the hydrocarbons from Albert coal and petroleum (the third series from petroleum, with the difference of 20°, had not then been reached), I began to surmise that this difference might be found to be common among all other series of hydrocarbons. In this connection my mind naturally reverted to the earlier determinations of the boiling-points of the members of the benzole series, some of which, especially those of benzole and toluole, which had been more studied than the others, indicated strongly that 30° might prove to be the true difference for the addition of C_2H_2 in this series. My confidence in Church's determinations thus began to diminish, and finally I undertook to make a thorough analysis of coal-tar naphtha, the results of which are given in Table 3. As there shown, the boiling-point difference in the benzole series is also 30°, and the number of its members is reduced to four, in place of five, as alleged by Church.

This difference of 30°, thus shown to be so common with the hydrocarbons, is so much larger than the difference of 19° which Kopp had found so frequent in other classes of substances, that the discrepancy cannot be regarded otherwise than as conclusive evidence, if such were wanting, that all liquid bodies do not obey the same law in this regard, but that there are unquestionably those series in which the boiling-point difference for the elementary difference of C_2H_2 may be greater than 19°, of which Kopp has already furnished some examples.

That the difference may also be *less* than 19° in some series receives confirmation from the facts presented in the following tables.

6. *Of the Nitro-compounds derived from the Hydrocarbons of the Benzole Series.*

Name of Substance.	Formula.	Boiling-point.	Elementary Difference.	Difference of Boiling-point.
Nitro-benzole,	$C_{12} H_5 NO_4$	212°.1		
Nitro-toluole,	$C_{14} H_7 NO_4$	225.9	$C_2 H_2$	13°.8
Nitro-xylole,	$C_{16} H_9 NO_4$	239.3	$C_2 H_2$	13.4
Nitro-isocumole,	$C_{18} H_{11} NO_4$		$C_2 H_2$	

7. *Of the Alkaloids derived from the Hydrocarbons of the Benzole Series.*

Name of Substance.	Formula.	Boiling-point.	Elementary Difference.	Difference of Boiling-point.
Aniline,	$C_{12} H_7 N$	184°.6		°
Toluidine,	$C_{14} H_9 N$	201.7	$C_2 H_2$	17.1
Xylidine,	$C_{16} H_{11} N$	216.0*	$C_2 H_2$	
Iso-cumidine,	$C_{18} H_{13} N$		$C_2 H_2$	

In regard to the results presented in the last two tables, it may be remarked, first, that of the difference shown in the table of nitro-compounds, viz. an average of 13°.6, the discrepancy between this and the number 19°, being 5°.6, is so large as to leave no room for reasonable doubt that this is one of those exceptional series in which the boiling-point difference is less than 19° for the elementary difference of $C_2 H_2$. As this series does not appear to have been examined by Kopp, I have taken care to make as accurate a determination of the difference as circumstances would allow. The boiling-points were corrected as usual for pressure and the upper mercurial column. The boiling-points which have already been published of these bodies, so far as I have noticed, appear to have been given in the observed, i. e. uncorrected temperatures. The quantities of nitro-benzole and nitro-toluole which I operated upon were sufficiently large, and of a high degree of purity, presenting perfectly constant boiling-points. The quantity of nitro-xylole, however, was not so large as would have been desirable. Although the boiling-point of this body is doubtless very nearly correct, those of nitro-benzole and nitro-toluole are more to be relied upon; and omitting the fraction, the number 14° may, I think, be safely taken as the true boiling-point difference in this series. Secondly, that the less striking difference presented in the series of alkaloids, being only 2° under the number 19°, cannot reasonably justify the assumption that this small discrepancy of 2° is attributable to impurity of the substances, or to

* Not corrected.

inaccuracy in the determination of the boiling-points, when it is considered that great care was taken to obtain a high degree of purity and accuracy, and when it is considered also that previous observers have made this discrepancy larger than mine. It was on account of the fact that so small a discrepancy would naturally raise a doubt as to the reliability of the determinations, and for the reason that Kopp* has considered this series of alkaloids as agreeing tolerably well with his general law, that special care was taken on my part to arrive at a correct result. I am confident, therefore, that the boiling-point difference here will not be found to vary more than a fraction from 17°. Of the absolute accuracy of the boiling-points themselves I do not speak so confidently, since these depend so much on the accuracy of the thermometer at these high temperatures; but the correction of any errors which may have arisen from this source would not be likely to alter the relation, and the difference between the boiling-points would still remain about the same. This remark applies with equal force as to the reliability of the other boiling-points presented in this paper, especially of those of high temperatures.

It remains now to consider the foregoing facts with reference to the other theories mentioned.

Of the Calculated Boiling-points of Hydrocarbons by Schröder's Theory.

The subjoined tables exhibit the theoretical boiling-points of the above-mentioned hydrocarbons,† as calculated according to Schröder's last theory, in comparison with the boiling-points actually found. By this theory, as already stated, each double atom of carbon (C_2) contained in a body is supposed to influence the boiling-point by 30°, and each double atom of hydrogen (H_2) to influence the same —10°; from the sum of these influences the number 70° is in all cases to be deducted, in order to find the boiling-point.

1. *Hydrocarbons from Pennsylvania Petroleum.*

1st Series.

Formula.	Determined Boiling-point.	Calculated Boiling-point by Schröder's theory.	Difference between Calculated and Determined Boiling-point.
	°	°	°
C_8H_{10}	0.0 (?)	0	
$C_{10}H_{12}$	30.2	20	10.2
$C_{12}H_{14}$	61.3	40	21.3
$C_{14}H_{16}$	90.4	60	30.4
$C_{16}H_{18}$	119.5	80	39.5
$C_{18}H_{20}$	150.8	100	50.8

* Annalen der Chemie und Pharmacie, 1855, XCVI. 24.

† To avoid useless repetition, the hydrocarbons from Albert coal-oil will be omitted in this series of tables, they being considered identical with the corresponding bodies from petroleum.

2D SERIES.*

Formula. (?)	Determined Boiling-point.	Calculated Boiling-point by Schröder's theory.	Difference between Calculated and Determined Boiling-point.
$C_8\ H_{10}$	$8-9^{\circ}$	0°	$8-9^{\circ}$
$C_{10}\ H_{12}$	37.0	20	17.0
$C_{12}\ H_{14}$	68.5	40	28.5
$C_{14}\ H_{16}$	98.1	60	38.1
$C_{16}\ H_{18}$	127.6	80	47.6

3D SERIES. (*Not completed.*)

Formula.	Determined Boiling-point	Calculated Boiling-point by Schröder's theory.	Difference between Calculated and Determined Boiling-point.
$C_{20}\ H_{20}$	174.9°	130°	44.9°
$C_{22}\ H_{22}$	195.8	150	45.8
$C_{24}\ H_{24}$	216.2	170	46.2

2. *Hydrocarbons from Coal-tar Naphtha.*

Name of Substance.	Formula.	Determined Boiling-point.	Calculated Boiling-point by Schröder's theory.	Difference between Calculated and Determined Boiling-point.
Benzole,	$C_{12}\ H_6$	80.0°	80°	0.0°
Toluole,	$C_{14}\ H_8$	110.3	100	10.3
Xylole,	$C_{16}\ H_{10}$	139.8	120	19.8
Isocumole,	$C_{18}\ H_{12}$	169.9	140	29.8

3. *The Homologous Hydrocarbons from Oil of Cumin and Cuminic Acid.*

Name of Substance.	Formula.	Determined Boiling-point.	Calculated Boiling-point by Schröder's theory.	Difference between Calculated and Determined Boiling-point.
Cumole,	$C_{18}\ H_{12}$	151.1°	140°	11.1°
Cymole,	$C_{20}\ H_{14}$	179.6	160	19.6

It appears, therefore, that the theory of Schröder finds no support from any one of the different series of hydrocarbons presented in these tables. The discrepancy between the observed and calculated boiling-points, as shown, varies from about 10° to 50° C. This discrepancy is found to increase pretty uniformly by about 10° as we rise from one member to the next higher in the ascending series. In the series of the formula $C_n\ H_n$, however, the discrepancy is nearly a constant one, viz. about 46°. I would not overlook the fact, that the calculated boiling-point of benzole is absolutely identical with that found by experiment; nor the remarkable coincidence, that the agreement is almost perfect between the probable boiling-point, and that obtained by

* See foot-note on page 167.

calculation for the body of the probable formula $C_8 H_{10}$ in the 1st Series from petroleum. It is obvious, however, that these are merely accidental circumstances, to which no importance can attach.

Of the Calculated Boiling-points of Hydrocarbons by Löwig's Theory, viz. that One Atom of Carbon (C) raises the Boiling-point 38°.4, and One Atom of Hydrogen (H) lowers it 29°.2.

Hydrocarbons from Pennsylvania Petroleum.

1st Series.

Formula.	Determined Boiling-point.	Calculated Boiling-point by Löwig's theory.	Difference between Calculated and Determined Boiling-point.
$C_8 H_{10}$	0°.0 (?)	15°.2	°
$C_{10} H_{12}$	30.2	33.6	3.4
$C_{12} H_{14}$	61.3	52.0	9.3
$C_{14} H_{16}$	90.4	70.4	20.0
$C_{16} H_{18}$	119.5	88.8	30.7
$C_{18} H_{20}$	150.8	107.2	43.6

2d Series.*

Formula. (?)	Determined Boiling-point.	Calculated Boiling-point by Löwig's theory.	Difference between Calculated and Determined Boiling-point.
$C_8 H_{10}$	8°–9	15°.2	6°.7
$C_{10} H_{12}$	37.0	33.6	3.4
$C_{12} H_{14}$	68.5	52.0	16.5
$C_{14} H_{16}$	98.1	70.4	27.7
$C_{16} H_{18}$	127.6	88.8	38.8

3d Series. (*Not yet completed.*)

Formula.	Determined Boiling-point.	Calculated Boiling-point by Löwig's theory.	Difference between Calculated and Determined Boiling-point.
$C_{20} H_{20}$	174°.9	184°.0	10°.9
$C_{22} H_{22}$	195.8	202.4	6.6
$C_{24} H_{24}$	216.2	220.8	4.6

A cursory examination of the last three tables will suffice to show that, so far as regards the hydrocarbons of the formulæ $C_n H_n$ and $C_n H_{n+2}$, the theory of Löwig also has no foundation in fact. That his theory did not hold good with the hydrocarbons of the formula $C_n H_{n-6}$ was observed by Löwig himself, who found that it would place the boiling-point of benzole at 285°.6, i. e. 205° above its actual boiling-point.

* See foot-note on page 167.

Of the Calculated Boiling-points of Hydrocarbons by Gerhardt's Theory.

We come finally to test the law of Gerhardt, above mentioned. Inasmuch as this law was specially designed to apply exclusively to the hydrocarbons, — upon the observed boiling-points of some of which it was indeed founded, — we should naturally expect to find this more in accordance with facts than either the hypothesis of Schröder or that of Löwig, both of which were claimed to be of more general application, and were framed more especially with reference to other classes of substances. The facts presented in the following tables will show, however, that this is far from being the case; and that the theory of Gerhardt, as well as those of Schröder and Löwig, so far as these relate to the hydrocarbons, was by no means legitimately drawn from nature, but is altogether artificial.

1. *Hydrocarbons from Pennsylvania Petroleum.*

1st Series.

Formula.	Determined Boiling-point.	Calculated Boiling-point by Gerhardt's theory.	Difference between Calculated and Determined Boiling-point.
$C_8 H_{10}$	0.0° (?)	—8.0°	°
$C_{10} H_{12}$	30.2	12.5	17.5
$C_{12} H_{14}$	61.3	33.0	28.3
$C_{14} H_{16}$	90.4	53.5	36.9
$C_{16} H_{18}$	119.5	74.0	45.5
$C_{18} H_{20}$	150.8	94.5	56.3

2d Series.*

Formula. (?)	Determined Boiling-point.	Calculated Boiling-point by Gerhardt's theory.	Difference between Calculated and Determined Boiling-point.
$C_8 H_{10}$	8 – 9°	—8°	16.5°
$C_{10} H_{12}$	37.0	12.5	24.5
$C_{12} H_{14}$	68.5	33.0	35.5
$C_{14} H_{16}$	98.1	53.5	44.6
$C_{16} H_{18}$	127.6	74.0	53.6

3d Series.

Formula.	Determined Boiling-point.	Calculated Boiling-point by Gerhardt's theory.	Difference between Calculated and Determined Boiling-point
$C_{20} H_{20}$	174.9°	130.0°	44.9°
$C_{22} H_{22}$	195.8	150.5	45.3
$C_{24} H_{24}$	216.2	171.0	45.2

* See foot-note on page 167.

2. *Hydrocarbons from Coal-tar Naphtha.*

Benzole Series.

Name of Substance.	Formula.	Determined Boiling-point.	Calculated Boiling-point by Gerhardt's theory.	Difference between Observed and Calculated Boiling-point.
Benzole,	$C_{12}H_6$	80.0°	93.0°	13.0°
Toluole,	$C_{14}H_8$	110.3	113.5	3.5
Xylole,	$C_{16}H_{10}$	139.8	134.0	6.0
Isocumole,	$C_{18}H_{12}$	169.9	154.5	15.5

3. *Hydrocarbons from Oil of Cumin and Cuminic Acid.*

Name of Substance.	Formula.	Determined Boiling-point.	Calculated Boiling-point by Gerhardt's theory.	Difference between Observed and Calculated Boiling-point.
Cumole,	$C_{18}H_{12}$	151.1°	154.5°	+3.4°
Cymole,	$C_{20}H_{14}$	179.6	175.0	—4.6
Cumo-oil of turpentine,	$C_{20}H_{16}$	155.4	160.0	+4.6

The chief conclusions deduced from the foregoing facts and considerations may be briefly summed up as follows: —

1. That the boiling-point difference for the addition of C_2H_2 in homologous series of hydrocarbons is generally 30° C., which is a much larger difference than has been commonly supposed.

2. That of the five series of hydrocarbons examined, only one series was found exceptional to the rule just stated, and this presented the boiling-point difference of about 20°, which is but little larger than the number 19°, which Kopp found so common with other classes of substances.

3. That certain series of derivatives from the benzole series of hydrocarbons present boiling-point differences, corresponding to the elementary difference of C_2H_2, considerably smaller than the number 19° of Kopp.

4. That the formulæ of Schröder, Löwig, and Gerhardt for the calculation of boiling-points, so far as these may be supposed to relate to the hydrocarbons, are incorrect and purely artificial.

5. That the custom of taking boiling-points with the bulb of the thermometer in the vapor is more liable to lead to an erroneous determination, at least in certain cases, than if the bulb be placed in the liquid.

VIII.

Examination of a Hydro-carbon Naphtha, obtained from the Products of the Destructive Distillation of Lime-soap.

By C. M. WARREN and F. H. STORER.

Communicated, August 9th, 1865.

In the winter of 1859, when the supply of coal-oil in the Atlantic States was altogether inadequate to meet the daily increasing demand for that article, and before the discovery of the fact that abundant supplies of petroleum could be obtained by sinking artesian wells in proper localities, it occurred to one of us that a burning-oil, equal to that from cannel-coal, could be readily obtained from the cheap fish-oils of commerce, by saponifying these with hydrate of lime and then subjecting to destructive distillation the lime-salts thus obtained.

In acting upon this conception, several trial experiments were conducted by its author, upon a somewhat extended scale, in the manufactory in New York, which was at that time under his control. These trials were as follows: — In a shallow wooden tub, eight or ten feet in diameter, at the bottom of which was a coil of metallic perforated pipe, for the introduction of steam, there was first prepared a quantity of milk of lime, and to this was added some two hundred or more gallons of commercial "menhaden-oil." This menhaden-oil is manufactured upon the large scale by boiling and expressing the common fish, *Alosa menhaden,* a sort of herring, which is known popularly in some localities as the menhaden. Steam being then blown into the mixture of oil and lime, saponification was effected in the course of a few hours. After the glycerine-water had been drawn off from the finished lime-soap, the latter was shovelled out into a bin and there left to dry. The amount of quick-lime employed having been about 25 per cent. of the weight of the fish-oil, the finished soap was of course mixed with a considerable excess of hydrate of lime. A ten-barrel cast-iron still was in the next place charged with the dry mixture of soap and hydrate of lime, and the whole heated so strongly that the bottom of the retort was finally red-hot. The distillation proceeded quietly and regularly, the matter in the retort exhibiting no tendency to froth or boil

over; but in the still employed, which was of the ordinary pot-shape, the operation was somewhat tedious, on account of the difficulty of heating the interior of so large a mass of lime. Portions of the products of distillation also condensed at first in the upper layers of the lime, and were driven off with difficulty. Both these impediments could however undoubtedly have been avoided by employing a common iron gas-retort instead of the still.

As a distillate there was obtained a mixture of hydro-carbon oils, of a dark brown color, and a peculiar, disagreeable odor. In consistency this mixture did not differ much from the crude coal-oil which is obtained by distilling rich cannel-coals. The distillate in question came over mixed with some water, which, however, immediately separated as a layer beneath the oil.

The calcareous residuum in the retort was usually colored more or less strongly with carbonaceous matter.

The crude hydro-carbon oil was rectified by first distilling it in a slow current of steam, then treating the distillate successively with oil of vitriol and a solution of caustic soda in the usual way, and again distilling in steam, as before. The refined product so closely resembled refined coal-oil and petroleum in odor, color, and illuminating properties, that it could hardly be distinguished from these.

The yield of refined, merchantable hydro-carbon oil amounted to 60 % or more of the menhaden-oil from which it was derived. Refined coal-oil was at that time selling for $1 to $1.25 per gallon, while the cost of the menhaden-oil was only 25 cents per gallon. But the discovery of petroleum in Pennsylvania, that is, of the method of obtaining petroleum by boring, of course destroyed the technical value of these results.

The residuum of the first rectification of the crude oil was a thick grease, from which large quantities of a colorless crystalline compound were easily separated; but of this solid matter we have as yet made no examination.

Preparation and Investigation of the Naphtha.

The naphtha which we have subjected to particular examination, and of which alone we propose to speak in the following memoir, was obtained from the crude hydro-carbon oil above described, as follows: After having stood enclosed in air-tight iron tanks during four years, that is, until the autumn of 1863, a part of the crude oil which had undergone no preliminary treatment whatsoever, was subjected to the process of distillation, and fractional condensation devised by one of us, which is described in detail in the ninth volume of the memoirs of this Academy.*

* Warren, Memoirs of American Academy, [N. S.] IX. 121.

Of the crude oil in question, ten separate portions, each measuring 6000 *c. c.* were distilled from a copper retort through the hot condenser, as described in the cited Memoir. The oil began to boil in the retort at about 140° C. and the first portions of distillate passed through the hot condenser when this had risen to about 120°; the temperature of the oil in the retort gradually rising to 250° or more, and that of the hot condenser to 220°, at which point the process was interrupted, and the residue in the retort thrown aside. During this first distillation the temperature of the hot condenser was maintained on the average from 30° to 50° below that of the boiling liquid in the retort. The total amount of distillate, that is, of naphtha, obtained was equal to about 20 % of the crude oil; by far the larger portion of the latter being composed of difficultly volatile substances incapable of distilling over at 220°. The naphtha is a mobile liquid, of light, lemon-yellow color, and peculiarly nauseous odor. As in the crude oil, so here the odor of acetone is noticed among others. But in this connection it will be well to remark that by far the larger portion of the naphtha appears to consist of hydro-carbon oils, the oxygenated compounds, like acetone, &c., which have been noticed by previous observers, being present in altogether subordinate proportion, and merely as impurities, as it were, or contaminations.

This first distillate or naphtha was now repeatedly redistilled, — on an average about seventy-five times, — from glass retorts through hot condensers as before, until portions of constant or nearly constant boiling points were obtained; and until the quantities of material lying between these "heaps" or fixed points had become so small as to leave no doubt of the absence of other bodies. During the progress of these operations, which lasted nearly a year, the more offensive element of the odor of the hydro-carbons, and the odor of acetone also, gradually disappeared in great measure; the yellowish color of the first products also diminished as the work went on, all of the pure hydro-carbons finally obtained being perfectly colorless. Considerable amounts of solid matter also collected in the retorts, especially during the distillation of the bodies of higher boiling points, being formed most probably from the oxidation of impurities with which the crude hydro-carbons were contaminated.

When the process of distillation was completed it appeared that at least sixteen bodies of constant boiling points had been obtained. But up to that time not the slightest clew to the composition of any of these bodies had been observed. So far as had been noticed their odor was unlike that of any bodies with which we were familiar, while the action which some of them had been found to exert upon sodium indicated the presence of substances very different from the hydro-carbons which had previously fallen under our notice.

Trial-analyses were now made of several of these products, from which it appeared that they were really hydro-carbons, though still all more or less impure. The analysis of the body boiling at 81°–82° (uncorrected) may be cited as an example of the results obtained at this stage: 0.2685 grm. of substance gave 0.8849 grm. carbonic acid and 0.2203 grm. water; or carbon 89.87% and hydrogen 9.13%.

These figures point at once towards the members of the benzole series, and in fact the body in question was benzole itself. Upon examination, it was found that even the impure substance analyzed had the characteristic odor of benzole, and that after agitation with a little concentrated sulphuric acid, the odor was identical with that of pure benzole, while, on being immersed in a mixture of ice and salt, the liquid crystallized readily, in the same manner as benzole. This result was particularly interesting to us, since we had not anticipated that benzole or its homologues would be found among our products, though a moment's reflection suggested that the conditions under which the lime-soap was heated were such as might give rise to the production of some of these highly carbonized bodies. Moreover, in pointing out the probable presence of its homologues, the benzole here went far toward accounting for several of our unknown bodies. This presumption was subsequently realized, the presence of each of the four members of the benzole series having been proved: and it may here be remarked that as the final result of our investigation it appeared that of the other twelve bodies, four were of the olefiant series ($C_n H_n$), being probably identical with those recently obtained by A. Wurtz * among the products of the action of chloride of zinc upon amylic-alcohol; that four others were members of another $C_n H_n$ series, isomeric with the above, and identical with those previously obtained by one of us, † from American petroleum, and that the remaining four belonged to that series of hydrides specially studied by Schorlemmer ‡ and by Warren, § whose members boil at degrees the names of which end in "eight" or "nine." ||

In view of the impure condition of our products, as indicated by these preliminary analyses, and by the peculiar action upon sodium,** already alluded to, it was deemed

* Bulletin de la Société Chimique de Paris, 1863, p. 300. † Warren, Memoirs of the American Academy [N. S.], IX. 167.

‡ Journal of the Chemical Society of London, 1862, XV. 419. § Memoirs of the American Academy [N. S.], IX.

|| Isolated members of this series had previously been encountered by Greville Williams, Philosophical Transactions, 1857, CXLVII. 461; and Journal of the Chemical Society of London, 1862, XV. 130.

** When a bit of metallic sodium is thrown into the crude hydro-carbon oil it is at once acted upon, becoming bright and lustrous while bubbles of gas are slowly evolved from the liquid so long as any of the metal remains. A flocculent, viscid, alkaline sediment at the same time separates out, which, on being collected and treated with water, behaves like a highly alkaline soap.

The action of sodium upon the isolated heaps composed of members of the $C_n H_n$ series was similar in kind to its action upon the crude oil, and quite unlike anything which we have noticed in studying bodies obtained from petroleum or any other source. In the case of some of the $C_n H_n$ products in question, it was found to be necessary to boil them repeatedly with sodium

best first to subject them to a slight chemical treatment before proceeding to determine their composition. Most of them were consequently treated with a mixture of two volumes of monohydrated sulphuric acid and one volume of water, then washed with a dilute solution of hydrate of potash, — here avoiding agitation which is liable to give rise to emulsions, — then dried over chloride of calcium, or, better, solid hydrate of potash, and distilled repeatedly over metallic sodium before being subjected to analysis. In other cases a treatment with undiluted oil of vitriol was resorted to as will be described further on.

The diluted acid above mentioned was added to the hydro-carbon, by successive small portions, each portion of acid amounting to perhaps one-fiftieth or one hundredth of the bulk of the hydro-carbon, the two liquids being violently agitated together during five or ten minutes, and the acid sediment finally drawn off after having been allowed to settle. As a general rule the first portion of acid became very dark colored and slightly viscid, although the hydro-carbon did not become colored to any extent; the second and third portions of acid behaved in a similar manner, though each was less strongly colored than the preceding, while the fourth, fifth, and sixth portions were only slightly colored. Since the hydro-carbon itself usually began to become colored on the addition of the fourth or fifth portion of acid, the acid treatment was rarely pushed beyond this limit. The caustic potash appeared to exert little or no action upon the hydro-carbons, serving only to remove the last traces of the acid employed. In practice it was found that while the bodies boiling at 35° and other degrees of temperature, the names of which end in five, that is, those of the formula $C_n H_n$ could be obtained in a state of tolerable purity by the treatment just described, the members of the other series required further purification before being fit for analysis, as will appear in the sequel.

Before proceeding to describe in detail the several bodies which we have isolated from the lime-soap naphtha it should be remarked that all statements of temperature refer to the uncorrected indications of ordinary thermometers, and make no claim to special accuracy, excepting when followed by the word "corrected," in which event they refer to the indications of the best Fastrè thermometers, corrected for atmospheric pressure, and for the upper column of mercury by H. Kopp's formula. The method of de-

in order to obtain purity. The more volatile members of this series ceased to act upon sodium much more quickly than those of higher boiling points. The precipitates produced by the action of sodium upon these products, or probably upon impurities contained in them, were white and flocculent, and closely resembled in appearance hydrate of alumina. It is worthy of remark that the heaps composed of members of the benzole series and of Schorlemmer's hydrides did not thus act upon sodium to any appreciable extent. But on the contrary exhibited the same deportment with this metal which we have been accustomed to witness when operating upon these and other products obtained from petroleum, etc.

termining these corrected boiling points being the same as that already described by one of us.* Most of the statements concerning "heaps" and quantities of products obtained refer to the series of distillates, — each representing one degree centigrade of temperature, — which were in our possession at the moment, already mentioned, when the process of fractional condensation ceased to be employed.

Amylene = $C_{10}\ H_{10}$. — The most volatile product obtained from the naphtha, boiled at 33.5°, and the fraction collected between 33½° and 35°, persisted, after repeated redistillation, in commencing to boil at 33½°, there being evidently no appreciable quantity of any substance more volatile than this in the naphtha. The quantity of this product was small, the sum of all the final fractions between 33½° and 55°, not amounting to 100 *c. c.* The summit of the heap was at 35°–37°, this fraction amounting to about 25 *c. c.*; the fractions 37°–39° and 39°–41° were also tolerably large, the latter being larger than the former. But above 41° all the fractions were exceedingly small, as was also the fraction 33½°–35.°

The fraction 35°–37° was selected for analysis. After being treated with diluted sulphuric acid, and alkali, as above described, it was boiled upon metallic sodium.

The dark sulphuric acid liquor which was obtained during the purification, became milky from separation of an oil, and evolved an agreeable, fruity odor when mixed with water.

After the purification, the body boiled, in an ordinary retort containing bits of sodium, at 34.5°–35.6° (corrected).

On combustion, 0. 1215 grm. of the substance gave 0.1682 grm. water, and 0.3796 grm. carbonic acid, or

	Found.		Theory.
Carbon	85.18	C_{10}	85.71
Hydrogen	15.30	H_{10}	14.29
	100.48		100.00

Hydride of Amyl = $C_{10}\ H_{12}$. — The fractions 37°–39° and 39°–41° most probably contained a portion of that variety of hydride of amyl, obtained by Schorlemmer † and by Warren, ‡ which boils at 38.° But the quantity of material at our disposal is so small that we have made no attempt to purify and analyze it.

Cavroylene = $C_{12}\ H_{12}$ and *Hydride of Caproyl* = $C_{12}\ H_{14}$. Next above the prod-

* Warren, Memoirs of American Academy, [N. S.] IX. 159.
† Journal of the Chemical Society of London, 1862, XV. 421.
‡ Memoirs of American Academy, [N. S.] IX. 167.

ucts at 35°–41°, a heap was obtained at 64½°–66½,° which, roughly estimated, amounted to about 250 *c. c.* This heap was well defined, the quantities obtained for each degree above it diminishing rapidly towards 70,° while on the other side the fractions fell away to almost nothing at 63.° At 70°–71° was another well-marked though smaller elevation, amounting to about 180 *c. c.* This diminished rapidly on either hand, but especially on the side towards 80.° The product at 70°–71° had an odor like petroleum, quite unlike the odor of acetone, possessed by the fraction 92°–93,° or the odor of the fraction 64½°–65½°, which though hardly to be compared with that of acetone still reminded one of the latter. Each of the fractions between 64½° and 72° was separately treated with diluted sulphuric acid and subsequently boiled over sodium. The sodium was at first acted upon to a considerable extent, even in the cold, but after three or four redistillations this action ceased almost entirely. The dark sulphuric acid liquor from 70°–71° evolved no such ethereal odor as did that from 92°–93°.

The purified fractions between 64½° and 72,° were now all redistilled some eighteen times through Warren's hot condenser, new fractions being taken off for every half degree. The thermometer employed was graduated to fifths of a degree, and the most scrupulous care was constantly exercised, in the belief that the two heaps might be made to coalesce into one. But the longer the distillatory process was conducted so much the more clearly did the two heaps stand out, their summits being respectively at 64°–65° and 67°–68°. About one-half of the material taken for purification wasted away during the operations here recorded.

Allowing for the elevating influence of the second (68°) body, we estimate the true (corrected) boiling point of the first, when pure, to be 65°.

On combustion, 0.1896 grm. of the purified fraction 65½°–66°, gave 0.254 grm. water and 0.5921 grm. carbonic acid; another portion, not weighed, gave 0.2978 grm. water, and 0.7085 grm. carbonic acid, or

	Found. I.	Found. II.	Theory.	
Carbon	85.18	85.37	C_{12}	85.71
Hydrogen	14.87	14.63	H_{12}	14.29
	100.05	100.00		100.00

A determination of the vapor density of the hydro-carbon boiling at 65½°–66°, afforded the following result.

Temperature of balance,	18°
Temperature of oil-bath,	128°
Excess of weight of balloon,	0.2965

Capacity of balloon,	217 *c. c.*
Air remaining in balloon,	5 *c. c.*
Height of barometer,	755.4mm at 19°
Density of vapor found,	3.001.
" " theoretical, ($C_{12}\,H_{12}$),	2.9046.

The sp. gr. of the liquid was found to be 0.6938 at 0.° A portion of the fraction 67½°–68°, purified as above, and now boiling at 68.5°(corrected), being analyzed, afforded the following result: 0.159 grm. of the hydro-carbon gave 0.2176 grm. water, and 0.4935 grm. carbonic acid, or carbon 84.65 % and hydrogen 15.22 %. Taken in connection with the boiling point of this body, and its petroleum-like odor, the analysis points at once to that hydride of caproyl which boils at 68°–69°, and which has been isolated from coal-oil by Greville Williams,* Schorlemmer, and others.

The method of purification with monohydrated sulphuric acid was here resorted to, in the hope that by this means the caproylene with which the body was supposed to be contaminated might be removed. The action of the concentrated acid, so far as the destruction of impurities is concerned, was apparently feeble; the acid did not even blacken, but only became yellow, though some warmth was evolved, and hence only a single portion of it was employed. But the acid evidently combined with a considerable portion of the hydro-carbon, a certain quantity of a compound much less volatile than the hydro-carbon being formed. After having been decanted from the acid sediment, washed with caustic alkali, and dried over chloride of calcium, and then heated in an ordinary retort, the hydro-carbon began to boil at 72°, the temperature gradually rising, as the distillation proceeded, to 81°, at which point the operation was interrupted and the oily residue in the retort put aside. On redistilling this distillate upon sodium almost all of it came over at 69.5°(corrected).

The last named product was now analyzed with the following result: 0.1111 grm. of the hydro-carbon gave 0.1633 grm. water, and 0.3417 grm. carbonic acid, or

Found.		Theory.	
Carbon	83.89	C_{12}	83.72
Hydrogen	16.38	H_{14}	16.28
	100.27		100.00

It is evident, therefore, that this body, boiling at 68°–69° (corrected), is probably identical with the hydride of caproyl of Schorlemmer, and of Warren; and that the concentrated sulphuric acid did really remove caproylene from the product first analyzed.

* Philosophical Transactions, 1857, CXLVII. pp. 452, 461.

Benzole = C_{12} H_6. Next in order was a very decided heap at 80°–81°; the quantities of the degree-fractions falling off immediately to almost nothing upon either side of the fraction in question. The quantity of liquid in this heap was rather less than 200 *c. c.* The odor of this body was that of benzole. When plunged in a mixture of ice and salt, the liquid did not congeal; but after having been treated with a single portion of monohydrated sulphuric acid, it crystallized at once, almost completely, when immersed in the freezing mixture. When ignited upon a wick, it burned with an exceedingly smoky flame, unlike that of the bodies previously described; when the latter are burning, no such abundant flakes of soot are disengaged. In the cold it had but little action upon metallic sodium, there being, at all events, nothing like the action which it exerted thereupon by the C_n H_n bodies hitherto in question.

An analysis of the product obtained by distillation has already been stated. (See p. 180, and No. I. below.) Attempts were now made to purify this impure material by chemical treatment. In the first place it was treated with the diluted sulphuric acid and alkali, and then repeatedly redistilled over sodium. The first portions of the diluted acid became very dark-colored, but the fourth and fifth portions exerted but little action. For an analysis of the liquid resulting from this purification, see No. II, below.

Monohydrated sulphuric acid was then resorted to, a portion of the product purified as above being treated therewith. The first portion of this strong acid blackened very much, a thick, viscid matter separated out, while some heat was evolved and a slight odor of sulphurous acid was manifested. The second portion of acid had but little action, and the third and fourth portions hardly became colored. These last portions of acid, however, caused the hydro-carbon itself to became rather dark-colored, though on adding dilute alkali, this color changed to a light-yellow.

The dark acid liquor resulting from the treatment with concentrated sulphuric acid, on being mixed with water, became turbid, from separation of a sort of tar; an abundance of sulphurous acid was also evolved on this addition of water, even after the acid liquor had stood at rest for a long while. This remark is true as well for all the other bodies which were treated with monohydrated acid, and in no instance did the addition of water to these dark acid liquors give rise to the evolution of agreeable ethereal odors such as were obtained from the product of the action of diluted acid upon the members of the C_n H_n series.

After having been dried over chloride of calcium, the purified hydro-carbon was heated in an ordinary retort containing pieces of sodium. After a small quantity

of the hydro-carbon had distilled over, the thermometer in the retort rapidly rose to above its upper limit, 120°, the liquid in the retort became black, and sulphurous acid was evolved. The distillation was at once interrupted and the residue put aside. The distillate, being now repeatedly redistilled over sodium, came over clear at 79.5° (corrected). For an analysis of this sample, see below, No. III.

A second portion of the product resulting from the treatment with diluted acid having been treated, as above, with monohydrated sulphuric acid, the first distillation over sodium was now more carefully watched than before. The retort being heated with a small flame, its contents distilled over freely at first, and without coloration, at 79°; after a while the temperature of the retort rose slowly, and at 87° scarcely anything came over. At this point the distillation was stopped, the liquid in the retort being quite oily, though still light-colored. On redistilling the distillate, upon sodium, it came off at 79.9° (corrected).

The bottle containing the product (No. III.) resulting from the treatment with monohydrated acid, was now immersed in a mixture of ice and salt until the moment when crystals began to form, when it was quickly removed and the still liquid portion of the hydro-carbon poured off, the bottle being inverted and the crystals allowed to drain as they melted until only a comparatively small portion of the solid remained. This last was then subjected to analysis; see below, No. IV.

The quantity of material at our disposal being small, we were unable to carry out any systematic course of purification by crystallization, and a single operation like the preceding could hardly be expected to augment the purity of our product to any great extent. But the tendency of this experiment is none the less worthy of being noted; its result still points in the direction indicated by the preceding trials. Each step in the series of treatments above recorded brings us a little nearer to the pure benzole of which, as we have ourselves no doubt, the product (80°–81°), obtained by distillation, is mainly composed.

I. As has already been stated, 0.2685 grm. of the unpurified hydro-carbon, obtained by distillation and fractional condensation, gave 0.2203 grm. water, and 0.8849 grm. carbonic acid.

II. 0.1914 grm. of the hydrocarbon, after treatment with diluted acid, gave 0.1593 grm. water, and 0.6386 grm. carbonic acid.

III. 0.1485 grm. of the hydrocarbon, after treatment with monohydrated sulphuric acid, gave 0.113 grm. water, and 0.4985 grm. carbonic acid.

IV. 0.1962 grm. of the hydrocarbon, after crystallization, gave 0.1484 grm. water, and 0.6631 grm. carbonic acid. Or,

	Found.				Theory.	
	I.	II.	III.	IV.		
Carbon	89.87	90.96	91.55	92.15	C_{12}	92.31
Hydrogen	9.13	9.25	8.48	8.41	H_6	7.69
		100.21	100.03	100.56		100.00

Omitting No. I., these results correspond with the following formulæ: —
II. $= C_{12}\ H_{7.32}$; III. $= C_{12}\ H_{6.67}$; IV. $= C_{12}\ H_{6.57}$, — instead of $C_{12}\ H_6$, as required by theory.

The sp. gr. of No. II. was found to be 0.8697 at 0°, and that of No. III. 0.8882 at 0°.

A portion of this benzole (No. III.) having been converted into nitro-benzole and anilin, there was at once obtained from the latter the purple reaction with hypochlorite of lime. Several portions of this anilin having been heated with arsenic acid, there were obtained in each instance decided manifestations of the color of anilin-red, though the red thus obtained was by no means so brilliant as that subsequently obtained from the toluol-fraction 110°–111° (*vid. inf.*).

A sample of anilin, prepared from a portion of the impure fraction 83°–84°, which fraction had never received any chemical treatment, gave the violet coloration with hypochlorite of lime, but it yielded no red color on being heated with arsenic acid.

Œnanthylene $= C_{14}\ H_{14}$. Above the benzole heap (81°–82°), the quantities of the degree-fractions were very small, until at 90° they began to increase again, there being a prominent heap between 90° and 94°, which amounted to about 300 *c. c.* The summit of this heap was at 92°–93°. On treating it with diluted sulphuric acid, the acid became dark colored, an aromatic odor being at the same time manifested, while a slight odor of acetone, which had previously been present, now disappeared altogether. On adding water to the dark sulphuric acid liquor, after this had been separated from the hydro-carbon, a very penetrating ethereal odor was evolved, while a small quantity of oil, of a reddish color, rose to the surface of the water. After the acid treatment and the subsequent washing and drying, the hydro-carbon was distilled eight times upon metallic sodium, through Warren's hot condenser. The sodium was very strongly acted upon at first, but on the fourth distillation this action had well-nigh ceased. At the close of these operations the summit of the heap was at 93°–94°, this fraction amounting to about 45 *c. c.*, the next fraction (94°–95°), being almost as large (42–43 *c. c.*) The fraction 91°–92° was now very small, the heap only beginning to show itself at 92°–93°, which was equal to 28 *c. c.* The fraction 95°–96° amounted to only about 12 *c. c.*, and the residues, at 96°, were each very small.

Boiled upon sodium, in an ordinary retort, the fraction 93°–94° came over at 94.1°

(corrected); hence, when the quantity of liquid in the fraction 94°–95° is considered the corrected boiling point of the hydro-carbon may be estimated at something less than 95°.

On combustion, 0.1297 grm. of the purified hydro-carbon gave 0.1688 grm. water, and 0.4081 grm. carbonic acid. Or,

	Found.		Theory.
Carbon	85.81	C_{14}	85.71
Hydrogen	14.42	H_{14}	14.29
	100.23		100.00

A determination of the vapor density of the purified fraction 93°–94°, gave the following result: —

Temperature of balance,	= 17°
" " oil-bath,	= 136°
Excess of weight of balloon,	= 0.3845 grm.
Capacity of "	= 219 *c. c.*
Air remaining in "	= 0 "
Height of barometer,	= 757.2mm. at 18°
Density of vapor found,	= 3.4445
" " " theoretical (C_{14} H_{14})	= 3.389

Hydride of Œnanthyl. = C_{14} H_{16}. At 97°–98°, as a summit, was a very well-marked heap, the adjacent fractions, 96°–97° and 98°–99°, being also large, while those next in order fell off gradually upon either hand, — more rapidly, however, above 99° than below 96°. Roughly estimated, the heap from 96° to 100° amounted to about 450 *c. c.*

The fraction 97°–98° having been analyzed before it had been subjected to any chemical treatment, afforded the following result: —

0.2252 grm. of the hydro-carbon gave 0.2913 grm. water, and 0.6964 grm. carbonic acid. Or,

	Found.
Carbon	84 32
Hydrogen	15.19
	99.51

These figures agree with the improbable formula C_{14} $H_{15.2}$.

The fraction 97°–98° was now at once treated with monohydrated sulphuric acid. The first portion of the acid blackened considerably, though the hydro-carbon itself was at first only slightly colored; this blackening was, however, not nearly so decided as was the case with benzole. A portion of viscid matter also separated out and adhered to the sides of the bottle; but only a slight amount of heat was evolved. The

second portion of acid blackened like the first, and considerable heat was now evolved, but less of the viscid matter was formed. The third portion of acid was still considerably colored, though much less than the preceding portions. Abundant fumes of sulphurous acid were now disengaged, and, as a very considerable proportion of the hydro-carbon had been destroyed, the treatment with strong acid was here stopped. The hydro-carbon was now washed with diluted sulphuric acid, with chlorhydric acid (which caused the hydro-carbon to assume a beautiful purple coloration, which was at once destroyed on the addition of potash), and with a solution of potash; it was then dried by means of sticks of hydrate of potash. By this time the portion of hydro-carbon operated upon, which originally amounted to 125 *c. c.*, was reduced to 65–70 *c. c.*

On distilling the purified product upon sodium, from a retort connected with Warren's hot condenser, the temperature of the liquid rose to 102°, and nothing came over until the hot condenser had attained a temperature of 94°. When the temperature of the retort had reached 108°–112°, that of the hot condenser being 96°, the contents of the retort became very black, and torrents of sulphurous acid were evolved; the thermometer in the retort then suddenly rose to 140°–145°, and a very violent reaction occurred in the retort. Much water was at this time evolved, a portion of it having condensed upon the upper part of the retort as soon as this had been removed from the fire. The water was evidently generated by the decomposition of a portion of the liquid contents of the retort. But, in spite of the water and of the sulphurous acid already alluded to, a great part of the sodium in the retort remained unacted upon.

From the foregoing it is evident that during the chemical treatment of this hydrocarbon a portion of it combines with the elements of sulphuric acid to form a compound of high boiling point, and decomposable at the temperature of ebullition.

The distillate was at first distributed as five fractions of nearly equal size, each representing two degrees centigrade, between 102° and 112°; but, on redistilling, the first fraction began to boil at 98,° and the last had all come over before the temperature of the retort reached 100.° After the two fractions thus obtained had been twice redistilled, it was found that nearly all of the hydro-carbon had collected again as a heap at 98°–99.° This product was, however, still highly charged with sulphurous acid gas, in spite of the sodium, which had all the while been present; but, on being washed with caustic alkali, the odor of sulphurous acid was at once removed. After having been dried with hydrate of potash, the product was distilled upon sodium, in an ordinary retort, from which it came over at 97.8° (corrected).

On combustion, 0.1464 grm. of the purified hydro-carbon gave 0.1897 grm. water,

and 0.4144 grm. carbonic acid (I.). Another portion of 0.1168 grm. gave 0.1654 grm. water, and 0.3601 grm. carbonic acid (II.) — or,

	Found. I.	Found. II.	Theory.	
Carbon	84.27	84.08	C_{14}	84
Hydrogen	15.74	15.75	H_{16}	16
	100.01	99.83		100

A determination of the density of its vapor resulted as follows: —

Temperature of balance,	18°
Temperature of oil bath,	150°
Weight of balloon,	0.3795 grm.
Capacity of balloon,	221 *c. c.*
Air remaining in balloon,	3 *c. c.*
Height of barometer,	757.2mm. at 18°
Density of vapor found,	3.5616
" " " theoretical ($C_{14} H_{16}$),	3.458

The sp. gr. of the liquid was 0.7085 at 0°, and 0.6942 at 17.5°.

Toluole $= C_{14} H_8$. Next above the hydride of œnanthyl (98°), was a singularly well-defined heap at 110°–111°. This body was more readily isolated — that is to say, brought into such a state of equilibrium that its boiling point was almost absolutely constant — than any of the other hydro-carbons which we have obtained from the lime-soap-naphtha. Altogether, from 109°–112°, this heap amounted to about 440 *c. c.* Its odor was that of toluole.

The fraction 110°–111° was treated at once with monohydrated sulphuric acid. The first portion of acid made the whole liquid dark-colored and became itself very viscid, some heat being evolved. The second and third portions of acid also became dark-colored, as did the fourth, though to a somewhat less extent. The fifth, sixth, and seventh portions of acid were each allowed to act during twenty-four hours, but they appeared to affect the hydro-carbon very little. The hydro-carbon was now very dark-colored, but on the addition of a dilute alkaline solution it cleared up to a light-yellow color. It was dried over chloride of calcium and distilled in an ordinary retort without sodium. After about two-thirds of the liquid had come over colorless, the residue suddenly became black, its temperature rose rapidly, and much sulphurous acid was evolved. The distillate being twice redistilled over sodium, the greater portion of it came over at 111° (corrected) in each instance. The product was, however, evidently still impure, for in each case a small residue of higher boiling point was obtained. The boiling point above given is doubtless too high; but we have deferred any reconsideration of this re-

sult until such time as a method of properly purifying our first product shall have been discovered. The impurity of the material in question appeared, moreover, from the following analysis: — 0.1656 grm. of the hydro-carbon gave 0.152 grm. water, and 0.5504 grm. carbonic acid, or

	Found.
Carbon	90.64
Hydrogen	10.20
	100.84

This result indicates that the product still contained some sulphurated compound, which, on combustion in the oxygen constantly present in the tube during the analysis, forms sulphuric acid; the latter condenses in the neck of the chloride of calcium tube and so vitiates the hydrogen determination.

A portion of the product just analyzed was now digested during twenty-four hours with a quantity of concentrated chlorhydric acid, by which it was at once rendered milky. After decanting the acid and washing with water, the hydro-carbon was dried over hydrate of potash, and finally distilled upon sodium. On combustion, an unweighed portion of it gave 0.1679 grm. water, and 0.6345 grm. carbonic acid, or

	Found.		Theory.
Carbon	90.25	C_{14}	91.3
Hydrogen	9.75	H_8	8.7
	100.00		100.00

The liquid which had been treated with chlorhydric acid was now distilled in vacuo. It began to boil at 66°, between which point and 68° a fraction was collected (No. I.). Another small fraction was then taken off above 98° (No. II.).

On combustion, 0.1985 grm. of fraction No. I. gave 0.1765 grm. water, and 0.6572 grm. carbonic acid.

0.2595 grm. of fraction No. II. gave 0.2244 grm. water, and 0.8579 grm. carbonic acid.

	Found. I.	Found. II.		Theory.
Carbon	90.07	90.17	C_{14}	91.3
Hydrogen	9.87	9.60	H_8	8.7
	99.94	99.77		100.0

These results correspond respectively with the formulæ: — I. C_{14} $H_{9.2}$; II. C_{14} $H_{8.97}$; the previous analysis, see above, agrees with the formula C_{14} $H_{9.09}$. The body

is doubtless toluole mixed with a portion of one of the more highly hydrogenized substances which occur with it in the crude naphtha.

Attempts to obtain a purer product by first converting portions of the crude fractions 108°–109° and 111°–112° into nitro-toluole led to no useful result, the mixture of nitro-products, etc., obtained being quite unmanageable, at least when in small quantity, as in the present case.

A portion of the product boiling at 111° (corrected) was converted into nitro-toluole and toluidin, and the latter was then heated with arsenic acid, a magnificent product of anilin-red being thus obtained. This toluidin gave no reaction for anilin on being tested with hypochlorite of lime. It may here be remarked that we have in the same way repeatedly obtained anilin-red from toluidin prepared from samples of toluole obtained from coal-tar naphtha by the process of fractional condensation.

Caprylene $= C_{16} H_{16}$ and *Hydride of Capryl* $= C_{16} H_{18}$. At 117°–127° there was a large heap, amounting to about 800 c. c. It had two principal summits, one at 123°–124°, the other at 126°–127°, and a subsidiary elevation at 118°–120°; but this last was probably caused by some irregularity in the conditions under which the distillation was conducted, the work of two operators having overlapped at this point. Moreover, as will be seen directly, analyses of the subsidiary heap, after purification, indicated that it, as well as the fraction 123°–124°, belongs to the $C_n H_n$ series.

All of the fractions from 117° to 127° constituting the great heap were treated with diluted sulphuric acid in the usual way, after which the fractions belonging to each of the three summits were separately and repeatedly distilled through Warren's hot condenser; the same thermometers, condenser, and retort being used in the distillation of each of the summits. The relative importance of the fraction 123°–124° was maintained throughout, this fraction being far larger than any of the others. The fractions 125°–126° and 126°–127° also held their own during these redistillations; but the position of the lower subsidiary summit changed materially, the greater portion of it having finally collected at 121°–122°. It is probable that by continued redistillation this might all have been collected at 123°–124°, or rather that these two heaps could have been made to coalesce at some point between 122° and 124°; but the quantity of material being small, we have made no special effort to effect this result.

Previous to the treatment with acid, the heap 117°–127° had a remnant of the offensive odor of the original crude hydro-carbon oil together with a trace of the odor of acetone; but on the addition of diluted sulphuric acid, an odor like that of mint was

developed together with an ethereal odor. The first portions of the diluted acid became dark-colored and rather thick; but the fifth and last portion of acid was but little colored, though still by no means colorless. After washing with a solution of caustic alkali, and drying over sticks of hydrate of potash, it was found necessary to boil the product with sodium during a long time, and to distil it repeatedly from the sediment which formed, before its action upon this metal became in some degree moderate.

The dark sulphuric acid liquor became turbid, as usual, when mixed with water, and evolved an agreeable ethereal odor.

Distilled over sodium from an ordinary retort, the boiling point of the purified fraction 123°–124° was 125.2° (corrected); and that of the fraction 121°–122° was 123.8° (corrected).

On combustion, 0.2278 grm. of the purified fraction 123°–124° gave 0.2994 grm. water, and 0.7114 grm. carbonic acid (No. I.). Another portion of 0.19 grm. gave 0.2504 grm. water, and 0.5917 grm. carbonic acid (No. II.).

Analyses of the purified fraction 121°–122° afforded the following results: 0.1894 grm. of the hydro-carbon gave 0.2466 grm. water, and 0.5925 grm. carbonic acid (No. III.). Another portion, not weighed, gave 0.1685 grm. water, and 0.406 grm. carbonic acid (No. IV.). Or,

	Found.				Theory.	
	I.	II.	III.	IV.		
Carbon	85.16	84.95	85.32	85.55	C_{16}	85.71
Hydrogen	14.62	14.63	14.47	14.45	H_{16}	14.29
	99.78	99.58	99.79	100.00		100.00

A determination of the vapor density of fraction 123°–124° gave the following result: —

Temperature of balance,	22°
" " oil-bath,	178°
Excess of weight of balloon,	0.3843 grm.
Capacity of "	201 c. c.
Air remaining in "	0 "
Height of barometer,	758mm. at 21°
Density of vapor found,	3.9756
" " theoretical (C_{16} H_{16}),	3.873

The sp. gr. of fraction 123°–124° was 0.7396 at 0°; that of fraction 121°–122° was 0.7433 at 0°, 0.735 at 12°, 0.7321 at 15°, 0.7305 at 17°.

After treatment with diluted sulphuric acid, the fraction 125°–126° afforded the following results on being analyzed: 0.1829 grm. of the hydro-carbon gave 0.2432 grm.

water, and 0.5687 grm. carbonic acid (I.). Another portion, not weighed, gave 0.2398 grm. water, and 0.5647 grm. carbonic acid (II.). Or,

	Found. I.	Found. II.
Carbon	84.80	85.27
Hydrogen	14.76	14.73
	99.56	100.00

From these analyses the improbable formulæ C_{16} $H_{16.7}$ and C_{16} $H_{16.6}$ are derived. The excess of hydrogen, however, indicates the presence of a member of the hydride series, and to obtain this the degree-fractions about 128° were treated with monohydrated sulphuric acid. The first and second portions of acid blackened instantly and much viscid matter was deposited; heat was also evolved. The third, fourth, fifth, and sixth portions of acid each became less black than the preceding portion; but a large proportion of the hydro-carbon disappeared during this treatment. The operations of washing, drying, and boiling with sodium were conducted in the usual way.

On combustion, after this treatment with strong acid, 0.1253 grm. of the hydrocarbon gave 0.1746 grm. water, and 0.3895 grm. carbonic acid. Or,

	Found.		Theory.
Carbon	84.75	C_{16}	84.2
Hydrogen	15.48	H_{18}	15.8
	100.23		100.0

The upper portion of this heap consequently contains hydride of capryl, the true boiling point of which is 128°–129°.

Xylole = C_{16} H_{10}. Between 140° and 144° was a large heap of 840–850 *c. c.*, the well-defined summit of which was at 142°–142½°. Upon either side of this point the size of the degree-fractions rapidly diminished, these being all very small below 140° and above 144°.

The fraction 142°–142½° was treated at once with monohydrated sulphuric acid, the first portion of which became very dark and viscid, some heat being at the same time evolved; the second and third portions of acid also became very dark, though less viscid than the first; the fourth portion of acid became somewhat dark, and the hydrocarbon itself now began to be colored, and the odor of sulphurous acid was perceptible.

After having been washed and dried in the usual way, the hydro-carbon was redistilled through Warren's hot condenser, when it appeared that, by combination of a portion of the hydro-carbon with the elements of sulphuric acid, there had been formed

during the acid treatment a quantity of a difficultly volatile compound, which, at a temperature between 150°–160°, becomes black and undergoes decomposition, while much sulphurous acid is given off.

The first portion of distillate from the above, having been washed with an alkaline solution to remove sulphurous acid gas, was analyzed, with the following result: 0.199 grm. of the hydro-carbon gave 0.2226 grm. water, and 0.6403 grm. carbonic acid. Or,

	Found.
Carbon	87.74
Hydrogen	12.41
	100.15

These numbers correspond with the improbable formulæ C_{16} $H_{13.58}$. We entertain, however, little doubt that the substance analyzed is really a mixture of xylole (C_{16} H_{10}), the boiling point of which is at 140°, and of a hydro-carbon, of the C_n H_n series, boiling at 155°, or thereabouts, which will be described directly.

An attempt was made to separate a purer sample of xylole by repeatedly redistilling the heap and collecting apart the portion more volatile than 140°, this being subsequently reworked, together with the small fractions which had previously been left between 130° and 140°. By this means a small heap was finally obtained, the summit of which was 135°–136°. This heap was treated with diluted sulphuric acid, the first portion of which blackened much, the second portion to a less extent, the third still less, and the fourth but little. After washing, drying, and distilling with sodium, in the usual way, a portion was analyzed with the following result: 0.181 grm. of the hydro-carbon, purified with dilute acid, gave 0.1905 grm. water, and 0.5858 grm. carbonic acid. Or,

	Found.
Carbon	88.29
Hydrogen	11.71
	100.00

This result corresponds with the formula C_{16} $H_{12.74}$; the body still containing far more hydrogen than pure xylole. But as this subsidiary heap is probably contaminated with hydride of capryl (boiling at 128°), we have made no further attempt to purify it by treatment with acids.

Pelargonene = C_{18} H_{18}. About 148°–150° was a rather large heap, amounting to some 500 *c. c.* This retained a little of the offensive odor of the crude lime-soap naphtha,

but had none of the odor of acetone with which the offensive odor had hitherto been accompanied. It was treated with six successive portions of diluted sulphuric acid. The first portions of acid became very black, but the fifth and sixth portions were only slightly colored. After two or three additions of acid, the hydro-carbon itself became rather dark-colored; but on treating it with a solution of caustic alkali the color changed to a light-yellow. After having been dried over hydrate of potash, the hydro-carbon was repeatedly distilled through Warren's hot condenser, being boiled the while over sodium, upon which it continued to act to a considerable extent for a long while.

At the close of these operations the fractions 148°–149° and 149°–150° retained their former prominence, either of them being more than three times as large as the adjacent degree-fractions; 148°–149° was rather larger than 149°–150°, standing to it in the ratio of 5 : 4.2.

Distilled in an ordinary retort over sodium, the fraction 149°–150°, boiled at 153° (corrected).

On combustion, 0.1499 grm. of the purified hydro-carbon gave 0.1965 grm. water, and 0.4705 grm. carbonic acid. Or,

	Found.		Theory.
Carbon	85.59	C_{18}	85.71
Hydrogen	14.54	H_{18}	14.29
	100.13		100.00

Determinations of the density of its vapor gave the following results: —

	I.	II.
Temperature of the balance,	22°	24.5°
" " oil-bath,	187°	186°
Excess of weight of balloon,	0.5493	0.4873
Capacity of "	239 *c. c.*	210 *c. c.*
Air remaining in "	0 "	0 "
Height of barometer,	766.6mm. at 22°	765mm. at 24°
Density of vapor found,	4.557	4.561
" " theoretical ($C_{18}\ H_{18}$),	4.357	

Its sp. gr. was found to be 0.7618 at 0°.

Isocumole = $C_{18}\ H_{12}$ and *Rutylene* = $C_{20}\ H_{20}$ } At 165°–173° was a very large heap amounting to 1200–1500 *c. c.*; being by far the largest heap obtained from the lime-soap-naphtha. Though perfectly well defined as regards the bodies next above (195°) and below (155°) it, this heap, nevertheless, exhibited no clearly-marked summit, each of the degree-fractions within the above-mentioned limits

being about as large as the others, and with the exception of its unusual lateral extension and of a slight depression at the fraction 167°–168° there was nothing in the appearance of the heap to denote any one of its component fractions as the point of culmination, or to indicate the presence of more than one body. The heap in question did, however, really contain two separate substances, being composed principally of a body (C_{20} H_{20}) boiling at 174°–175° (corrected), or thereabouts, together with a quantity of isocumole the boiling point of which is at 170°, as has been shown by one of us.

Analyses of several of the unpurified fractions afforded the following results:— 0.2689 grm. of the fraction 164°–165° gave 0.3038 grm. water, and 0.8264 grm. carbonic acid, or carbon 83.82 %, hydrogen 12.53 %. 0.1345 grm. of the fraction 166°–167° gave 0.1609 grm. water, and 0.4157 grm. carbonic acid, or carbon 84.12 % and hydrogen 13.31 %; another portion of 0.2732 grm. gave 0.3245 grm. water, and 0.8429 grm. carbonic acid, or carbon 84.15 % and hydrogen 13.18 %. 0.1734 grm. of the fraction 170°–171° gave 0.2118 grm. water, and 0.535 grm. carbonic acid, or carbon 84.20 %, and hydrogen 13.55 %.

Despairing of being able to find any central point in this heap, and acting upon the hint furnished by the aforesaid depression, at 167°–168°, we proceeded to purify and examine fractions at each of the extremities. The fractions 170°–171° and 171°–172° were treated in the usual way with diluted sulphuric acid. The first two portions of acid blackened very much; the third, fourth, fifth and sixth also became dark, but each less so than the portion which preceded it. The hydro-carbon itself finally becoming dark-colored, the acid treatment was stopped, and the hydro-carbon, washed with water and alkali, was dried over hydrate of potash and boiled repeatedly upon sodium.

As thus purified it boiled at 174.6° (corrected).

On combustion, 0.1706 grm. of it gave 0.214 grm. water, and 0.5373 grm. carbonic acid, or

	Found.		Theory.
Carbon	85.93	C_{20}	85.71
Hydrogen	13.95	H_{20}	14.29
	99.88		100.00

A determination of the density of its vapor gave the following result:—

Temperature of the balance,	25°
Temperature of the oil-bath,	229°
Excess of weight of balloon,	0.4714 grm.
Capacity of balloon,	208 *c. c.*

Air remaining in balloon,	0 *c. c.*
Height of barometer,	$769.3^{mm.}$ at 25°
Density of vapor found,	4.9166
" " theoretical (C_{20} H_{20}),	4.841

Its sp. gr. was found to be 0.7912, at 0°.

The fraction 165°–166° was treated at once with monohydrated sulphuric acid. The first portion of acid blackened very much, and heat was evolved, but no very great amount of viscid matter separated; the second portion of acid also blackened very much, but was unusually free from viscidity; the third portion acted much less than the second, and the fourth and fifth portions much less than the third, though considerable heat was evolved throughout. The hydro-carbon at last became strongly colored, and much sulphurous acid was evolved. It was washed, dried, and boiled upon sodium in the usual way.

On combustion, 0.2294 grm. of the hydro-carbon gave 0.285 grm. water, and 0.7264 grm. carbonic acid, or

	Found.
Carbon	86.36
Hydrogen	13.77
	100.13

These numbers give the empirical formula C_{18} $H_{17.2}$.

Previous to the above-mentioned examination of the two extremities of this heap, but not until long after the heap itself had been obtained, an attempt was made to hasten operations by working upon it alone; the ten or twelve most prominent fractions of the heap having been selected and repeatedly redistilled, to the exclusion of all other fractions, both above and below, in the hope that products of nearly constant boiling point might thus be more quickly procured. At the commencement of this special operation the common difference between each of the fractions was about one degree, that is, at each successive distillation each of the fractions began to boil about one degree lower than it did in the preceding distillation, and the distillatory process was continued until this common difference had been reduced to one-third of one degree; the portions of distillate more volatile than the selected fraction of lowest boiling point, and of residue less volatile than the selected fraction of highest boiling point, being meanwhile added to the fractions in the old series, now excluded from the distillation, which had been taken off at similar temperatures. It may here be said that this operation was not found to be advantageous, and we do not in any way commend it, at least when applied, as above, to bodies which have already been brought to such a condition that their boiling points are tolerably constant.

As a result of this trial, however, we were led to appreciate more clearly the importance of conducting the entire series of distillations, from first to last, in a systematic and methodical manner, and of avoiding as far as possible interruptions and irregularities of every kind; for we have observed how easily one might fall into error by pursuing the opposite course. Thus, after the prominent fractions composing the heap 165°–173°, had been repeatedly redistilled by themselves, and the products had been finally set aside as completed, our attention was next directed to the intermediate fractions lying between the heap in question and that next below, namely, at about 155°, which fractions had latterly been untouched, excepting for the purpose of adding to the highest among them the most volatile portions of distillate obtained from the selected fractions. It will be observed that, by these additions, the last or highest of the outlying fractions had become quite large, and that the hydro-carbons composing it were undoubtedly mixed in proportions very different from those of the old fractions next below. On being now repeatedly redistilled, together with the old fractions, the position of this factitious heap gradually changed to lower degrees of temperature. This result was of course to be expected, since the accumulated distillate from the special series of fractions would naturally contain much of a comparatively volatile hydrocarbon. This change of position was rapid at first, but soon became less marked, and after a while a sort of temporary equilibrium was attained at 157°–160°, about which point a small heap maintained itself during several distillations. Though this heap was evidently still to be regarded as a mixture, both in view of its previous history and of the fact that it continued all the while to give up considerable quantities of its material at each successive distillation, it was nevertheless thought best to ascertain something concerning its composition, rather than to continue the process of distillation until the heap should be completely destroyed. The propriety of analyzing the compound was, moreover, especially indicated, since there was no apparent improbability that a hydride, homologous with those already described, might be found at 158°–159°. The heap in question was consequently treated with diluted sulphuric acid, precisely as has been described under pelargonene, and again distilled several times through a hot condenser. As before, it soon fell into a condition of equilibrium, — a long flat heap, rising gently to a decided summit at 158°–159°, being constantly obtained. The size of the fractions near the summit remained almost absolutely the same during any two or three distillations, although considerable quantities of the fraction 150°–151° and of residue at 161° were taken off at each successive distillation, and the general behavior of the heap indicated that it was still a mixture.

On combustion, 0.1195 grm. of the purified hydro-carbon gave 0.1348 grm. water and

0.3832 grm. carbonic acid (I.). Another portion, not weighed, gave 0.18 grm. water and 0.5138 grm. carbonic acid (II.).

	Found.	
	I.	II.
Carbon	87.46	87.51
Hydrogen	12.47	12.49
	99.93	100.00

From these results we derive the formula $C_{18}\ H_{15.4}$. It appears, then, that this spurious heap is composed in great part of a member of the benzole series, — undoubtedly of the one which boils at 170° (isocumole); indeed, the formula last given is much nearer that of isocumole than the one previously derived from an analysis of the fraction 165°–166°. During the redistillation of the intermediary fractions the volatile matter, which, as fast as it was eliminated from the products 165°–173°, had been heaped up at the upper extremity of the intermediary series [namely, at 163°–164°], gradually came forward towards its own proper place at 153°, or thereabouts, and in so doing dragged along with it a quantity of the isocumole properly belonging at 170°, until a point was reached at which the tendency of the isocumole to go back nearly balanced the power of the 153° body to go forward, and at this point a temporary heap of course arose. At the moment of the analysis, this heap had been operated upon so long that the isocumole was largely in excess; but if an analysis had been made of the heap as it existed a week earlier, a different result would undoubtedly have been obtained.

Such temporary adjustments, or, as it were, balancings of the opposing forces exerted by two bodies of different degrees of volatility, are noticed not unfrequently in the course of the earlier series of distillations of a mixture of crude hydro-carbons. Soon after definite heaps first begin to appear, there will be seen for a time, at points about midway between the real, permanent heaps, small temporary elevations, which subsequently disappear again as the distillation progresses. But as at this stage of operations all of the fractions are far from possessing constant boiling points, no question as to the lack of individuality of these half-way heaps can well arise. It is probable that the danger of mistaking a counterfeit for a real heap can only occur by virtue of some such cause as in the instance above cited, or where one of the components of a mixture of two bodies is in large excess as compared with the other. Perhaps the substance encountered by Pelouze and Cahours * at 136°–138°, and described by them as hydride of pelargyl $= C_{18}\ H_{20}$, may have been nothing more than a spurious heap, such as

* Bulletin de la Société Chimique de Paris, 1863, pp. 235, 238.

we have just now alluded to; this compound, of P. & C., is in any event unconformable with either of the series of hydrides which are known to exist in petroleum.

It is of course always possible that cases may occur where, from insufficient quantities of material, it will be impracticable to continue the process of distillation until a constant boiling point has been reached; indeed, this inability will in most instances occur, in due course, at either end of every long series of fractions which have been obtained as in the present case from a complex mixture of substances; we would insist only upon the fact that doubts as to the definite character of any small heap will be far less likely to arise when the process of distillation has been carefully and methodically conducted from beginning to end.

With regard to the bodies which we have obtained at or near 140° and 170°, it is no doubt still conceivable that they are not really impure xylole and isocumole, as we suppose, but new compounds, and the observations of Tollens and Fittig,* upon mixed radicals of the ethyl and phenyl series, would seem to strengthen this thought; but in our opinion the weight of evidence is decidedly in favor of the view which places these 140° and 170° compounds in the benzole series. As we understand it, our own experience indicates that the members of the benzole series are peculiarly liable to retain a certain portion of the more highly hydrogenized hydro-carbons so forcibly that these cannot be readily separated by fractional condensation.

Margarylene = C_{22} H_{22}. At 193°–196° was a heap of about 650 *c. c.* Its summit was well defined at 194°–195°, from which point it fell away gradually on either hand through several degrees. It was treated with diluted sulphuric acid in the usual way, the first portion of acid becoming quite dark, and the second, third, and fourth portions each less dark than the preceding. The hydro-carbon itself began to be colored on the fourth addition of acid.

In an ordinary retort upon metallic sodium it boiled at 195.4° (corrected).

On combustion, an unweighed portion of it gave 0.2721 grm. water, and 0.6478 grm. carbonic acid. Or,

	Found.		Theory.
Carbon	85.40	C_{22}	85.71
Hydrogen	14.60	H_{22}	14.29
	100.00		100.00

A determination of the density of its vapor gave the following result:—

Temperature of balance, ...	24.5 °

* Annalen der Chemie und Pharmacie, 1864, CXXIX. 369 and CXXXI. 303.

Temperature of oil-bath,	234°
Excess of weight of balloon,	0.557 grm.
Capacity of balloon,	213 *c. c.*
Air remaining in balloon,	0 *c. c.*
Height of barometer,	765 mm. at 24°
Density of vapor found,	5.471.
" " theoretical,	(C_{22} H_{22}) 5.325.

Two determinations of its sp. gr., at 0,° gave respectively 0.7902 and 0.7916.

Laurylene $= C_{24} H_{24}$. After the first distillate from the crude lime-soap naphtha had been eight or ten times redistilled, the quantities of residue at 200° were so small in comparison with the large retorts employed that it was found to be impracticable to continue the distillation to 220°, as had been originally proposed; the larger portion of the matter less volatile than 200° which was contained in the first distillate, was consequently set aside as a highly impure residue at an early period. After a while, however, when smaller retorts were employed, the distillation of the products then in hand was carried as high as possible, and a small, very flat heap was finally obtained at 208°–212°.

Although there was still considerable doubt whether this heap had been distilled sufficiently, it was thought best to examine it as it stood. It was consequently treated with the diluted sulphuric acid, and subsequently analyzed, as follows: The first, second and third portions of acid blackened very much, the third portion less than the others, a faint odor of mint being meanwhile developed; the fourth and fifth portions of acid blackened much less than the preceeding, and the sixth was but little colored. The hydro-carbon was then washed, dried, and boiled with sodium, as usual.

The largest degree-fraction of the purified product boiled at 212.6° (corrected.)

On combustion (I.), 0.1481 grm. of it gave 0.1735 grm. water, and 0.4715 grm. carbonic acid. After having been again distilled over sodium several times, it was again analyzed (II.) as follows: 0.1463 grm. of it gave 0.1680 grm. water, and 0.468 grm. carbonic acid. Or,

	Found. I.	Found. II.	Theory.	
Carbon	86.83	87.22	C_{24}	85.71
Hydrogen	13.03	12.78	H_{24}	14.29
	99.86	100.00		100.00

A determination of the density of its vapor gave the following result:—

Temperature of the balance,	25.5°
" " oil-bath,	248°
Excess of weight of balloon,	0.5853 grm.
Capacity of the balloon,	217 *c. c.*
Air remaining in "	0 *c. c.*
Height of barometer,	769.3mm. at 25°
Density of vapor found,	5.7314
Theoretical ($C_{24} H_{24}$),	5.8092

Its sp. gr. was found to be 0.8361 at 0°.

We suppose this body to be that member of the $C_n H_n$ series which boils at 215°, but contaminated with some less highly hydrogenized substance. That this contaminating substance is naphthalin we entertain but little doubt, since we have encountered a case almost precisely similar to this when studying the hydro-carbons from Rangoon petroleum, and in that instance were fortunate enough to crystallize out from the hydro-carbon, which corresponds to the one now under consideration, so much naphthalin, that we were able to prove its identity by an analysis and by the examination of its properties. In the case in hand, however, we could obtain no deposit of napthalin on cooling the fractions 202°–203° and 204°–205° in a mixture of ice and salt. It may here be stated that no crystals of any kind separated from any of the products which have been described above, although all of these were maintained during several days at temperatures below 0°.

Mention has already been made, when specially treating of each of these bodies, of the fact that the hydro-carbons boiling at 175°, 195°, and 215° do not readily collect in abrupt heaps at a single fraction, but remain dispersed in nearly equal quantities through a range of several degrees. This comparative flatness of the heaps of high boiling points is in striking contrast with the clearly defined summits of the bodies which boil at low temperatures, that is, below 140°. The constituents of Pennsylvanian and of Rangoon petroleum, which boil at 175°, 195°, and 215°, exhibit the same characteristic flatness; which fact, so far as it goes, would tend to indicate their identity with the corresponding bodies from the lime-soap naphtha. In the same way it may be counted as one item of difference in distinguishing the upper from the lower series of the formula $C_n H_n$. The tendency of these hydro-carbons to form flat heaps undoubtedly explains one part of the difficulty of removing them from isocumole and xylole, to which allusion has already been made.

It has occurred to us that it is not altogether unlikely that the flatness of the heaps at these comparatively high temperatures may be occasioned by the partial decomposition, during distillation, of the hydro-carbons of which the heaps are composed; if such

decomposition occur, it would be attended with the formation of small quantities of bodies boiling at lower temperatures than the substance sought for. It should be remarked, however, that this view has been suggested to us, not so much by any observations peculiar to ourselves, as by the consideration of the well-known fact that paraffine and those portions of petroleum which boil at very high temperatures can be broken up by repeated distillation, with formation of much lighter and more volatile oils, — a fact which has frequently been acted upon by the manufacturers of coal-oil for illuminating purposes, and which, when carried out upon the large scale, is technically known as "cracking" the paraffine or heavy oil.

In brief, then, it appears as the result of our examination of the naphtha from lime-soap that this mixture contains: —

Amylene, ...	$C_{10} H_{10}$
Hydride of Amyl, ...	$C_{10} H_{12}$
Caproylene, ...	$C_{12} H_{12}$
Hydride of Caproyl, ...	$C_{12} H_{14}$
Benzole, ...	$C_{12} H_6$
Œnanthylene, ...	$C_{14} H_{14}$
Hydride of Œnanthyl, ...	$C_{14} H_{16}$
Toluole, ...	$C_{14} H_8$
Caprylene, ...	$C_{16} H_{16}$
Hydride of Capryl, ...	$C_{16} H_{18}$
Xylole, ...	$C_{16} H_{10}$
Pelargonene, ...	$C_{18} H_{18}$
Isocumole, ...	$C_{18} H_{12}$
Rutylene, ...	$C_{20} H_{20}$
Margarylene, ...	$C_{22} H_{22}$
Laurylene, ...	$C_{24} H_{24}$

Or, arranged in homologous series: —

FORMULA.	OBSERVED BOILING POINT.[1]	FORMULA.	OBSERVED BOILING POINT.[1]
$C_{10} H_{10}$	. 34.5°–35.6°	$C_{10} H_{12}$	. . . . About 39°
$C_{12} H_{12}$	. About 65°	$C_{12} H_{14}$	. . . . 68.5°–69.5°
$C_{14} H_{14}$	. Something less than 95°	$C_{14} H_{16}$	. . . . 97.8°
$C_{16} H_{16}$	. 123.8°–125.2°	$C_{16} H_{18}$	. . . . 128°–129°
$C_{18} H_{18}$	. . . 153°		

[1] With regard to these two series we are still uncertain whether or no the true boiling points may not in the one case be a trifle, say a fraction of one degree, lower than the degrees the names of which end in five; and in the other case be more nearly expressed by the degrees the names of which end in nine than by those which end in eight. These points we hope to elucidate thoroughly at some future time. But with regard to the difference in boiling point of almost precisely 30° C. for each addition of $C^2 H_2$, there can be no room for doubt.

FORMULA.	OBSERVED BOILING POINT.	FORMULA.	OBSERVED BOILING POINT.
$C_{12}H_6$	79.9°	$C_{18}H_{18}$	153°
$C_{14}H_8$	111.°	$C_{20}H_{20}$	174°–175°
$C_{16}H_{10}$		$C_{22}H_{22}$	195.4
$C_{18}H_{12}$		$C_{24}H_{24}$	212.6° [1]

It will be noticed that the observed boiling points of all these bodies go to corroborate the results previously published by one of us.[2] A difference of 30° C. for the addition of C_2H_2 being the rule in each of the series, — with the single exception of the higher C_nH_n series, in which the difference is only 20°, as has been previously stated. See pp. 168, 176 of the memoir cited. It will be noticed, moreover, that in this research we have encountered, for the first time in our experience, the lower C_nH_n series (homologues of olefiant gas), and that its members follow the rule of 30°; hence the addition of still another series to the list of those known to conform to this law.

It is a curious fact that the two C_nH_n series unite at 155°. We are still ignorant whether the body, "pelargonene," which boils at about that temperature, belongs to the lower or to the higher series; or whether, as is possible, it does not belong to both series. In petroleum we have found it as a member of the higher series. Since the two series thus coalesce, or at all events since there is no absolute breach between them, it seems to be proper enough to derive the names of the members of the higher series in accordance with the rules which have hitherto been employed by chemists in designating the members of the lower or olefiant series. In view of the greater simplicity of this course, we have preferred to adopt it, rather than to leave the substances without names, or to attempt to base a system of nomenclature upon the somewhat discordant data concerning diamylene, triamylene, etc., which have been published. Further researches are of course called for before the true character of these new bodies can be definitely determined.

It would be premature, at this time, to offer any speculations as to the precise manner in which the hydro-carbons which we have here examined are formed during the destructive distillation of the lime-soap. But it is important to bear in mind the facts that while in coal-tar naphtha we find members of the benzo leseries, and in coal-oil naphtha and petroleum two series of hydrides and the higher series of C_nH_n, we have here in the products of distillation of lime-soap a third naphtha which stands in some respects midway between the other two; for it contains both benzole, Schorlemmer's hydrides, and the higher series of C_nH_n, though it is at the same time specifi-

[1] Known to be impure. [2] Warren, Memoirs of American Academy (N. S.), IX. 170.

cally characterized by the presence of the olefiant series of hydro-carbons, that is, the lower series of $C_n H_n$. A recent examination by A. Wurtz,[1] of the naphtha obtained by distilling a mixture of amyl-alcohol and chloride of zinc, goes to show that in this case the olefiant series is accompanied by that series of hydrides the several members of which boil at 0°, 30°, 60°, 120°, and 150°. We have proposed to ourselves to re-examine this naphtha from amyl-alcohol at an early day, for the purpose of proving whether or no it contains also members of the other series of hydrides (Schorlemmer's) which boil at 8°, 38°, 68°, 98°, and 128°, as well as for the sake of obtaining the paramylene compounds, which we desire to compare with some of the members of our higher series of $C_n H_n$.

As has been stated in the text above, the quantity of liquid contained in each of the heaps was roughly measured. On summing up these several amounts it appeared that there was altogether a total of nearly 6400 *c. c.* of liquid. Of this amount about 0.8% consisted of amylene and hydride of amyl.

3.9%	of Caproylene.
2.8%	of Hydride of Caproyl.
3.1%	of Benzole.
4.7%	of Œnanthylene.
7.6%	of Hydrate of Œnanthyl.
6.9%	of Toluole.
12.5%	of Caprylene and Hydride of Capryl.
13.3%	of Xylole.
7.8%	of Pelargonene.
23.5%	of Isocumole and Rutylene.
10.2%	of Margarylene.
3.1%	of Laurylene.

With regard to this table, it must be understood that it refers only to the quantities of liquid contained in the actual heaps, and does not include the numerous small fractions lying between the heaps. It is offered merely as an approximative indication of the relative proportions in which the several hydro-carbons were obtained. In the present condition of chemical science there can be of course no thought of attempting the quantitative analysis of a crude naphtha like the one now under consideration. Indeed, it has been to ourselves a matter of surprise and gratulation that we have been able successfully to effect the qualitative separation of the hydro-carbons from a mixture so extremely complex as this. The actual isolation of fifteen or sixteen different

[1] Bulletin de le Société Chimique de Paris, 1863, V. 300; compare Bauer, Sitzungsberichte der math. naturwissen. Klasse der Akademie zu Wien, 1861, vol. XLIV. part II. p. 89.

hydro-carbons, belonging to four distinct series, and all boiling within a range of 180° centigrade, is a result in obtaining which we feel amply repaid for all the time and labor which this exceedingly tedious research has cost us. Certainly no more stringent test of the efficiency of the process of "Fractional Condensation," as contrasted with the old methods of distillation, need for the present be required.

Boston, August, 1864.

IX.

Examination of Naphtha obtained from Rangoon Petroleum.

BY C. M. WARREN AND F. H. STORER.

Communicated August 9, 1865.

SEVERAL years since, Warren De La Rue and Hugo Müller [1] attempted to determine the chemical composition of the petroleum from Rangoon. But the results obtained by these distinguished chemists were exceedingly unsatisfactory.

De La Rue and Müller operated upon the large scale, having started with a stock of several tons of the crude petroleum; but in so far as concerns the hydro-carbons, which, as they admit, constitute the chief part of the naphtha, these observers confess their inability to separate the mixture into compounds of fixed boiling points.

So soon as the method of separating volatile hydro-carbons by fractional condensation had been successfully employed by one of us,[2] the desire naturally arose to apply this method to the elucidation of problems which the best chemists of the day had failed to solve. The labors of De La Rue and Müller at once occurred to us as furnishing an extreme instance, and it was determined to test the new process with materials which, as these chemists had shown, could not be unravelled by the old processes of analysis. With this view a sample of native Rangoon Petroleum was obtained, in Jan. 1862, from Price's Patent Candle Co., of London, it being well known to chemists that this firm was at that time constantly importing the petroleum in question. The sample received from Price's Co. amounted to "five imperial gallons;" it was contained in a well-secured vessel, and was accompanied by a certificate of the company to the effect that the petroleum was in the condition in which it is imported into England, — that is, "just as we receive it from Burmah."

The package containing this sample remained in our possession unopened until the autumn of 1863, when the investigation now to be described was commenced. Upon examination the petroleum was found to be a thick, greasy matter, not sufficiently

[1] Proceedings of the Royal Society of London, VIII. 221; or Phil. Mag. 1857, [4.] XIII. 512.

[2] Warren, Memoirs of American Academy, [N. S.] IX. 121.

liquid to admit of being poured from the can which contained it, when the temperature of the air was 25° C.; but, upon being heated, it flowed readily at 30°–33°, and became perfectly fluid at 38°–40°. The color of the mass was yellowish-green. It emitted the odor common to the purer varieties of native petroleum; though its odor was but slight and in no wise offensive. The specific gravity of this native petroleum was 0.875 at 29°.

Four separate portions of the crude petroleum, each about 5600 *c. c.*, were distilled in a common copper retort without the interposition of any hot condenser. A few drops of the liquid began to distil over at about 140°–150°, and the process of distillation was continued until the temperature had reached 270°–300°. The distillate obtained amounted, all told, to a little more than 7000 *c. c.*, or 30.46% of the crude petroleum.

The volatile product, or *naphtha,* thus obtained was now subjected to the process of distillation and fractional condensation, as described in Vol. IX., p. 130, of the Memoirs of this Academy. This naphtha began to pass through the hot condenser at about 125°, the liquid in the retort then boiling at about 165°. During a dozen or more operations the distillation was carried up to 260°; afterwards, as the quantities became smaller, only up to about 250°. The naphtha contained only a very small quantity of easily volatile products, nothing having been collected, in a second cold receiver surrounded with ice, either during the preliminary distillation from the copper retort or during the first series of fractional distillations.

Fractions of the naphtha were taken off for every ten degrees of temperature at first, then for every five degrees, then for every two degrees, and, finally, for each single degree, — by far the larger part of the work having, of course, been done in fractions of one degree. The greatest care was constantly exercised in order to lessen as far as possible the loss by evaporation of the more volatile products. It is, however, impossible to avoid a great waste of these matters. With the same regard to economy of liquid, the size of the glass retorts and of the worms employed was reduced to the lowest practicable limit.

After the distillatory process had been continued until products of definite boiling points had been obtained, or until, in the lack of this, the amount of liquid in each fraction had been so far reduced that there was no longer any hope of isolating pure substances in that part of the field, a survey of the work indicated that there had been obtained seven well-defined heaps[1] between the temperatures of 170°[2] and 250°; but

[1] For definition of this term see the preceding Memoir, p. 179 of this volume.

[2] All statements of temperature, when not followed by the word "corrected," refer to the indications of ordinary thermometers. Corrected temperatures are those taken in the manner described in Vol. IX., p. 143 of the Memoirs of this Academy, and corrected for atmospheric pressure, and the upper column of mercury in accordance with H. Kopp's formula.

that below 175° the quantities of liqiud had become so small that no definite results could there be obtained. It is true that several elevations existed in this range of small fractions, but to these we will refer hereafter.

It should here be mentioned that we had not been long at work upon this naphtha from Rangoon petroleum, before the conviction was forced upon us that we had started with an insufficient quantity of material. Application was therefore made in the winter of 1863–4 to Price's Patent Candle Company for a supply of the naphtha such as was formerly prepared by them by distilling Rangoon petroleum upon the large scale, but to our regret we learned that the naphtha in question was no longer manufactured by the company, and that it was out of their power to furnish us with any of it. In spite of this, and in fact while the negotiation was pending, we continued to work upon our naphtha as before, being animated by a determination to learn how much could be done with the process of fractional condensation when this is applied to so small a quantity of volatile material as that at our disposal.

Each of the isolated heaps of liquid was now worked by itself, over sodium, until this metal was no longer acted upon, after which the most prominent fractions were analyzed and otherwise examined, as is stated below. It should be noted that neither the crude petroleum, nor the naphtha, nor the finished heaps were ever subjected to the action of any chemicals other than this distillation from sodium which has just been alluded to.

Rutylene $= C_{20} H_{20}$. The heap at 170°–176° amounted to about 120 *c. c.* Its summit was very clearly defined at 172°–173°, this fraction being twice as large as those at 171°–172° or 174°–176°, and half as large again as that at 173°–174°.

The fraction 172°–173° boiled at 175.8° (corrected).

On analysis, 0.2036 grm. of it gave 0.255 grm. water, and 0.6421 grm. of carbonic acid. Or,

	Found.		Theory.
Carbon,	86.00	C_{20}	85.7
Hydrogen,	13.75	H_{20}	14.3
	99.75		100.00

Determination of vapor density:—

Temperature of balance,	12.5°
" oil-bath,	223°
Excess of weight of balloon,	0.5745
Capacity "	242 *c. c.*
Air remaining in "	0 "
Density of vapor found,	5.086
" " theoretical ($C_{20} H_{20}$),	4.841

Its specific gravity was found to be 0·823 at 0°.

Heap at 187° $= C_n H_n$. Immediately above the rutylene heap there was noticed a well-defined tendency toward persistency at 180°–184°, and upon finally working these fractions by themselves, it was found to be impossible to reduce them below a certain size, little or nothing coming off at 180°, and there being no residue worth mentioning at 185°. The summit of this spurious (?) heap remained constant at 182°–184°.

The fraction 182°–183° boiled at 187.4° (corrected).

On analysis, 0.234 grm. of it gave 0.289 grm. of water, and 0.736 grm. of carbonic acid. Or,

	Found.		Theory.
Carbon,	85.77	C_n	85.7
Hydrogen,	13.68	H_n	14.3
	99.45		100.0

Its specific gravity was found to be 0.8356 at 0°.

Margarylene $= C_{22} H_{22}$. Between 186° and 193° was a heap amounting to about 215 *c. c.* the summit of which stood out boldly at 188°–190°.

The fraction 189°–190°, which, by the way, was of precisely the same size of the 188°–189°, boiled at 195.9° (corrected).

On analysis, 0.1407 grm. of it gave 0.175 grm. water, and 0.4469 grm. carbonic acid. Or,

	Found.		Theory.
Carbon,	86.64	C_{22}	85.7
Hydrogen,	13.79	H_{22}	14.3
	100.43		100.0

A determination of vapor density resulted as follows:—

Temperature of balance,	13.5°
" oil-bath,	249°
Excess of weight of balloon,	0.5697
Capacity "	231 c. c.
Air remaining in "	0
Height of barometer,	759mm. at — 2°
Density of vapor found,	5.478
" " " theoretical ($C_{22} H_{22}$),	5.325

Its specific gravity was found to be 0.8398 at 0°.

Laurylene $= C_{24} H_{24}$. Between 200° and 214° there were three distinct summits amounting respectively to about 125 *c. c.*, 150 *c. c.*, and 150 *c. c.* These summits were well defined, particularly the lower one, the true boiling point of which was found to be 208.3° (corrected); the second summit boiled at 214.6° (corrected), and the third at 219.5° (corrected).

On analysis, the following results were obtained; No. I. refers to the fraction which boiled at 208.3° (corrected); No. II. to that boiling at 214.6°; and No. III. to that boiling at 219.5°. 0.1845 grm. of No. I. gave 0.2121 grm. water, and 0.5885 grm. carbonic acid (*a*); a second sample of 0.1326 grm. of No. I. gave 0.1525 grm. water, and 0.4235 grm. carbonic acid (*b.*); 0.1387 grm. of No. II. gave 0,1654 grm. water, and 0.443 grm. carbonic acid (*a*); a second sample, of 0.1433 grm., of No. II. gave 0.1728 grm. water, and 0.4552 grm. carbonic acid (*b*); while 0.194 grm. of No. III. gave 0.2407 grm. water, and 0.6106 grm. carbonic acid. Or,

	Found.					Theory.
	I.		II.		III.	$(C_{24} H_{24})$
	a.	*b.*	*a.*	*b.*		
Carbon,	86.99	87.10	87.09	86.60	85.83	85.7
Hydrogen,	12.79	12.82	13.27	13.40	13.87	14.3
	99.78	99.92	100.36	100.00	99.70	100.0

These figures accord with the formulæ I. (*a*) $C_{24} H_{21.19}$, (*b*) $C_{24} H_{21.20}$: — II. (*a*) $C_{24} H_{21.95}$, (*b*) $C_{24} H_{22.29}$: — III. $C_{24} H_{23.28}$; and point clearly to the presence of some substance less highly hydrogenized than laurylene. In a subsequent paragraph it will be shown that this disturbing element was naphthalin. That the naphthalin should have given rise to the formation of three heaps is a matter of no surprise in the present instance, since unfortunately the work of two operators happened to overlap at this very point. One obtained the greater part of the naphthalin, the other most of the laurylene; while between the two a spurious heap[1] was formed.

A determination of the vapor density of Nos. II. and III. gave the following results: —

	II.	III.
Temperature of balance,	12.5°	13°
" " oil-bath,	256.5°	262°
Excess of weight of balloon,	0.6247	0.6238
Capacity "	228 *c. c.*	226.5 *c. c.*
Air remaining in "	0 "	0 "
Height of barometer,	759mm. at — 2°	748.6mm. at 3°
Density of vapor found,	5.980	6.051
" " theoretical $(C_{24} H_{24})$,	5.809	5.809

The specific gravity of No. I. was found to be 0.8654 at 0°; that of No. II. to be 0.8548 at 0°; and that of No. III. to be 0.8453 at 0°.

Naphthalin = $C_{20} H_8$. In the course of the winter it happened that the temperature of the apartment in which the products from the Rangoon naphtha were kept, fell to 1° or 2° below zero, and remained at that point during several days. It was then

[1] See this volume, p. 200.

noticed that an abundant crop of crystalline plates had separated out in the fraction which boiled at 208° (corrected), and the fraction next above, namely, 209° (corrected). The quantity of these crystals was large as compared with the amount of liquid from which they had been deposited. After they had been removed from the liquid, a second crop of crystals was obtained from the same fractions, by cooling them in a mixture of ice and salt. These crystals were specially abundant, as before, in the fraction 208° (corrected); but no crystals were obtained, by the treatment with ice and salt, from any of the neighboring fractions, or from any of the fractions in the heap at 214.6° (corrected). Nor were any crystals deposited during the continuance of the cold weather from any of the other products which we have obtained from the Rangoon naphtha. The crystals from fractions 208°, 209° (corrected), were allowed to drain and were then pressed gently between folds of filter paper so long as any oil could be thus removed from them. During these operations the crystals remained unchanged; excepting in so far as they developed the unmistakable odor of naphthalin, Like those of pure naphthalin from coal-tar, the crystals were not very soluble in cold spirit, but on warming the spirit they dissolved in the same manner as crystals of pure naphthalin. In cold ether they dissolved in the same way as the crystals of naphthalin, and crystallized out again like naphthalin; a portion of the crystals in each case, that is, both the crystals from Rangoon naphtha and those from coal-tar, sublimed into the upper part of the test-tube in which the solution was effected, and were deposited there in the well-known characteristic plates. The crystals from fraction 208° (corrected) melted at 74°. A portion of them having been maintained during some time at a temperature slightly superior to 90°, a sublimate of the characteristic plates already alluded to was deposited upon the cold upper part of the tube above the source of heat.

On analysis, 0.167 grm. of the crystals first deposited from fraction 208° (corrected) gave 0.1015 grm. water and 0.5729 grm. carbonic acid. Or,

	Found.		Theory.
Carbon,	93.53	C_{20}	93.75
Hydrogen,	6.70	H_8	6.25
	100.23		100.00

Cocinlyene = $C_{26}H_{26}$. The last heap in our series was at 226°–234°, the summit of it being at 229°–232°, each of the fractions 229°–230°, 230°–231°, and 231°–232 being very nearly of equal size, though each was considerably larger than any others in the heap. The fraction 230°–231° was a trifle larger than the others. The quantity of liquid in the whole heap 226°–234° amounted to about 325 *c. c.* Above 234

the quantities of liquid in the degree-fractions fell away to almost nothing; there is evidently no compound present between 234° and 250°.

The fraction 230°–231° boiled at 232.75° (corrected).

On analysis, 0.2871 grm. of the fraction 230°–231° gave 0.3541 grm. water, and 0.91 grm. carbonic acid. Or,

	Found.		Theory.
Carbon,	86.38	C_{26}	85.7
Hydrogen,	13.69	H_{26}	14.3
	100.07		100.0

In determining the vapor-density of this substance the balloon was filled with an atmosphere of carbonic acid[1] after the introduction of the liquid. The following result was obtained: —

Temperature of balance,	22°
" " oil-bath,	274°
Excess of weight of balloon,	0.6863
Capacity "	233 *c. c.*
Air remaining in "	0
Height of barometer,	760.5 m. m. at 21°
Density of vapor found,	6.4225
" " theoretical ($C_{26}H_{26}$),	6.2940

The specific gravity of the fraction 230°–231° was found to be 0.8445 at 0°.

The attempts which we have made to isolate the constituents of that portion of Rangoon naphtha which is more volatile than the hydro-carbons above described, were unsuccessful; the quantity of naphtha boiling at temperatures lower than 175° having been so small that it could not be thoroughly analyzed by the process of fractional condensation. After protracted efforts to separate these volatile hydro-carbons from one another by means of a diminutive apparatus, we were at last reluctantly forced to abandon the attempt, and to acknowledge our inability to obtain satisfactory results from such small quantities of the complex material.

Indeed, the quantity of volatile naphtha at our disposal was so small that although at the last it was divided only into fractions of wide range, each of them representing three or more degrees of temperature, these portions in several instances soon became too minute to be operated upon at all, even in the smallest practicable apparatus. But since these volatile products had been subjected, first and last, to a large number of distillations and fractional condensations, each of the fractions finally obtained must have

[1] In a previous attempt to determine this vapor-density in the usual way, without employing carbonic acid, the mixture of air and vapor in the balloon took fire with a slight explosion, the temperature of the oil-bath being then at 321°.

been tolerably well purified from all substances, excepting those whose boiling points are not widely different from its own. We have, therefore, taken pains to analyze some of the more prominent among the fractions into which the volatile portion of the naphtha had been divided, in order to learn whether there might not thus be obtained a general idea of the composition of this part of the naphtha.

The following is a record of the analyses in question, — all statements of degrees of temperature here referring to "corrected" boiling points: —

I. One portion (*a*) of the fraction 98°–109°, this being the most volatile[1] of all the products which we have obtained from Rangoon petroleum, gave 0.3294 grm. water and 0.771 grm. carbonic acid; another portion (*b*) gave 0.4533 grm. water and 1.0673 grm. carbonic acid.

II. 0.194 grm. of the fraction 121.6°–123.5° gave 0.2589 grm. water and 0.6022 grm. carbonic acid.

III. 0.2987 grm. of the fraction 142.3°–144.3° gave 0.3799 grm. water and 0.9416 grm. carbonic acid.

IV. 0.1583 grm. of the fraction 151.6°–153.7° gave 0.2048 grm. water and 0.4952 grm. carbonic acid; another portion (*b*) gave 0.1346 grm. water and 0.3245 grm. carbonic acid.

V. A portion of the fraction 154.7°–155.7° gave 0.2002 grm. water and 0.491 grm. carbonic acid.

VI. 0.1554 grm. of the fraction 158.8°–159.8° gave 0.1986 grm. water and 0.488 grm. carbonic acid.

VII. 0.1714 grm. of the fraction 164°–165° gave 0.2174 grm. water and 0.5422 grm. carbonic acid.

VIII. 0.1899 grm. of the fraction 172.3°–173.8° gave 0.2398 grm. water and 0.5992 grm. carbonic acid.

Reducing these results to per cents., we have

	I. *a*	I. *b*	II.	III.	IV. *a*	IV. *b*
Carbon,	85.18	85.27	84.64	85.97	85.34	85.59
Hydrogen,	14.82	14.73	14.85	14.13	14.40	14.41
	100.00	100.00	99.49	100.10	99.74	100.00

	V.	VI.	VII.	VIII.
Carbon,	85.79	85.65	86.23	86.04
Hydrogen,	14.21	14.16	14.06	14.01
	100.00	99.81	100.29	100.05

[1] It should be remembered, in this connection, that De La Rue and Müller, by operating upon large quantities of the petroleum, obtained products boiling as low as 50°.

From these analyses the following formulæ are derived:—

I. $(a) = C_{14}\ H_{14.62}$; $(b) = C_{14}\ H_{14.51}$. II. $C_{16}\ H_{16.85}$. III. $C_{16}\ H_{15.78}$. IV. $(a) = C_{18}\ H_{18.23}$; $(b) = C_{18}\ H_{18.19}$. V. $C_{18}\ H_{17.9}$. VI. $C_{18}\ H_{17.86}$. VII. $C_{18}\ H_{17.54}$. VIII. $C_{18}\ H_{17.59}$.

It will be observed that the fractions Nos. I. and II. are largely composed of hydrides,—doubtless those of œnanthyl and of capryl, which boil at 98°, 120°, and 128°,—though still contaminated with hydro-carbons belonging to one or both of the $C_n\ H_n$ series, or possibly even with toluole. Fraction No. III. is probably a mixture of xylole, the foregoing hydrides, and members of the $C_n\ H_n$ series, as before. The composition of fraction No. IV. indicates the probable presence of hydride of pelargonyl, boiling at 150°. Fractions Nos. V., VI., VII., and VIII. are probably composed for the most part of pelargonene ($C_{18}\ H_{18}$) contaminated with a little isocumole ($C_{18}\ H_{12}$).

The comparatively small proportion of hydrogen found in the fractions which boil in the vicinity of 140° and 170° (the boiling points of xylole and isocumole), goes to corroborate the opinion of De La Rue and Müller,[1] that Rangoon petroleum contains members of the benzole series, and is perhaps all the more pertinent in view of the fact that we have ourselves isolated naphthalin from the petroleum, as has been already stated. It is of course conceivable that the naphthalin alone may have contaminated the fractions in question, as well as the definite heaps which have been previously described, the analysis of all of which indicate the presence of a little less hydrogen than is contained in pure $C_n\ H_n$. But this conception seems to us improbable; the composition of fractions 142.3°–144.3° (No. III.) and 151.6°–153.7° (No. IV.), in particular, would appear to invalidate it. We may here say that on the whole our results have very much weakened the opinion, which at one time seemed to us to be not altogether improbable, that the benzole-homologues obtained by De La Rue and Müller might have resulted from the action of nitric acid in removing hydrogen from the more highly hydrogenized hydro-carbons, and might not have been contained in the native petroleum.

As the result of our examination thus far, it appears that the naphtha from Rangoon petroleum contains:—

Rutylene = $C_{20}\ H_{20}$	boiling at about	175°
Margarylene = $C_{22}\ H_{22}$	" " "	195°
Laurylene = $C_{24}\ H_{24}$	" " "	215°
Cocinylene = $C_{26}\ H_{24}$	" " "	235°
Naphthalin = $C_{20}\ H_8$.		

Also, probably, Pelargone = $C_{18}\ H_{18}$, boiling at about 155°, and members of one or both of the series of hydrides; it being a fair presumption that we have had in our hands the Hydrides of Œnanthyl, of Capryl, and of Pelargonyl. Our experiments also indicate the probable presence of Xylole and Isocumole.

[1] Proceedings of the Royal Society of London, VIII. 225.

BOSTON, June, 1865.

X.

A History of the Fishes of Massachusetts.

By DAVID HUMPHREYS STORER, M. D., A. A. S.

Continued from Vol. viii. p. 434.

ORDER II. PLAGIOSTOMI.

Gills fixed by their external edges, with five small external openings on each side. No opercle. Jaws represented by the palatine and postmandibulary bones, which alone are armed with teeth. Pectorals and ventrals always present, — the latter, in the male, furnished on their internal margins with long appendages.

FAMILY XXVIII. SQUALIDÆ.

Body elongated, cylindrical. Tail thick and muscular. Eyes lateral. Branchial openings on each side, never underneath.

GENUS I. CARCHARIAS. Cuv.

One anal and two dorsal fins; the first dorsal placed over the space between the pectoral and abdominal fins. Jaws and head depressed. Teeth flat, pointed, and cutting; serrated in the upper jaw, sometimes in both jaws. No temporal orifices in adults, but rudiments may be observed in the fœtus of some of the species.

Carcharias griseus, *Ayres.*

The Gray Shark.

(Plate XXXVI. Fig. 1.)

Carcharias griseus, Ayres, Bost. Journ. Nat. Hist., iv. p. 293, pl. 12, fig. 4.
" " Storer, Synopsis.

Color. The anterior and upper parts of the body are of a dark ash color; sides lighter; beneath white.

Description. Body much more elongated than that of the Lamna punctata; its greatest depth, across from the origin of the dorsal fin, is equal to about one fifth the entire length of the fish. The length of the head is equal to about one seventh the entire length. The eyes are horizontally oblong, — their longest diameter one inch and a half; the distance between the eyes is three and a half inches. The nostrils are large, situated half way between the eyes and the tip of the snout; similar in form to those of the Lamna punctata. Numerous minute black points, the orifices of mucous pores, are distributed along the under surface of the snout. Mouth of moderate size, situated beneath. Three rows of elongated, sharp, smooth teeth, with a minute denticulation on each side, at their base, in each jaw; about twelve teeth on each side of the median line, — those toward the angle of the mouth the smallest. The tongue is large, fleshy, smooth. The posterior of the branchial orifices is situated just anterior to the base of the pectoral fins.

The lateral line is scarcely discernible.

The first dorsal fin arises upon the anterior half of the body, nineteen inches posterior to the tip of the snout. It is slightly convex upon its anterior edge, rounded above, emarginated posteriorly. A portion of its base is free.

The second dorsal is situated about six inches back of the first, of the same form, and but a very little smaller than that fin.

The pectorals are broad and stout, and as high again as long.

The ventrals are semiquadrate; they are just back of the termination of the first dorsal fin; the distance between the ventrals and anal is less than the length of the ventrals.

The anal fin is of the form and size of the second dorsal, and arises on a line opposite the termination of that fin.

About three inches back of the second dorsal, the body terminates in a slight protuberance, directly back of which is a depression from which commences the caudal fin. This fin is thirteen inches in length; at its anterior inferior margin, it is similar in form to the dorsal and anal fins; this portion is rounded at its inferior posterior margin, emarginated upon its posterior edge, and is continued gradually elongating, its inferior edge being a mere fringe, and terminates within three inches of the extremity of the tail, which dilates into a triangular portion.

The specimen above described measured three feet and eleven inches.

Length, four feet.

Remarks. This species was first described and figured by Mr. William O. Ayres from

a specimen taken in Long Island Sound, on the north shore of Brookhaven. It is uncommon in our waters.

Massachusetts, STORER. Connecticut, AYRES.

CARCHARIAS OBSCURUS, *Storer*.

The Dusky Shark.

(PLATE XXXVI. FIG. 2.)

Squalus obscurus, Dusky Shark, LESUEUR, Acad. Nat. Science 1, p. 223, pl. 9.
Carcharias obscurus, " " STORER, Report, p. 184.
" " " Bost. Journ. Nat. Hist., II. p. 558.
" " DEKAY, Report, p. 350, pl. 61, fig. 201.
" " STORER, Synopsis.

Color. Above, a dark yet vivid blue, somewhat banded by lateral lines, yet gradually passing into the pure white of the abdomen; this tinge of the back extends low upon the sides. Upper part and sides of head, as well as caudal and most of dorsal fins, of a delicate steel color with coppery reflections. Pectorals above, very dark green; beneath, a dull white. Posterior border of dorsals covered with a black mucous slime. Pupils dark brown, irides golden.

Description. Body tapering gently posteriorly; its greatest depth, midway between the pectorals and the first dorsal fin, equal to a little more than one seventh the entire length. Head elongated, sharp, flattened above and below. Snout throughout studded with mucous pores arranged in lines or at random, some of which are very large. The length of the head is equal to little more than one sixth the length of the body; its greatest depth is about one third its length, and nearly equal to its greatest width. Eyes large; their longest diameter, the longitudinal, equals about one quarter the distance between them. Nostrils situated on the outer edge of the lower surface of the head, not quite midway between the eyes and end of the snout, nearer the eye, double; the anterior and outer, a narrow slit, entering downwards and inwards; the posterior, nearly circular. Mouth not very large; the outer edge of the upper jaw just beneath the centre of the eye; its outer angle about one third the distance between the eye and pectorals. Both jaws armed with short, triangular, and serrated teeth; those in the upper jaw curved backwards; in each jaw a single row behind, a double row in front. Branchial apertures, five, comparatively small; the posterior two just above the anterior margin of the pectorals.

The lateral line is indistinct, — high up on the back; mucous pores somewhat similar in appearance, scattered throughout upper back.

The first dorsal fin is small and subquadrangular; its height equalling its length; and each about one third the length of the head.

The second dorsal is very small, about one third as large as the first, from which it also differs in another respect, its posterior margin being the longer; whereas in the first it is the anterior. It is situated posterior to the middle point between the first dorsal and caudal fin.

The pectorals are very large; their length is nearly four times their height, reaching beyond the middle of the first dorsal, — triangular, slightly falciform; the apex and posterior angle being rounded. These fins are situated just posterior to the middle point, between the end of the snout and the first dorsal.

The ventrals are moderate sized; height and length about equal; placed on a line about midway between the first and second dorsals.

The anal fin is small, of the size of the second dorsal, and of same form as that fin, save that its anterior border is slightly more rounded, and its margin more deeply cleft directly beneath that fin.

The caudal fin is slender, elongated, about two ninths the entire length. The upper lobe is little more than twice as long as the lower, and less stout; the preceding carinæ but little marked; a well marked notch above and below, before the caudal.

Length, nine to ten feet.

Remarks. The specimen described by me in the Boston Journal of Natural History was captured at Nahant, July 10th, 1839, and measured nine and a half feet. The one above described was sent to me from Provincetown, by Captain Atwood, October 30th, 1851. This is not a common species in the waters of Massachusetts. It sometimes floats ashore in the night, like the Goose-fish, — Lophius Americanus, — or becomes entangled in mackerel-nets, like the mackerel shark — Lamna punctata.

Massachusetts, STORER.

CARCHARIAS VULPES, *Cuv.*

The Thresher. Fox Shark.

(PLATE XXXVI. FIG. 3.)

Squalus vulpes, GMEL., LIN., Syst. Nat. I. pt. 3, p. 1496.
Long-tailed Shark, PENN., Brit. Zoöl., III. p. 110, pl. 14.
Squalus vulpes, Fox Shark, SHAW., Gen. Zoöl., V. p. 333.
Carcharias " " " *or Thresher*, GRIFFITH'S, CUV., X. p. 599.
Thresher, MITCH., Medical Repository, VIII. p. 77.
Squalus vulpes, Thresher or Long-tailed Shark, MITCH., Trans. Lit. and Phil. Soc. of N. Y., I. p. 482.
" " *Sea Fox or Thresher*, JENYNS, Brit. Vert., p. 498.
Carcharias vulpes, Fox Shark, STORER, Report, p. 182.
Alopias vulpes, Sea Fox, Thresher, Sea Ape, YARREL, Brit. Fishes, 2d edit., II. p. 523, fig.
Carcharias " *Thresher or Fox Shark*, LINSLEY, Cat. of Fishes of Connecticut.
Carcharias vulpes, Thresher Shark, DEKAY, Report, p. 348, pl. 61, fig. 199.
Alopias vulpes, STORER, Synopsis.

Color. All the upper part of the body, together with the fins, a dark bluish lead; beneath white. Pupils blue-black, edged with golden.

Description. Surface of the skin rough when hand is passed toward the head. The depth of the body, at the origin of the dorsal fin, is equal to a little more than one eighth of the length of the fish; the length of the tail, from its origin to its extremity, is rather more than one half the entire length of the fish; the distance from the tip of the snout to the origin of the dorsal fin nearly one fifth the length of the fish. Length of the head, from the tip of the snout to the first branchial aperture, nearly equal to the greatest depth of the body.

Occiput slightly convex. Eyes situated vertically, very movable in their sockets. In the specimen now before me, a female, their longest diameter is one seventh the length of the head; whereas in a male specimen I formerly described, it was about one tenth the length of the head. Snout blunted; distance from its tip to the mouth two thirds of the length of the head. Gape of mouth moderate; three rows of very small teeth in each jaw, smooth on their edges; the two first rows nearly perpendicular, the back row recurved; teeth in the upper jaw rather the longer. Nostrils beneath, nearer the mouth than the snout. Five branchial apertures placed vertically, the posterior the smallest.

The body of this fish is terminated on the back by a slight ridge; just back of this, is a depression between it and the tail, at the origin of which is quite a concavity.

The first dorsal fin is triangular, as long as high; convex anteriorly, rounded above.

The second dorsal is a mere finlet, quadrangular, with its posterior superior angle projecting backwards.

The pectorals are large, stout, falciform; their posterior bases are free.

The ventrals are shaped like the first dorsal, and are of the same length.

The anal fin is of the same size and form as the second dorsal, and is situated just posterior to it.

The caudal fin is very strong and powerful; its inferior base is triangular; back of this portion it gradually diminishes in thickness and terminates obtusely; just anterior to the extremity of the tail, upon its inferior edge, is a small triangular portion. A fleshy membrane margins the entire inferior edge of this fin.

Length, twelve to fifteen feet.

Remarks. This species, which sometimes weighs from one hundred and fifty to two hundred pounds, is known by our fishermen as the *Thresher* or *Swingle-tail*, from the motions of its tail, which is often used with great force. It is met with in our waters in summer, not often however, pursuing mackerel and manhaden, upon which it feeds. Small numbers are yearly captured in the fall of the year in nets set for mackerel, and occasionally a specimen is taken with the hook while fishing for dog-fish. When thus caught, it is secured with much difficulty on account of the constant and powerful thrashing of its tail.

This fish is considered almost valueless. Its liver, however, contains a small quantity of oil, and when an individual is accidentally taken, this is preserved and sold with the oil from the livers of other species.

Massachusetts, STORER. New York, MITCHILL, DEKAY.

CARCHARIAS ATWOODI, *Storer*.

The Man-eater Shark.

(PLATE XXXVI. FIG. 4.)

Carcharias Atwoodi, STORER, Proceed. Bost. Soc. Nat. Hist., III. p. 72.

Color. A leaden gray upon back and sides, and white beneath. The lower portion of the tips and edges of the pectorals are black.

Description. Depth across from the origin of the dorsals, twenty-three inches; across from the origin of the pectorals, twenty-six inches; across from the first branchial orifice, twenty-five inches; across from the extremities of the ventrals, fourteen inches; from the tip of the snout to the first branchial orifice the distance is equal to the greatest depth of the fish. The cheeks are very prominent. The eyes are perpendicularly oblong, their greatest diameter being two inches, their shorter diameter an inch and a half;

the distance between the eyes is eleven inches; the distance from the eyes to the tip of the snout is ten inches. The nostrils are situated three and a half inches in front of the eyes, and six inches from the tip of the snout. The gape of the mouth is very large. Both jaws are armed with five rows of large, triangular, serrated teeth,—the front teeth of the upper jaw about an inch and a quarter long; toward the angles of the jaw they are smaller. The teeth in the lower jaw are less wide than those of the upper. About twenty-five teeth can be counted in each row.

The branchial apertures vary from twelve to fifteen inches in length.

The first dorsal fin is just back of the pectorals; it is eighteen inches high, measured over its outer edge, and thirteen inches long, four inches of its base being unattached; it is slightly emarginated posteriorly.

The second dorsal fin arises thirty-one inches back of the posterior edge of the first dorsal. This fin is four inches high and five inches long, three and a half inches being unattached.

The pectorals are thirty-two inches high, and rounded over their outer edge; they are fourteen inches long at their base, six inches of which are unattached.

The ventrals are eight inches high at their outer edge, three at their middle, and five at their posterior portion. They are nine inches long at their base, four inches of which are unattached.

The anal fin arises eleven inches posterior to the extent of the ventrals, on a line opposite the posterior portion of the second dorsal; it is three and a half inches high, and five inches long at its base, three inches of which are unattached; its posterior edge is highly emarginated.

Just anterior to the caudal fin, upon the dorsum, is a groove two inches across, and half an inch deep; beneath this, upon each side, a prominent carina passes to the base of the caudal fin.

The caudal fin is large and strong; it measures thirty-three inches over its upper lobe, and twenty-six over its inferior lobe; eight and a half inches anterior to the tip of the larger lobe is a small triangular posterior. This fin measures thirty-three inches across from the tip of its lobes.

The specimen here described measured twelve feet eleven inches in length, and weighed about fifteen hundred pounds.

Length, thirteen feet.

Remarks. That this is an exceeding rare species along our coast, is obvious from the fact that I can learn of but three individuals having been seen by our fishermen during

the last fifty years. One of these measured six feet; a second, nine feet; the third, thirteen feet. My specimen was captured at Provincetown, June 16th, and was brought to this city for exhibition. When first seen, it was swimming in about ten feet of water on the Long-Point side of Provincetown harbor. A boat's crew having given chase, a harpoon was thrown into it, when it instantly turned toward the boat, and seized it with great ferocity near the bows, in which act several of its teeth were broken off. It was eventually killed by being frequently lanced.

I know of no species which resembles this, unless it be the great white shark, — carcharias vulgaris, — and it certainly cannot be identical with that. When I presented the generic characters of this species to the Boston Natural History Society, October 18th, 1848, I made the following remarks: "The absurd notion of indiscriminately annexing the names of individuals to objects of Natural History has been almost discarded, unless in cases where the persons so specified have in some way advanced the boundaries of science. In the instance before us, I feel you will all agree with me in acknowledging that the compliment here offered is deserved, when I remind you that the hardy fisherman referred to, while constantly engaged in the fatigues of his exceedingly laborious profession, has transmitted me within the last two seasons, besides the species here described, a species of *Blennius* and *Motella*, both of which genera were new to our waters; besides a specimen of the *Somniosus brevipinna*, previously only known by a description of a stuffed specimen met with by Lesueur, at Marblehead, thirty years ago; and a specimen of *Aspidophorus monopterygius*, never but once previously met with south of Greenland; without referring to numerous specimens of our most common species. I would at the same time reiterate, what you have repeatedly heard me state, that he is more conversant with the history and habits of the fishes north of Cape Cod, than any individual with whom I am acquainted, or in other words that he is our best practical ichthyologist." Eighteen years have elapsed since the above words were spoken, and my debt to the individual referred to has immensely increased, and can never be repaid. Whatever other genus this species may be hereafter arranged in, whether it be Carcharodon or some one yet unformed, unless it be ascertained to have been previously described, I implore succeeding ichthyologists to hesitate before they expunge it. Let his name, who has done so much to enable me to present this final report on the fishes of Massachusetts, be indelibly associated with the science to which he is an honor.

Massachusetts, STORER.

GENUS II. LAMNA, Cuv.

Muzzle pyramidal, under the base of which are the nostrils. Branchial apertures all in front of the pectorals.

Lamna punctata, *Storer*.

The Mackerel Shark.

(Plate XXXVII. Fig. I.)

Squalus punctatus, Green-backed Shark, Mitchill, Trans. Lit. and Phil. Soc. of N. Y., i. p. 483.
Lamna punctata, Mackerel Shark, Storer, Report, 185, pl. 3, fig. 2.
" " " " " Bost. Journ. Nat. Hist., ii. p. 534.
" " *Mackerel Porbeagle,* Dekay, Report, p. 352, pl. 63, fig. 206, 207.
Lamna punctata, Mackerel Shark, Storer, Synopsis, p. 252.

Color. All the upper part of the body is greenish, which becomes of a slate color after death; lighter upon the sides; white beneath. Pupils black, irides dusky.

Description. Head small, its length is nearly equal to one seventh the length of the entire fish. Eyes nearly circular, very movable in their orbits; distance between the eyes equal to three times their diameters. Nostrils large, in front of eyes and inferior to them; the posterior opens forward, and is the larger; a semicircular groove passes forward and downward to the inferior which opens posteriorly. On a line above the eyes, are seen a series of mucous pores, resembling black orifices, running toward the snout; another series between the eyes and the snout. These are also distributed upon the under portion of the snout. Each jaw is furnished with three rows of small, sharp, triangular teeth, smooth at their edges; the two first rows straight, the back row recurved; the three teeth on each side of the middle of the lower jaw, the largest. Tongue large, rough, fleshy. Five large branchial apertures situated vertically; the distance between the anterior greater than that between the posterior. The depth of the fish, in front of the dorsal fin, is less than one quarter the length of the fish; the distance from the extremity of the snout to the dorsal fin is less than one third the length of the fish.

The first dorsal fin is somewhat triangular, with a fleshy horizontal process pointing backward from its base posteriorly, higher than long, emarginated posteriorly, rounded above.

The second dorsal fin is adipose, rhomboidal; its height is equal to one fourth the length of the first dorsal.

The pectorals are quite strong, falciform, higher than the length of the head, and connected posteriorly by a membrane to the body.

The semiquadrate ventrals are situated far back on the body; anus large, situated between the ventrals.

The anal fin is formed like the second dorsal, and is opposite it.

A double series of mucous pores point out the lateral line. On a line with the origin of the second dorsal fin, continuous with the lateral line, a wide carina runs on each side to the centre of the tail. The space between the second dorsal and the tail is equal to the length of the pectorals; at the posterior portion of this space is a crescent-shaped ridge with a groove behind.

The lobes of the caudal fin are unequal; the upper is much the larger, with a slight emargination at the superior posterior portion. This emargination is not referred to by Dekay in his description; and it is omitted in his figure. The specimen which I described in my "Report," measured eight feet, and near its anus, embedded in the flesh, was a specimen of the "Anthosoma Smithii"—Leach.

Length, three to ten feet. Weight, between two and four hundred pounds.

Remarks. This is the most common species of shark found in Massachusetts. It is met with during the summer and autumn. The fishermen are much annoyed by having their hooks and lines bitten off by this species while fishing for cod and mackerel, and their nets seriously injured, and not unfrequently ruined by them, while fishing for the latter species. It is more plenty upon some portions of the coast of Maine than in our bay. Captain Atwood informs me that while he was fishing for mackerel with nets at Monhegan, Maine, in September, 1845, his boat's crew of four men took twelve individuals; and another boat's crew of six men captured nineteen in a single night; and he adds, he should judge that one hundred and fifty at least, were taken during three weeks he continued to fish there. Except for the oil furnished by this species it is worthless to the fisherman. Seven gallons of oil are frequently extracted from the liver of a single fish, and eleven and a half gallons have been taken from one. Of late years this fish has yielded less oil than formerly, so that they are now scarcely thought worth saving. Formerly, a barrel of oil was frequently made from the livers of eleven fish, and Captain Atwood tells me that, many years since, his father even procured a barrel of oil from eight livers; not selecting the largest but employing large and small indiscriminately; but now, at least one hundred livers would be required to furnish this amount of oil. So that the procuring oil from this fish, which was once a regular business, has been almost entirely abandoned. When this oil is carefully prepared by boiling the fresh liver,

it is less valuable than whale oil to burn. It is a usual practice, however, among the fishermen to mix all the common fish oils together, when they are sold in Boston market under the name of shore oil. The curriers use the greater portion of this oil. This species feeds upon many different kinds of fish; but as it is generally met with while following shoals of mackerel, it is generally known as the mackerel shark. At Provincetown, it is called blue shark.

Maine, Massachusetts, STORER. New York, MITCHILL, DEKAY.

GENUS III. MUSTELUS, CUV.

Teeth blunt, forming a closely compacted pavement in each jaw; with temporal orifices. First dorsal in front of the ventrals. Lower lobe of the caudal short. No spines.

MUSTELUS CANIS, *Dekay*.

The Smooth Hound.

(PLATE XXXVII. FIG. 2. 2 *a*. Head beneath.)

Squalus canis, Dog-fish, MITCHILL, Trans. Lib. and Phil. Soc. N. Y., I. p. 486, pl. 64, fig. 209.
Mustelus canis, American Hound-fish, DEKAY, Report, p. 355, pl. 64, fig. 209.
" " STORER, Synopsis, p. 258.

Color. All the upper part of the body is of a uniform slate color; the sides are lighter; the abdomen of a dirty white.

Description. Of an elongated form, gradually sloping upward from just back of the eyes, to the origin of the dorsal fin, beyond which it tapers to the tail. Skin smooth. The individual before me, which is a male, is three feet and three inches in length; the width of the body at the ventrals, is five inches; the greatest height is at the origin of the first dorsal fin, about three inches. The length of the head is eight and a half inches; the distance between the eyes is two and a half inches. The head is flattened between the eyes, which are longitudinally oblong; their greatest length is one inch. The temporal orifices are just back of the posterior angle of the eyes, on a line with them. The mouth is large, triangular when closed. The teeth like those of the rays. At the posterior angle of the upper jaw, a fleshy prolongation, half an inch in length, projects backward. The snout is obtuse; the nostrils are large, situated just in front of the mouth, on the edge of the base of the snout, and are covered by a valve.

The lateral line is quite prominent throughout the greater portion of its extent, and is continued in a straight course to the tail.

The first dorsal fin is subquadrangular, rather longer than high, deeply emarginated posteriorly, and terminating in an acute point.

The second dorsal fin is formed like the first, and is situated far behind it.

The pectorals are large and subtriangular.

The ventrals are subquadrangular; the claspers on each side of the ventrals are as long again as the fins themselves.

The anal fin is of the same form as the second dorsal, but smaller; it arises beneath the middle of that fin, and extends beyond it.

The caudal fin commences by a small elevation or crest, the prolongation, as it were, upwards of the cuticle, gradually becomes higher and is rounded at its posterior extremity; beneath, at its posterior extremity, is a triangular portion, which is partially separated at its base by a small fissure from the anterior portion, which is of a more elongated form.

Length, two to four feet.

Remarks. This species, which is called by the fishermen of Massachusetts Bay the smooth hound from its smooth skin, and dog-fish from its general resemblance to the dog-fish shark, I had not seen when my report on the fishes of Massachusetts was published. Since then I have examined several specimens taken in our bay and at Holmes Hole. This species sometimes runs ashore in great numbers. It is more numerous south of the Cape. Its liver yields about as much oil as that of the Acanthias Americanus. The largest I have met with measured forty-six inches.

Massachusetts, Storer. New York, Mitchill, Dekay.

GENUS IV. SELACHUS, Cuv.

Two dorsal fins, — the first placed but little behind the line of the pectorals, the second over the interval between the ventral and anal fins. The skin rough. Snout short and blunt. Temporal orifices very small. Teeth very small, numerous, conical, edges smooth, no lateral denticles. Branchial openings large, nearly encircling the neck.

SELACHUS MAXIMUS, *Cuv.*

The Basking or Elephant Shark.

(PLATE XXXVII. FIG. 3.)

Squalus maximus, LIN., Sys. Nat. I. p. 400.
" " *Basking Shark*, PENN., Brit. Zoöl. p. 134, pl. 16.
" " SHAW, Gen. Zoöl. v. p. 327, pl. 149.
" " FABRICIUS, Fauna Greenlandica, p. 130.
" " *Basking Shark*, JENYNS, Brit. Vert. p. 503, sp. 193.
Squalus peregrinus, BLAINVILLE, Ann. du Museum, XVIII. pl. 6, fig. 1.
Squalus maximus, *Basking Shark*, MITCH., Trans. Lit. and Phil. Soc. of N. Y. I. p. 486.
Squalus elephas, LESUEUR, Journ. Acad. Nat. Soc. II. p. 343, pl.
Squalus (*Selachus*) *maximus*, CUV., *Basking Shark*, RICH, Faun. Boreal, Americ. III. p. 291.
Selachus maximus, *Basking Shark*, *Sun-fish*, *Sail-fish*, YARRELL, Brit. Fishes (2d edit.) II. pl. 518, fig.
Selachus maximus, GRIFFITH, CUV. X. p. 603.
Squalus elephas, LES., STORER, Report, p. 407.
Selachus maximus, *Basking Shark*, DEKAY, Rep. p. 357, pl. 63, fig. 208.
" " STORER, Synopsis.

Color. The whole upper part of the body of a dark slate color; lighter beneath. Mouth white, mottled with fuliginous.

Description. The surface of the body throughout, divided into innumerable rugæ which are covered with minute sharp points, often collected into groups, resembling the discs of many of the echini, upon which are situated the spines by which they are ornamented; or, still more, the tubercles along the lateral line of some of our cotti, causing the skin to be exceedingly rough.

From the tip of the snout to the first branchiæ, four feet nine inches. Five very large branchiæ nearly surrounding the head, as the animal is lying; the first pair of branchiæ are separated on the neck, from each other, six inches; the second pair are separated, at the same situation, nine inches; the third pair, one foot three inches; the fourth pair, one foot nine inches; the fifth pair, two feet three inches; showing the first interval to be much the largest. The head is small; perfectly smooth for the most part in front of the eyes, and covered with circular and oblong mucous pores, which keep this portion constantly lubricated. Snout blunt. Nostrils five inches in front of the eyes, the lower portion upon the edge of the upper lips. Eyes very small; their diameter three inches; largest circumference of sclerotic coat when removed from the socket, eight and a half inches. Eyes very movable in their orbits; distance between the eyes two feet; distance between the tip of the jaws, as artificially raised, two feet; this vertical gap is undoubtedly as much again at least, in the living fish, which gives an opening of four feet.

Jaws furnished with a great number of small, incurved, pointed teeth. Six rows of these in the upper jaw, and seven rows in the lower jaw; the inner row in this jaw are hardly formed; each of the rows in this jaw, as I counted them in the mouth, contained one hundred teeth from the tip to the angle of the jaw, or two hundred, as counted from one angle to the opposite one; or, in a word, fourteen hundred teeth in this jaw. The teeth are conical, sharp, polished, with a sensible ridge upon each side, often roughened, almost serrated; the lower portion of the tooth striated; the teeth at the angles of the jaws, short and more compressed. The teeth in the centre of the jaw are three lines high above the jaw, and their base or root about the same length within the socket. Temporal orifices small; just back of the angle of the jaws.

The first dorsal fin is triangular; two feet ten inches long, four feet four inches high anteriorly, three feet posteriorly; distance between the first and second dorsal fins, six feet.

The second dorsal fin is ten inches long, sixteen high anteriorly, thirteen posteriorly.

The pectorals are falciform; one foot nine inches long, five and a half feet high; distance between pectorals and ventrals, eight feet.

Length of the ventrals, one foot eleven inches; height, two feet nine inches; length of the claspers, three feet three inches; width at their base, eight inches, from which they gradually pass to a point; they enclose a strong bony spine.

The anal fin commences opposite the second dorsal; its length is eleven inches, its height fourteen inches; across the top, ten inches; distance between the anus and the anal fin, three and a half feet.

Anterior to the caudal fin is a lunated depression; above and beneath the posterior extremity of the fish, at the base of the tail, is a carina upon each side, one foot eight inches long. The caudal lobes are unequal; the upper lobe, six feet six inches in length, measured over its curve, having at its extremity a small triangular lobe; the lower lobe, four feet two inches, measured in the same way; width of the extremity of the lower lobe, six inches; width at the base, two feet two inches; width of the extremity of the upper fluke or lobe, one inch; width of the base, two feet three and a half inches; from the lunated depression to the middle of the fin, two feet eleven inches.

Length, thirty-six feet.

Remarks. The specimen above described, measured thirty feet and three inches. It was harpooned in the harbor of Provincetown, in 1839, and being towed to Chelsea, was there exhibited. I visited it with my friend, Jefferson Wyman, M.D., who made a figure, while I prepared the description which accompanied my "Report," which I

have here transcribed, not having been able to see a second specimen. When I saw the fish, it was lying upon the beach, where it was entirely exposed at low tide, and nearly, if not altogether, covered by water when the tide was high. The tide was flowing in when I examined it, which compelled me to make a more rapid survey than could have been wished. It had been opened, and its viscera were removed. The liver filled eight barrels, and furnished six barrels of oil.

Among our fishermen this species is known as the Bone Shark. It is rarely observed on our coast, and when taken is generally harpooned. For my knowledge of it in our waters, I am almost entirely indebted to my old and tried friend, Capt. Atwood. Within his remembrance he has known but three to be captured in nets. In 1835, an individual became entangled in a mackerel-net, and was then harpooned. In 1836 or 1837, a second was caught in a net; and after being drowned, its carcase was freed by the fishermen from the net, and it afterward drifted ashore in a state of decomposition. After lying upon the beach several days, a fisherman visited it for the purpose of procuring a slice for his hens, as is the custom at Provincetown, he supposing it to be a dead whale. Ascertaining what the animal was, he removed the liver and sold the oil in Boston for *one hundred and three dollars*, it having produced five or six barrels of oil. In 1847, a third was captured, then harpooned and drawn ashore.

In 1848, a vessel going to the coast of Maine for humpback whales, fell in with many of this species off Cape Elizabeth, and secured several of them. A tradition exists among the fishermen, that this species was taken in quite large numbers one hundred years ago, in the spring, for their oil.

This species was described and figured by Lesueur, from a specimen taken near New York, in 1822, as being previously unknown to naturalists, under the name of *Squalus elephas*. The specimen seen by Lesueur was afterward examined by Dekay, who has given us Lesueur's figure with some alterations; having been taken from a preserved specimen it fails to give some of its characteristics. Some of the figures of this fish, found in different works of natural history, are exceedingly unnatural. This fact is thus accounted for by Yarrell in his description of the species: "The difficulty of obtaining a perfect view of this unwieldy fish, either when floating in water, or when, from its great weight, it lies partly imbedded in the soft soil of the sea-shore, has led to the differences which appear in the representations of it which have been published by different naturalists."

Greenland, Fabinius. Massachusetts, Storer. New York, Mitchill, Dekay. New Jersey, Lesueur.

GENUS V. ACANTHIAS. Risso.

Two dorsal fins, with a spine before each; first dorsal behind the line of the pectorals; the second dorsal over the space between the ventral and caudal fins; no anal fin. Skin rough in one direction; the scales heart-shaped, with a central spine directed backward. Temporal spiracles large. Several rows of teeth in both jaws, cutting and sharp, the points directed outward and backward.

ACANTHIAS AMERICANUS, *Storer.*

The Dog-fish.

(PLATE XXXVIII. FIG. 1. 1 *a.* JAWS.)

Spinax acanthias, Picked Dog-fish, STORER, Report, p. 187.
" " *Spinous Dog-fish,* DEKAY, Report, p. 359, pl. 64.
" " *Dog-fish,* AYRES, Bost. Journ. Nat. Hist., IV. p. 289.
Acanthias americanus, STORER, Synopsis, p. 506.

Color. All the upper part of the body is of a slate color, which is deeper upon the head, and lighter below the lateral line. Body beneath, white; a row of circular white spots are situated just under the anterior portion of the lateral line, and a few similar spots are irregularly distributed upon the back; these spots, in some specimens, are arranged with much more regularity than in others. The young of this species are much more spotted than the adults. In several fœtuses I have examined, there have been noticed several white spots on the tops of the shoulders, — two in front of, and two just behind, the first dorsal fin; also spots on the sides, which, becoming confluent, form a white band extending almost the whole length of the body.

In a male specimen, twenty-three inches in length, I could scarcely observe a spot upon its entire surface.

Description. Body elongated, cylindrical, with a slight ridge on the back, which is more perceptible between the dorsal fins. A distinct carina on each side of the abdomen, posterior to the second dorsal fin. The entire surface is rough. The head, which is flattened above, and tapers to a blunted snout, is equal to one seventh the entire length of the fish. The eyes are horizontally elongated; the pupils are small, black; the irides are silvery with a cupreous tint. The orbits are large, allowing great motion to the eyes. The distance between the eyes is equal to more than one half the length of the head. The temporal orifices are back of, and just above, the posterior angles of the

eyes; they are furnished anteriorly with a cartilaginous valve; their length is equal to the short diameter of the eyes. Between the eyes, are two longitudinal patches of numerous mucous pores, which are indistinctly continued nearly to the extremity of the snout. All the lower portion of the head, in front of the mouth, is covered with similar mucous orifices, which, like those just mentioned, exude, when pressed, a gelatinous secretion. The nostrils are double, and are situated nearer to the eye than to the snout; the outer orifice is circular, the inner transverse; they are situated transversely with regard to each other. The mouth is moderate, nearly circular when expanded. In the upper jaw, are three rows of teeth; in the lower jaw, are two rows; these teeth have very sharp edges, and their points are turned outwardly from the centre of the jaw. The tongue is large, rounded at its tip, and, like the whole interior of the mouth, is white. The branchial orifices, five in number, are situated directly in front of the pectoral fins; the posterior is rather the largest.

The lateral line, which is situated on the upper half of the side, pursues nearly a straight course to the extremity of the fleshy portion of the tail, from whence it passes obliquely upward to the outer edge of the fin.

The first dorsal fin arises on the anterior third of the body; it is convex before, emarginated above, and terminates posteriorly above in an acute angle. A strong triangular spine, almost black at its base and white at its tip in some specimens, nearly half the height of the fin, arises at its anterior base, and is concealed in nearly half its height by the fin.

The second dorsal fin of the same form as the first, but much smaller, is situated back of the first dorsal, at a distance from it equal to one fourth the whole length of the fish. A spine similar in its form and situation to that in the first dorsal, but nearly as high as the fin itself, is also here observed.

The pectorals are large, subtriangular, emarginated posteriorly; they commence at the last branchial orifice, their length is rather less than half their height. The ventrals are small and subtriangular; they are situated just anterior to the second dorsal, with the anus between them.

The caudal fin is very large and powerful; its upper portion is broad, and as long again as the lower.

Length, one to three feet; weight, eight to fifteen pounds.

Remarks. In the Spring and Autumn, this species makes its appearance in shoals in our bay; they are frequently met with in immense numbers. These shoals seldom remain in shallow water, or near the shore more than three or four days. They feed upon

mackerel and other fishes, and also upon the offal and garbage thrown upon the bottoms by the fishermen.

It is usually caught with the hook. On account of the sharpness of the teeth of this species, an ordinary line will not answer, as it would be severed at once; so that beneath the lead or sinker is suspended a piece of twisted line eight or nine inches in length, to which is attached, by a swivel, a firm leathern thong about twelve inches long on each side, supporting at each extremity a small chain about six or eight inches in length, each bearing a hook. Although it is not taken in quantities through the summer along the shore, yet so late as June 27, 1847, I noticed along the entire beach of Long Point, Provincetown, wherever the fishermen had cleared their nets of the Whiting they had caught the previous night, that more or less of this species also had been thrown away.

At their first appearance in May, they are quite abundant for about a fortnight at Chilmark, Martha's Vineyard, and the inhabitants take them in large quantities for their oil. During the spring of 1846, so numerous were they about Gay Head, that in half of a day, six hundred dog-fish were caught by the crew of a single boat by the hook. When this species comes into Massachusetts Bay in the early part of June, it tarries but for a few days; and as the fishermen at Provincetown are engaged in taking mackerel, they pay no attention to it at that time. But when they again appear in September, to remain until the middle of November, the fishermen being more at leisure, fit out their smacks for the sole purpose of capturing them for their livers. About one thousand livers furnish a barrel of oil, which is worth twelve dollars. When the livers are preserved, without being tried out, they are sold for about four dollars per barrel. After the fishery is over, the oil is boiled out of the livers and it is prepared for the market, where it will be worth from twenty-five to thirty cents per gallon; it is not very salable however, in cold weather, as it frequently becomes very hard when cold. The oil from this species is of an inferior quality, and is readily detected by its odor and lighter color; so that if a small quantity of dog-fish oil is mixed with shore oil it is condemned by the speculators. This shore oil is used by the tanners and curriers; it is prepared by putting the livers in barrels or butts in the sun; in a short time the water separates and sinks, and the oil is dipped out.

The fish itself on some parts of Cape Cod was formerly dried for fuel, and its skin was considerably used for polishing, by the mechanic.

These are some of the benefits derived from this species: but, upon the whole, those fishermen who catch mackerel in nets consider them very unwelcome visitors, as they not unfrequently swim near the surface of the water during the night and devour large

quantities of mackerel entangled in the nets, by biting them in pieces; they also become themselves entangled in the nets, and by their teeth and rough skin nearly destroy them.

Northerly, beyond the coast of Labrador, DEKAY. Massachusetts, STORER. Connecticut, AYRES. New York, DEKAY.

GENUS VI. SCYMNUS. Cuv.

All the fins small; two dorsal fins, the first but little before, and the second but little behind the line of the ventrals; no anal fin. Skin rough. Temporal orifices or spiracles large, placed rather high upon the head, above as well as behind the eyes. Teeth in the lower jaw crooked at the point, equilateral at the base; in the upper jaw lancet-shaped, but little curved; the points in both jaws diverging from the centre. Gill openings small.

SCYMNUS BREVIPINNA, *Dekay*.

The Nurse or Sleeper.

(PLATE XXXVIII. FIG. 2. *a.* Teeth of upper jaw. *b.* Teeth of lower jaw. *c.* Spine in skin.)

Somniosus brevipinna, Nurse or Sleeper, LESUEUR, Journ. Acad. Nat. Sciences, I. p. 222, pl.
" " " " STORER, Report, p. 189.
Scymnus brevipinna, Nurse, DEKAY, Report, p. 361. pl. 61. fig. 202.
" " " STORER, Synopsis, p.
Leiodon echinatum, WOOD, Proceed. Bost. Soc. Nat. Hist., II, p. 174.

Color. A purplish gray, with numerous white spots distributed over its surface.

Description. Body robust, subtriangular to the posterior line of the first dorsal fin, slightly convex in front of the dorsal fin, posterior to which it is cylindrical, and rapidly diminishes in its diameter. The entire length of the specimen before us is seven feet nine inches, measured from the tip of the snout to the extremity of the upper lobe of the tail. The depth of the body, across from the first dorsal fin, is eighteen inches; the depth at the anal, is five inches; the depth at the origin of the tail is three inches. The length of the head is fourteen inches; it is elongated, and terminates in a blunted snout, which is rounded above, somewhat flattened beneath, and six inches deep at its base. The eyes are circular, one and a half inches in diameter, situated seven inches posterior to the tip of the snout. The nostrils are large, situated beneath the base of the edge of the snout, four inches anterior to the eyes.

The mouth is of moderate size; the upper jaw is covered with five rows of small,

sharp, incurved, lancet-shaped teeth. The lower jaw has two rows of broad, quadrangular teeth, divided in their centres by a perpendicular ridge, and having their apices armed with a horizontally subtriangular cutting edge, directed, on each side of the centre of the jaw, toward the angle of the jaw. There are about twenty-six teeth on each side of the centre of the jaw. The temporal orifices are small; they are situated rather more than three inches posterior and superior to the eyes. The branchiæ, about two and a half inches in length, are eight inches back of the eyes. The entire surface of the body, including the fins, is thickly covered with minute conical recurved spines; these spines are grooved longitudinally, particularly upon their convex surfaces.

The lateral line is scarcely perceptible in the recent fish, but is readily traced on the dried specimen: it is a somewhat irregular black line, which, arising above the eyes, and passing along the whole length of the body, is lost upon the upper lobe of the caudal fin; from its under edge pass downward numerous lines of about one quarter of an inch in length, of the same color as the line itself, separated about a half inch from each other.

The first dorsal fin is subtriangular, — eight inches high from its base to its posterior tip; three inches long; the posterior portion of the fin is prolonged three inches beyond the base.

The second dorsal fin arises twenty inches back of the first; it is three inches high and eight inches long at its base; its posterior portion is elongated five inches beyond the base.

The pectorals are situated thirteen inches back of the angle of the jaws; their height is eleven inches, their length six inches, — they are rounded posteriorly.

The ventrals are subquadrangular, and are situated just in front of the second dorsal fin; their height is six inches, their length five inches, their posterior prolongation three inches in extent.

The caudal fin is emarginated; the height of the upper lobe is fifteen and a half inches; that of the lower lobe is twelve inches. The distance from the tip of the upper lobe to the lower edge of the lower lobe is twenty inches.

Length, eight to twenty feet.

Remarks. In the year 1818, Lesueur described and figured this species, from a stuffed specimen he saw at Marblehead. Never having been able to obtain a specimen, I was obliged to transcribe Lesueur's description into my "Report," published in 1839. Dekay followed my example in his "Report on the Fishes of New York," which appeared in 1842, in copying Lesueur; but not satisfied, with, to use his words, "an illy-constructed

genus," he referred the species to the genus Scymnus, which classification I accepted in my Synopsis.

A specimen of this species, sixteen feet in length, was taken on the coast of Maine, about eighty miles east of Portland, in August, 1846. After being skinned and stuffed, it was seen and described by William Wood, M. D., of Portland. He supposed it to be new, and called it *Leiodon echinatum.* His description appeared in the second volume of the "Proceedings of the Boston Society of Natural History." In the month of January, 1848, Capt. N. E. Atwood brought me, from Provincetown, a specimen he had taken the day previous while fishing for cod. I at once described it and had it figured, supposing it to be a new species. The accompanying description and figure give its characters while recent. When, however, it had been stuffed and dried, it proved to be Lesueur's species; its aspect being materially changed by the process of being skinned and preserved. Another specimen was caught at Nahant, in November, 1848. It was drawn upon the beach where it remained alive during the night. At its death it was brought to the city for exhibition. A third was harpooned at Provincetown in April, 1849, at Long Point, fifteen feet long; and still a fourth was taken on the 24th of April, the same year, at Provincetown, near the Long Point light-house. These are the only instances with which I am acquainted of its capture. I have learned from conversation with an intelligent fisherman, however, that individuals are captured every winter, and that it is more numerous than is generally supposed. Sometimes it is very large — measuring twenty feet in length, and weighing two tons or more, on these the cutaneous spines attain a great size. In the vicinity of Provincetown, its most common resort is near Race Point, in a gully famous for halibut and star-fish. The liver furnishes five or six gallons of oil — in one case, a single half lobe filled a flour barrel, and yielded fifteen gallons of oil. It is called by the fishermen *gurry* or *ground shark*, from its feeding on the offal which is thrown overboard from the smacks. It is sometimes attracted, like other species of sharks, by the carcasses of whales killed in Massachusetts Bay.

There is a description of a species of Scymnus, accompanied by a figure by Valenciennes in the "Nouvelles Annalles du Museum," tom. I, 1832, which he calls micropterus. The fish was taken near the mouth of the Seine. He considered it distinct from the dried specimen of Lesueur. There is a very strong resemblance, however, between the descriptions of the recent fish.

Massachusetts, Lesueur, Storer.

GENUS VII. ZYGÆNA. Cuv.

Head depressed, more or less truncated in front; the sides extend horizontally to a considerable length, with the eyes at the external lateral extremity. Teeth of the same shape in the upper and lower jaw, namely, the points directed toward the corner of the mouth, with a smooth edge when young, but distinctly serrated in adult specimens. Branchial openings, five. Two dorsal fins, — the first in a line close behind the pectorals; the second, over the anal fin.

Zygæna Malleus, *Val.*

The Hammer-headed Shark.

(Plate XXXVIII. Fig. 3.) *a.* Head beneath.)

Squalus Zygæna, Hammer-headed Shark. Mitch. Trans. Lit. and Phil. Soc. of N. Y., I. p. 284.
Zygæna Malleus, Val. Mem. du Mus. D'Hist. Nat. v. 9. p. 288., pl. 2, fig. a, 1, 6.
" " *The Hammer-headed Shark,* Storer, Bost. Journ. Nat. Hist. IV. p. 185.
" " Val., Synopsis, p. 256.
" " Dekay, Report, p. 362. pl. 62, fig. 204.

Color. The upper part of body a dark grayish brown, lighter on sides; beneath, white; posterior inferior parts of head, bluish; anterior margin of head tinged with white.

Description. Head somewhat rounded anteriorly, semicircular directly in front, with a smaller curve on each side of this, widely expanded and much compressed at sides. The external margin of the sides of the head rounded, having the eyes situated at their anterior extremity; the anterior angle in front of the eyes is very prominent. The width of the head is equal to twice its length, and is also about the length of the upper lobe of the caudal fin, and one fourth the length of the fish. The expanded sides of the head are two thirds the length of the head. The nostrils are situated beneath and front of this angle, in the extremity of an emargination that extends along the smaller curves previously mentioned. The posterior portion of the head is bordered by a stout, fleshy, and concave membrane. The eyes are large and prominent. The mouth is situated beneath; its posterior angles on a line with the posterior edges of the head. Several rows of sharp teeth are seen in each jaw, — their points are directed toward the sides, and have a prolonged base. Branchial apertures, five. The under surface of the head abounds in mucous pores, disposed in patches, the largest of which is of a triangular form, and directly in front.

The lateral line, which is quite indistinct, commencing on the side of the occiput, passes obliquely backward to a line above the third branchial orifice, and then assuming a straight line, runs the whole length of the body, and is lost upon the posterior extremity of the caudal fin.

The first dorsal fin is somewhat triangular, rounded anteriorly, with its posterior base elongated and free. It is situated in the anterior half of the body.

The second dorsal fin arises just anterior to the caudal fin. It is quadrangular, quite small, having its anterior margin slightly rounded, and its posterior extremity prolonged into a filament.

The pectorals arise at the base of the fourth branchial orifice, and are rounded anteriorly.

The ventrals, of a quadrangular form, commence just back of the middle of the body.

The anal fin commences on a line just in front of the second dorsal, resembling somewhat that fin in shape, but more deeply emarginated posteriorly.

The upper lobe of the caudal fin is very long, and curved at its extremity, its lower portion, a thin membrane, ends posteriorly in a small triangular expansion. The lower lobe is much smaller than the upper, and triangular.

Length, two to twelve feet.

Remarks. Although some slight discrepancies might be pointed out between our species and that described by Valenciennes, I have but little doubt that they are identical, and as such classify them. This species is exceedingly rare in our waters. In my "Report," published in 1839, I observed that "Dr. Yale had informed me that a species of Zygæna was found at Holmes Hole." In October, 1841, I had an opportunity to see a specimen which had been brought to this city by Winslow Lewis, Esq., from Chatham, Cape Cod, at which place it had been taken with a second specimen in a net. This individual I described in the second number of the fourth volume of the Boston Journal of Natural History, for September, 1842. In 1851, I received a specimen from Provincetown, from Mr. Jonathan E. Smith, taken accidentally in a net. Capt. Atwood also saw some half of a dozen of this species taken that season, one being seven feet in length. Previous to these, he had never seen but two or three, and those very small, in Massachusetts Bay. He had, however, seen them south of Cape Cod in some abundance. In September, 1857, I received a specimen from Capt. Atwood, taken at Provincetown, which is above described.

Massachusetts, STORER, New York, MITCHILL, DEKAY. Caribbean Sea, BANCROFT. "From Nantucket to Brazil," DEKAY.

FAMILY XXIX. RAIIDÆ.

Body very much flattened out, resembling a disk. Pectorals very large, uniting in front with the snout, and extending backward to near the base of the ventrals. Tail, more or less long and slender. Mouth, nostrils, and branchial openings, beneath. Eyes and temporal orifices, above. Dorsals (when present), almost always on the tail.

GENUS I. RAIA. Lin.

Disk rhomboidal. Tail slender; with two small dorsals near the tip, and sometimes the vestige of a caudal fin. Teeth slender, close set, arranged in quincunx.

Raia diaphanes, *Mitchill.*

The Clear-nosed Ray.

(Plate XXXIX. Fig. 1.)

Raia diaphanes, Clear-nosed Ray, Mitch., Trans. Lit. and Phil. Soc. N. Y. i. p. 478.
Raia ocellata, The Ocellated Ray, Mtch., Storer, Report, p. 191.
Raia diaphanes, Clear-nosed Ray, Dekay, Rep. p. 366, pl. 77, fig. 215.
" " " Linsley, Cat. of Fishes of Connecticut.
" " " Storer, Synopsis, p. 510.

Color. The body above is of a light brown color, thickly sprinkled over its entire surface with more or less circular black spots or blotches, varying in their size from one half of a line to two lines in diameter; beneath, white. Pupils black, irides golden and stellated.

Description. In this species the pectorals are rounded; in front of them is a concavity on the sides of the head, which is preceded by a slight convexity of the margin. Snout obliquely projecting, blunted at extremity, with an emargination on each side. The length of the head is equal to about one-seventh the length of the body; its width across the pectorals more than half the length of the body; its width directly back of the eyes across occiput, as long again as the length of the head; the distance between the eyes is equal to one third the length of the head. The eyes are prominent, horizontally oval. The temporal orifices are situated obliquely, directly back of the eyes, and shorter than they. The length of the mouth is rather more than one third the length of the head. An aperture extends from the exterior angle of the mouth to the nostrils, which are situated directly in front of the mouth, large, and protected by fleshy elongations. The

branchial openings are situated at equal distances from each other. In front of, and at the sides of the mouth, and at the anterior portion of the pectoral fins, are seen a large number of minute black points which are mucous pores. The space between the anterior orbitar ridge and the snout is naked and diaphanous. Minute sharp spines upon the snout, from which extend a series of spines on each side to the anterior orbitar ridge; numerous exceedingly minute spines occupy the space between the eyes. From the anterior edge of the emargination on the sides of the snout, along the edge to the posterior portion of the head, are situated several rows of prominent recurved spines. Two rows of very prominent sharp spines, about a dozen in a row, are seen toward the outer portion of the pectoral fins. Two rows of spines on each side of the back of the tail; those at the posterior extremity are the largest, and between the two central rows is a naked groove. The remainder of the upper part of the surface of the body is destitute of spines. The anterior portion of the pectorals is reddish at the edge; posterior portion bordered with white rays very numerous and easily distinguished. The ventrals are quite large, containing about twenty-four rays; those next to the pectorals are very strong and lobed; these fins resemble very much the posterior wings of some of the Phalænæ.

Two subtriangular dorsal fins, of nearly equal size, and rough upon their surface, are situated a short distance in from the extremity of the tail; they are united to the tail by a delicate transparent membrane.

The tail is more than half the length of the entire fish, and is bordered by a membranous expansion upon its inferior edge.

The sexes are readily distinguishable by the ventral fins. From the ventrals in the male, extends a cylindrical appendage about half the length of the tail, measuring from the anal orifice, which is called the clasper; at its posterior outer portion it is fissured, and contains on its lower division a large falciform bony hook; and on the upper, a small projecting tooth, somewhat like a shark's tooth; the ventrals of the female are destitute of these appendages. Besides this sexual difference, in the female there are fewer spines upon the surface of the body generally, and particularly upon the fleshy portions of the pectorals. The male has fewer spots; his teeth are less prominent and less sharp than those of the female; the anal orifice of the male is circular, in the female it is a simple incision.

Length, two to three feet.

Remarks. This species, which grows to the length of three feet, is found along our whole coast; it is frequently taken while fishing for other species, and is found along the

beaches, swimming in shallow water. I have seen it at Provincetown, in great numbers, thrown upon the shores. It is so common at Nahant, that Mr. Jonathan Johnson, a fisherman of that place, informs me that it is not an unusual circumstance when a large number of hooks attached to the same line are baited, for a skate to be captured by each of them.

The specimen from which the above description and accompanying drawing were made, was taken with a hook from one of the bridges over Charles River.

I have never known them to be used as an article of food by our people, although Dekay says they are eaten in New York, by the poorer classes.

Massachusetts, STORER. Connecticut, LINSLEY. New York, MITCHILL, DEKAY.

RAIA LÆVIS. *Mitch.*

The Smooth Skate.

(PLATE XXXIX. FIG. 2.)

Raia lævis, Smooth-backed Skate, MITCH., Amer. Month. May 11, p. 327.
" *batis, Skate,* STORER, Report, p. 193.
" *lævis, Smooth Skate,* DEKAY, Report, p. 370.
" *batis, Skate,* LINSLEY, Cat. of Fishes of Connecticut.
" *lævis,* STORER, Synopsis, p. 511.

Color. Above, of a light ash color, with a few indistinct symmetrically arranged yellowish ocelli; one in front of, and exterior to each eye; two posterior and exterior to each eye; two at the posterior base of each pectoral fin, and one on each ventral fin. All the under portion of the body is of a dingy white color. Pupils black; irides silvery, with a beautiful golden fringed curtain suspended from above.

Description. Rhomboidal. The entire length of the head is nearly equal to one fourth the length of the fish; the head is much compressed, with a furrow between the eyes, which extends to the posterior portion of the snout. Snout slightly blunted. The eyes are oblong and moderate in size. The distance between the eyes is less than one fourth the length of the head. The gape of the mouth is large. The jaws are composed of compact, hexædral teeth, forming almost a plane surface, the inner angle of the innermost middle ones, beginning to become acute. The nostrils are a short distance in front of the mouth. Branchial apertures situated obliquely, the anterior the longest. The greater part of the body is smooth above. A strong spine, naked at its tip, is situated at the anterior angle of each eye, and a smaller one exists at the posterior angle; back of the latter, is a strong spine at the posterior inner edge of the temporal orifices. A series of very minute spines along the inner edge of the orbit. The top of the snout is

covered with small, sharp, flexible spines; their extremities are naked; similar spines are continued along the edge of the head to the base of the pectoral fins; upon the anterior edge of the pectorals are numerous very minute spinules. On the upper portion of the pectorals, toward the lateral angle, are four or five longitudinal rows of very sharp, incurved, erectile spines, — some of the rows containing a dozen or more spines. A few short spines are distributed upon the posterior inferior base of the pectorals, — and some very minute asperities may be felt upon the ventrals. A row of prominent strong spines,— ten, twelve, or more in number,—commence just back of the eyes, and extend along the dorsum to the first dorsal fin. In the intervals between these, are numerous minute spinules; on each side of the central row is a perfectly regular series of small spines extending from the shoulder to the first dorsal fin. Two spines are situated upon each shoulder, — the posterior of which is the longer. On each side of the dorsum, passing from the shoulders to the posterior portion of the body, and also exterior to each shoulder, are seen lines resembling the lateral line, passing down to, and bifurcating upon the pectorals, from which, small mucous ducts are distributed at regular intervals. Body beneath perfectly smooth, with the exception of a very few spines scarcely perceptible, unless by the touch, on each side of the commencement of the caudal fin, and a small patch of equally minute spinules upon the middle of the tail, just in front of the termination of the ventral fins. Mucous pores are scattered over the greater portion of the under surface, appearing like black dots, in most instances distributed in a regular manner, although a longitudinal line of these is seen toward the middle of the pectorals, and another transverse one at the base of the ventrals.

The dorsal fins are of equal length; the first, a little the higher and connected at its base to the tail by a membranous prolongation; both dorsals rounded above. These fins are separated from each other by a short interval. The posterior terminates near the extremity of the tail.

Length, two to five feet.

Remarks. This species is common in Massachusetts Bay.

Massachusetts, Storer. Connecticut, Linsley. New York, Mitchill.

GENUS II. PASTINACA. Cuv.

Tail slender, without fins; but armed with one or more long spines, which are dentated on the edges.

Pastinaca hastata, *Dekay*.

The Whip Sting-Ray.

(Plate XXXIX. Fig. 3.)

Raia centroura, Prickly-tailed Sting-ray, Mitchill, Trans. Lit. and Phil. Soc. N. Y. i. p. 479.
Pastinaca hastata, Whip Sting-ray, Dekay, Report, p. 373. p. 65, fig. 214.
Trygon centroura, " " " Linsley, Cat. of Fishes of Connecticut.
" *hastata,* Dekay, Storer, Synopsis, p. 261.

Color. Body, pectorals, and ventrals, light brown above, whitish beneath; tail, dark brown throughout.

Description. Body ovoid, its lateral margins blending imperceptibly with the pectorals, which, continued anteriorly, form with the snout an almost unbroken curve. Greatest width of body, about equal to its length exclusive of the tail, and about five times the greatest depth.

Eyes oval, of moderate size, situated obliquely, the greatest divergence posteriorly; distance between them being about three quarters that to tip of snout. The temporal orifices are large, just behind the eyes, the anterior edge coming to a line with their middle point; situated obliquely, in opposite direction to that of the eyes. Nostrils small, their alæ, large and projecting. Mouth moderate, transverse, curved anteriorly; teeth sessile and smooth.

Surface of body without spines or projections, save a single row of large, irregular, horny tubercles posteriorly. These commence on the median line of the back, at its last quarter, and extend upon the tail. This organ, which is studded throughout with similar projections, is long, flexible and tapering. At the end of its anterior fourth there arise in succession, a pair of strong, elongated and pointed spines, flattened above and beneath and serrated upon their lateral edges. These spines are each received into a smooth longitudinal groove when at rest. Their number undoubtedly varies. Dekay speaks of three being present, — and in the specimen from which his description is drawn, but one is present, although the stump of a second, the anterior one and its distinctive groove, afford evidence of its former existence.

Anus longitudinal, near the origin of the tail.

The greatest depth of pectorals on a line with the temporal orifices.

The ventrals are fan-shaped, emarginated posteriorly. The claspers are rather more than one half the length of the ventrals.

Length of specimen here described, a male, from the snout to the commencement of the tail three feet and a half; from the snout to the end of the tail, nine feet.

Length, five to nine feet.

Remarks. Previous to the publication of my "Report on the Fishes of Massachusetts," I was aware that a species of sting-ray inhabited our waters. Thus, Dr. Yale, then of Holmes Hole, wrote me: "I have seen frequently in this harbor and have assisted in taking them; but, owing to their poisonous nature when wounded by their sting, we have been rather cautious about taking them into the boats, so that we seldom see one on shore. One or two individuals in this vicinity have come well nigh losing their lives by a wound from them. In July and August they are abundant on the flats in the harbor here." When captured they are taken by the harpoon. In October, 1840, I received from Dr. Yale the head and tail of a species, which I supposed to be Mitchill's Raia centroura, — and afterwards another tail of this species was sent me from Holmes Hole. In September, 1857, my friend E. W. Carpenter, M. D., of Chatham, Cape Cod, sent me the fine specimen from that place which has enabled me to present the above description and accompanying figure.

Massachusetts, Storer. Connecticut, Linsley. New York, Mitchill.

GENUS III. MYLIOBATIS. Dum.

Head projecting from the pectorals, and the latter more broad transversely than in the other Rays, which gives them somewhat the appearance of a bird of prey with the wings extended, and has caused them to be compared to the eagle. Their jaws are furnished with broad, flat teeth, arranged like the squares of a pavement, and of different proportions, according to the species; their tail, extremely long, is terminated in a point, and is armed like that of a trygon, with a strong spine, serrated on both sides, and has above, toward its base, in front of the spine, a small dorsal. Sometimes there are two or more spines.

Myliobatis acuta, *Ayres.*

The Smooth-tailed Sting-Ray.

(Plate XXXIX. Fig. 4.)

Myliobatis bispinosus, Storer, Proceed. Bost. Soc. Nat. Hist.
" " " Bost. Journ. Nat. Hist. iv. p. 187.
" " " Ayres, Bost. Journ. Nat. Hist. iv. p. 290, pl. 13, fig. 1.
Myliobatis acuta, Linsley, Cat. of Fishes of Connecticut, Silliman, Journ.
" " Storer, Synopsis.

Color. The whole body and head above, reddish brown; tail lighter at the base, but nearly black toward the tip; beneath, whitish.

Description. Body above, smooth, entirely destitute of spines, even on the dorsal ridge. Entire length, three feet eleven inches; length, exclusive of the tail, one foot six inches; breadth across the pectorals, two feet five inches. Distance between the eyes, four and a quarter inches; eyes vertical, elliptical, greatest diameter one inch, least diameter three quarters of an inch. On the summit of the orbit of each eye is a hard, blunt, vertical projection, about one eighth of an inch in height, nearly white at the extremity. The head is rounded anteriorly, and extends backward, widening but little, four and a half inches, until opposite the eyes, where it joins the body. Mouth two inches in breadth, situated four inches posterior to the snout; jaws lined with numerous blunt, tesselated teeth. Nostrils about an inch and a half anterior to the mouth, each provided with a valve, and having a depression or channel leading back almost to the corner of the mouth. Branchial apertures five on each side; distance between the anterior pair, four inches. Spiracles situated behind the eyes, elliptical, one and a half inches in length.

On the tail are two reversely serrated spines, one situated directly above the other, of which the upper is the shorter; their lengths are two and three fourths and three and three fourths inches. Their insertion is at about five and a half inches from the origin of the tail; in color they are dingy white. Immediately before them is a small dorsal fin, one and a half inches in length, and one inch in height. Tail very slender, smooth to the tip, the inferior surface presenting no vestige whatever of fins. Anus beneath the origin of the tail; immediately posterior to it are two cylindrical, or slightly conic appendages, three and three fourth inches in length. As the specimen here described is the only one which has fallen under my notice, I am of course unable to determine whether all the characters which have been stated will prove to be constant. With respect to one, the relative length of the two spines upon the tail, variation may probably be expected. The upper spine will, in some instances, doubtless be the longer of the two. The specimen figured was apparently a male.

Length, three to four feet.

Remarks. In December, 1841, I presented to the Boston Natural History Society, the tail and portion of the jaw of a species of myliobatis which I had just received from Dr. Yale of Holmes Hole, — and from two spines which were situated upon the tail, I proposed the specific name of bispinosus. Mr. William O. Ayres, then of Hartford, Connecticut, afterwards found an entire specimen at Brookhaven, Long Island, and called it myliobatis acuta; this name, however, he withheld, and in his description of this species, prefixed the name I had indicated, — at the same time remarking, "It is a very clearly marked species, and as Dr. Storer was obliged to draw up his account from imperfect

materials, I have prepared a description and drawing." Yarrell in his generic characters of the genus myliobatis, which I had examined, speaks only of "a serrated spine" upon the tail; whereas, Dumeril, in his formation of the genus, says, "sometimes there are two or more spines." My specific name, heretofore, was evidently untenable,— and as Ayres published the first accurate description, his name *myliobatis acuta* should undoubtedly be acknowledged. I have never seen a perfect specimen, and have therefore given Ayres' description and figure.

Massachusetts, STORER. Connecticut, AYRES.

GENUS IV. TORPEDO. DUM.

The disk of the body nearly circular; pectoral fins large; two dorsal fins placed so far back as to be on the tail; surface of the body smooth; tail short and rather thick; teeth small and sharp.

TORPEDO OCCIDENTALIS, *Storer*.

The Cramp-Fish.

(PLATE XXXIX. FIG. 5.)

Torpedo occidentalis, STORER, Amer. Journ. of Arts and Sciences, 45, p. 165, pl. 3.
" " " Synopsis, p. 516.

Color. The whole upper surface of this species is of a dark brown, with a few almost black spots distributed over it; beneath, white.

Description. The entire length of the specimen before me, which is a female, is four feet and two inches, and its greatest breadth is three feet. The globe of the eye, which is circular, is an inch and a quarter in diameter; the cornea is oval; its longest diameter is one half of an inch, and is directed obliquely outward; its shortest diameter is three eighths of an inch.

The spiracles are oval and smooth at their edge; they are one and a quarter inches in their largest diameter, and one inch in their shortest diameter, and are directed outward and a little forward. On the anterior and inner surface of the spiracles, just within the orifice, is a plaited membrane, the folds of which resemble somewhat the nasal septa; the longest of these folds are next to the median line, and they gradually diminish in length as they recede from it. The mouth, when closed, measures six inches across from the angles, and when opened to its widest extent, measures from the middle of the upper to the middle of the lower jaw, five inches. The teeth are numerous, small, and sharp,—broad at their bases, and pointed at their extremities, like spines.

When the fish is placed upon its under side, and the anterior extremity of the disk is turned backward, the nostrils are observed about three inches beneath its edge; they are covered above by a membranous prolongation, formed by a fold of the skin which arises from their exterior angle and is continued to the median line; the free edge of this fold is five eighths of an inch wide at its greatest width. A second fold commences at their outer upper angle, and passes downward and inward to the middle of the lower edge of the aperture. A third fold commences near the middle of the second, and is directed outward and a little downward. The nasal cavity is divided by a horizontal plate into two portions, and at right angles to this proceed numerous small septa going to the upper and lower margin of the nostrils.

The first dorsal fin, which is three inches and a quarter long and five inches high, is situated at the posterior portion of the pectorals, one half of its base being posterior to those fins.

The second dorsal fin is two inches long, and two inches and three quarters high; it is two and a half inches back of the first dorsal, and three inches anterior to the commencement of the upper lobe of the caudal fin.

The greatest length of the pectorals is two feet, and their greatest breadth is fifteen inches.

The ventrals are ten inches long, and five and a half wide. The anus is large, and is situated beneath the middle of the ventrals.

The caudal fin is nearly triangular; its lower portion is the larger; the depth of this fin, at its posterior extremity when expanded, is eleven inches; its posterior margin is straight.

Length, two to five feet.

Remarks. In the January number of the "American Journal of Science and Arts" for 1843, I made a slight reference to a species of torpedo, which had been taken a few weeks previously upon the coast of Massachusetts. The description of a species captured on the coast of Ireland, published by William Thompson, Esq., Vice President of the Belfast Natural History Society, in the "Annals of Natural History," answered so well to my specimen, that I was led to suppose it must be the *nobiliana*, Buanaparte. When, however, I carefully compared with mine, the description and figure of the foreign species, contained in the second edition of Yarrell's British Fishes, I found no slight differences in the form of the disk of the body, in the size of the pectoral and caudal fins, and in the situation and form of the temporal orifices in the two specimens; and at once suspected the American fish must be an undescribed species. As Yarrell's figure was en-

graved from a dried specimen, and consequently might not perfectly represent the form of this fish, I wrote to Mr. Yarrell, stating to him my doubts of the identity of the two fishes and presenting him with my figure. His opinion coincided perfectly with mine. I have, therefore, the pleasure to present a description of a torpedo hitherto unknown to science; and as no other species of this genus is known to exist on the shores of our hemisphere, I shall call it *Torpedo occidentalis*." The above remarks, I have copied from my communication upon this subject, in the October number of the American Journal of Science and Arts, for 1843. A portion of the following observations also will be noticed to have been transferred from the same paper.

Dr. Mitchill introduced the *Raia torpedo* into his "Fishes of New York," published in 1815, upon the authority of several fishermen with whom he had conversed, who had been electrified by a species of ray, when they were detaching it from the hook with which it was taken. He had never seen a specimen, but had no doubt of its being the *common* torpedo, and consequently catalogued it as such. Since the appearance of Dr. Mitchill's paper, I can find no further notice of the existence of an electrical ray in our waters. In my "Report on the Ichthyology of Massachusetts," published in 1839, I cited the testimony of several observers to prove that an electrical fish, known as the *cramp-fish*, was occasionally taken on the shore of Cape Cod, but had never been seen by a naturalist. During the month of November, 1842, a specimen of this long-looked-for species was captured at Wellfleet by Mr. Seth N. Covell; I fortunately obtained it, and from it prepared the above description. In Massachusetts Bay, this species appears to have been met with only on the eastern shore of Cape Cod, between Provincetown harbor and Orleans, an extent of about thirty miles; and is found in greater numbers upon the eastern shore of Long Point, a narrow neck of land west of the town of Provincetown, than at any other place. In these localities, it is observed only in the months of September, October, and November. The greater number of those taken run ashore upon the sandy beaches. Captain Atwood informs me he has known three individuals to be taken with the hook, by persons fishing for other species; and that others, being discovered in the day-time near the shore, are harpooned and dragged on shore. In the year 1819, and for four or five years afterwards, this species was unusually common at Provincetown — from sixty to eighty being seen in a year; since that time they have been comparatively scarce, and for the ten years preceding 1845, not more than thirty were found; in that year, 1845, a dozen were noticed. While on a visit at Gay Head, in August, 1846, I was informed by Captain Leonard West, of Chilmark, and Mr. Samuel Flanders, keeper of the light-house of Gay Head, that in Chilmark, three miles from Gay Head, they had known

at least fifteen or twenty of this species to be taken by hook and line, and also in seines, for several years in succession, in the spring of the year. Although I had supposed it was a southern species, this was the first positive information I had received of its being taken south of Cape Cod. During the latter part of October, 1845, Captain Atwood brought from Provincetown to Boston a living specimen, weighing about sixty pounds. It was found near the shore apparently benumbed, and was readily dragged ashore by attaching a rope to its tail. In company with my friend, J. B. S. Jackson, M. D., I visited it while it was still alive in the well of the smack in which it had been brought. Upon its being lifted upon the wet deck, it gave a distinct shock. This shock was scarcely perceptible while the fish was quiet, and was most marked when the portion directly over the electrical organs was excited. The most powerful effect was produced by seizing the tail with one hand, and grasping that portion of the pectoral fins which is supplied with nerves from the fifth pair; here, quite a shock was perceived in the arms as high as the elbows. In some cases, the shock produced by this fish, when in the water, is irresistible. The following anecdote I copy from my Report: "Mr. Newcomb, senior, the oldest fisherman in Boston market, stated to me, that his father, who resided at Wellfleet, had a dog which frequently waded into the shallow water of the coves and brought out flounders which he had seized with his mouth. In one of his fishing excursions he attacked a torpedo, which perfectly convulsed him; he dropped the fish, and ran away howling most piteously, and could never be persuaded to resume his fishing." Captain Atwood informs me that he has received a great many very powerful shocks which have thrown him upon the ground as quick as if he had been knocked down with an axe. He has also received many shocks by taking hold of the pole of an harpoon, when he was at the distance of eight or ten feet from the fish; and he has also felt its effects when holding the rope attached to the harpoon; but in this, and in removing the liver from the fish when it is nearly dead, there is generally nothing more than a numbness felt in the fingers, and they seem inclined to straighten; so that he has known it to be difficult to grasp the handle of the knife while cutting the fish.

The smallest individuals do not exceed twenty pounds in weight, while Capt. A. thinks the largest may weigh from one hundred and seventy to two hundred pounds. The largest circumference of any of them, is about twelve feet. They are taken for their oil. The livers of the largest specimens yield about three gallons of oil; those of the smallest ones, a pint; the ordinary sized livers furnish from one to two gallons. Many of the fishermen have an opinion that this oil is serviceable in cases of cramp when exteriorly applied, and relieves cramp in the stomach when internally administered.

But it is principally valued as lamp-oil; for this purpose it is superior to the oil obtained from any other fish, and is equal to purified sperm oil.

Massachusetts, Storer.

ORDER III. CYCLOSTOMI.

Gills purse-shaped, fixed, opening outwards by several apertures. Jaws represented by an immovable cartilaginous ring, formed by the union of the palatine and mandibular bones. Intestinal canal straight and narrow.

FAMILY XXX. PETROMYZONIDÆ.

Body elongated, cylindrical, eel-shaped. No pectorals nor ventrals. Fins without rays.

GENUS I. PETROMYZON. Lin.

Seven branchial apertures on each side of the neck. Maxillary ring armed with strong teeth. Mouth beneath.

Petromyzon americanus. *Lesueur.*

The Great Lamprey.

(Plate XXXVIII. Fig. 4.)

Petromyzon marinus, Great Lamprey, Mitch., Trans. Lit. and Phil. Soc. N. Y. i. p. 461.
Petromyzon americanus, Lesueur, Trans. Amer. Phil. Soc., new series, i. p. 382.
" " *American lamprey,* Storer, Report, p. 195.
" " *American sea lamprey,* Dekay, Report, p. 379, pl. 66, fig. 216.
" " Storer, Synopsis, p. 265.

Color. Above, olive-brown, mottled with dark brown, almost black, confluent patches; beneath, of a uniform dull brown. Pupils black, irides golden.

Description. The anterior portion of the body is cylindrical; the posterior compressed. A slight carina is observed upon the back. Head rounded, somewhat flattened on the upper portion in front of the eyes. Snout obtuse. Eyes of moderate size. The distance of the eyes from the snout is less than one twelfth the length of the entire fish. A tubular orifice, a line in its longest diameter, is seen in front of, between the eyes. Posterior to each eye, are seven large branchial apertures, separated about a quarter of

an inch from each other, passing backward in nearly a straight line. When unattached the mouth of this fish is a longitudinal fissure; when attached it is circular, the lip forming a ring, within which are situated numerous hard, horny teeth of a yellow color. The anterior and the inner row of the lateral teeth are the larger; the posterior teeth are more numerous, and smaller. Mucous pores are seen upon the upper portion of the head.

The first dorsal fin arises posterior to the middle of the body; its height is less than one sixth its length.

The second dorsal fin is situated about an inch posterior to the first dorsal; its greatest height is equal to about one sixth its length. This fin is continued to, and united with, the caudal fin.

The anal fin is a mere fringe.

The caudal fin seems, like the extremity of the solid portion of the body, very much compressed, or is an expansion of the dorsal and anal fins.

Length, two to three feet.

Remarks. This species is occasionally taken in Massachusetts Bay attached to pieces of drift-wood and the bottoms of boats and larger vessels. In its spawning season, it ascends the mouths of rivers. In the Merrimac River, at Lowell, it is taken in large quantities. I am indebted to the late Elisha Bartlett, M. D., for the following interesting facts. He wrote me, that "they ascend the rivers a little earlier than the shad, and move mostly in the night. It is not known by the fishermen when they return, as they are never seen. There is a notion that they all die. They are often seen in the Summer in pairs at work together, constructing a little mound of stones. They build this about three feet in diameter at the base, and about two feet high, of stones from the size of an ounce bullet to that of the fish. They often aid each other in carrying the same stone. This is pretty evidently a *labor of love*, as they copulate once in five minutes, or so, during the whole time. The young go down the river when the water begins to freeze. They are from six to eight inches long."

Massachusetts, STORER. Connecticut, LINSLEY. New York, MITCHILL, DEKAY.

PETROMYZON NIGRICANS. *Lesueur.*

The Bluish Lamprey.

(PLATE XXXIX. FIG. 6. *a.* Mouth.)

Petromyzon nigricans, LESUEUR, Trans. Amer. Phil. Soc., new series, vol. I. p. 385.
" " *The bluish lamprey*, STORER, Report, p. 197.
" " *The bluish sea lamprey*, DEKAY, Report, p. 381, pl. 79, fig. 247.
" " " " " " LINSLEY, Cat. of Fishes of Connecticut.
" " " " " " STORER, Synopsis, p. 517, 265.

Color. The upper part of the body is of a deep bluish green color; beneath, bluish white. Pupils black; irides silvery. On the top of the head, between the eyes, a small white spot exists.

Description. The body is cylindrical anteriorly, compressed posteriorly, very much so at the tail, which terminates in a point. The head is oval, flattened on the top; the length of the head, measured from the snout to the posterior angle of the eye, is less than one eighth the entire length of the fish. The branchial orifices, of equal size, are situated obliquely back of the eyes. The eyes are of moderate size. The mouth is circular; its diameter is equal to two thirds the length of the head and surrounded by a fleshy margin; it is armed within by numerous incurved teeth or horny spines, projecting from widened bases resembling the spines with which the Raiæ are armed; these are much larger on the anterior portion of the disk, and quite small upon the posterior portion.

There are three teeth in the throat; two higher up than the third, which is in front of and between the others. Posterior to these, is a semicircular bony ridge similar to the jaws of the Orthagoriscus.

The first dorsal fin is situated on the posterior half of the fish; it is rounded posteriorly. The distance between the dorsals is equal to half the length of the first dorsal. The anterior portion of the second dorsal fin is considerably higher than the first dorsal. This fin is as long again as the first dorsal; it gradually diminishes in height toward the caudal fin, to which it is attached, and forms with it a continuous fin.

The caudal fin is a simple membrane, triangular at its termination, and uniting with the anal fin which is very small.

Length, three to seven inches.

Remarks. This species is found attached to other species of fishes. It is not unfrequently affixed to mackerel; less often to cod; and still less frequently to haddock.

Massachusetts, LESUEUR, STORER. Connecticut, LINSLEY.

After this memoir on the Fishes of Massachusetts was completed, aware that several species had been described during its preparation, which I had not included in my communication, I requested Mr. Frederick W. Putnam, Curator of Ichthyology in the Boston Society of Natural History, and also in the Essex Institute, to furnish me with a list of these. He has placed me under great obligations by sending me the accompanying catalogue.

1. *Grystes fasciatus*, Ag. (Black Bass of the Lakes.)

This species, which is the common lake bass and black bass of the Great Lakes, Lake Champlain, and several lakes in New York, and which also extends farther south, has been introduced into Great Sandy Lake, in Wareham. In the Summer of 1862, a specimen of this fish was caught in Massachusetts Bay, by one of the members of the state legislature, and is now in the state cabinet. The fish had evidently found the salt water not much to its liking, as it was much emaciated, and had changed so in its general appearance as at first sight hardly to be recognized.

2. *Priacanthus altus*, Gill, Procd. Philad. Ac. Nat. Sci., 1863, p. 332.

A specimen of this most beautiful little fish was found alive on Marblehead Beach by Miss Mary Nichols, of Salem, and is now in the Essex Institute. Only three other specimens are known of this species.

3. *Bryttus obesus*, Girard, Procd. Philad. Ac. Nat. Sci. Syn. *Pomotis obesus*, Gir., Procd. B. S. N. H.

Girard's specimens were from Framingham. I have found it quite common in several ponds in Essex County, and also in Fresh Pond (Cambridge), and in a pond in Malden. It is probably a widely distributed species, and may prove to have been described by some of the earlier authors. (See Gill in Procd. Philad. Ac.)

4. *Cryptacanthodes inornatus*, Gill, Procd. Philad. Ac. Nat. Sci., 1863, p. 332.

Have seen a specimen of this pure white Cryptacanthodes taken off Swampscott.

5. *Gasterosteus Wheatlandi*, Putn., Procd. E. I., v. p. 4, 1866.

Specimens of this species were taken at Nahant, on April 15, 1859, by the late Dr. R. H. Wheatland. The species is very strongly characterized.

6. *Zeus ocellatus*, Storer. *Zenopris ocellatus*, Gill, Procd. Philad. Ac., vi. p. 888.

7. *Trachynotus Corolinus*, Gill.
8. *Trachynotus ovatus*, Gunther.

These two species were collected at Wood's Hole by Prof. Baird. See Gill, Procd. Philad. Ac., 1863, p. 332.

9. *Blennius.* (*sp.?*)

A specimen of Blenny, of a species unknown to me, was found on a barnacle taken from a ship in Salem, just arrived from Africa. The specimen is in the Essex Institute.

10. *Cyprinodon variegatus*, La Cep.

I have seen specimens taken in several localities on Cape Cod. Gill (Procd. Phil. Ac. 1863, p. 332) mentions that Prof. Baird found species at Wood's Hole; and Lyman (Procd. B. S. N. H. vii. p. 76) states that he found it at Yarmouth; (Lyman gives it under the name of *C. ovinus.*)

11. *Centriscus scolopax*, Storer, Procd. B. S. N. H. v. p. 178.

12. *Salmo eryox.*

Prof Agassiz, in Procd. B. S. N. H., states that a specimen of this European trout or salmon was captured at the mouth of the Merrimac River. The *S. eriox* of Europe (Parnell and Kroger) is referred by Dr. Gunther to *S. trutta.*

13. *Salmo.* (*sp.?*)

I have seen specimens of a small headed trout, similar to the "Blue back" of the Richardson Lakes, taken in the western part of the state; but have no specimens, and do not know the exact locality. The specimens were not *S. fontinalis.*

14. *Ciliata argentata*, Gill, Procd. Philad. Ac. Nat. Sci., 1863, pp. 241 and 332.

During one tide in the Summer of 1860, Mr. Caleb Cooke, of Salem, found a large number of specimens of this species, on Nahant Beach; and in 1861, I found three specimens in the surf at West's Beach, Beverly. Gill also mentions it from Nahant. (Col. by Dr. Slack.)

15. *Euchalarodus Putnami*, Gill, Procd. Philad. Ac. Nat. Sci., 1864, p. 216, and p. 221.

This species is described by Prof. Gill, from two specimens belonging to the Essex

Institute, which were taken off Beverly Bridge, in Jan., 1858, by my brother, C. A. Putnam.

16. *Liparis*, (*sp.?*) allied to *L. arctica.*

Several years ago, living specimens of this species were in Mr. Cutting's Aquarial Gardens, and were dredged in the bay. Afterwards I obtained two specimens from Mr. Fuller, of Portland, who collected them in Portland harbor. (These specimens are now in the M. C. Z. at Cambridge.) There is also a specimen in the state cabinet, collected at Nahant, by Charles Flint, Esq., and Mr. Alex. Agassiz has also collected specimens at Nahant, which are in the Museum at Cambridge.

17. *Leptocephalus gracilis*, STORER.

Mr. Caleb Cooke, of Salem, found four specimens of this species on Nahant Beach, in the Summer of 1860. These specimens are now in the Essex Institute, Boston Soc. Nat. Hist., and Museum of Comp. Zoölogy.

18. *Ammodytes dubins*, REINH.

Dr. Gunther (Cat. Fish, IV. p. 387) states that there is a specimen of this species, in the British Museum, which was taken in Boston. I much doubt the species being found so far south, though I have seen large specimens of *A. americanus*, that might be easily mistaken for *A. dubius*, from our bay.

19. *Syngnathus*, (*sp.?*)

I have seen three or four specimens of a Syngnathus from Cape Cod, which were three or four times the size of *S. peckianus*, and different in other respects from that species. (Specimens in the Essex Institute, and Museum of Comp. Zoölogy.)

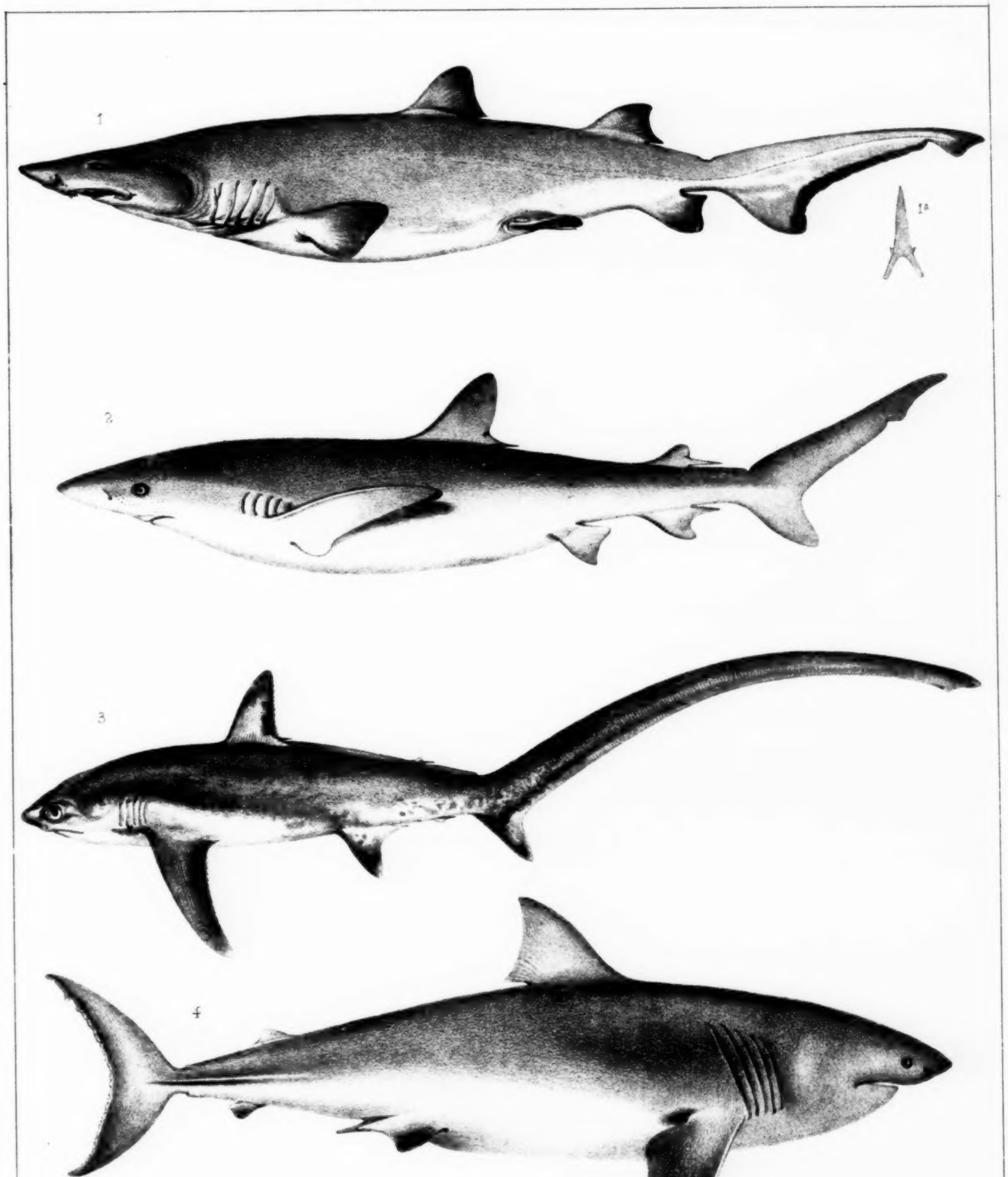
20. *Hololepis fusiformis*, PUTN., Bul. Mus. Comp. Zoöl., 1863. Syn. *Boleosoma fusiforme*, GIR.

Girard's specimens were from Framingham. I have found it very plenty in several ponds in Essex County, and also in other parts of the state.

21. *Semotilus corporalis*, ABBOTT, Procd. Philad. Ac. Nat. Sci., 1861, p. 154.

Have collected specimens of this fish in the brooks near Williams College.

Pl. 36

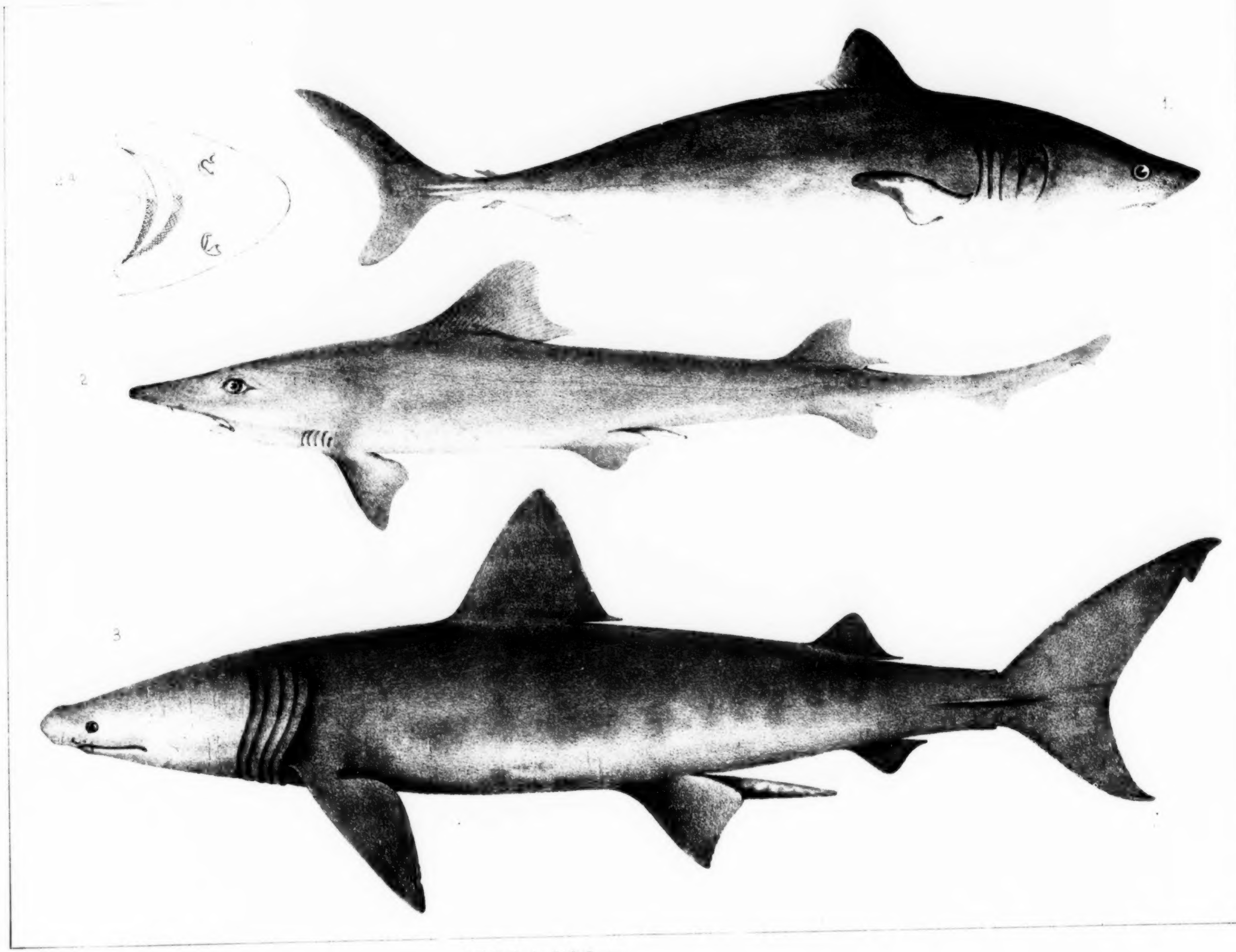


1. Carcharias griseus. *Ayres.*
1a. tooth.

2. Carcharias obscurus. *Lesueur.*
3. Carcharias vulpes. *Cuv.*

4. Carcharias Atwoodi. *Storer.*

Pl. 37



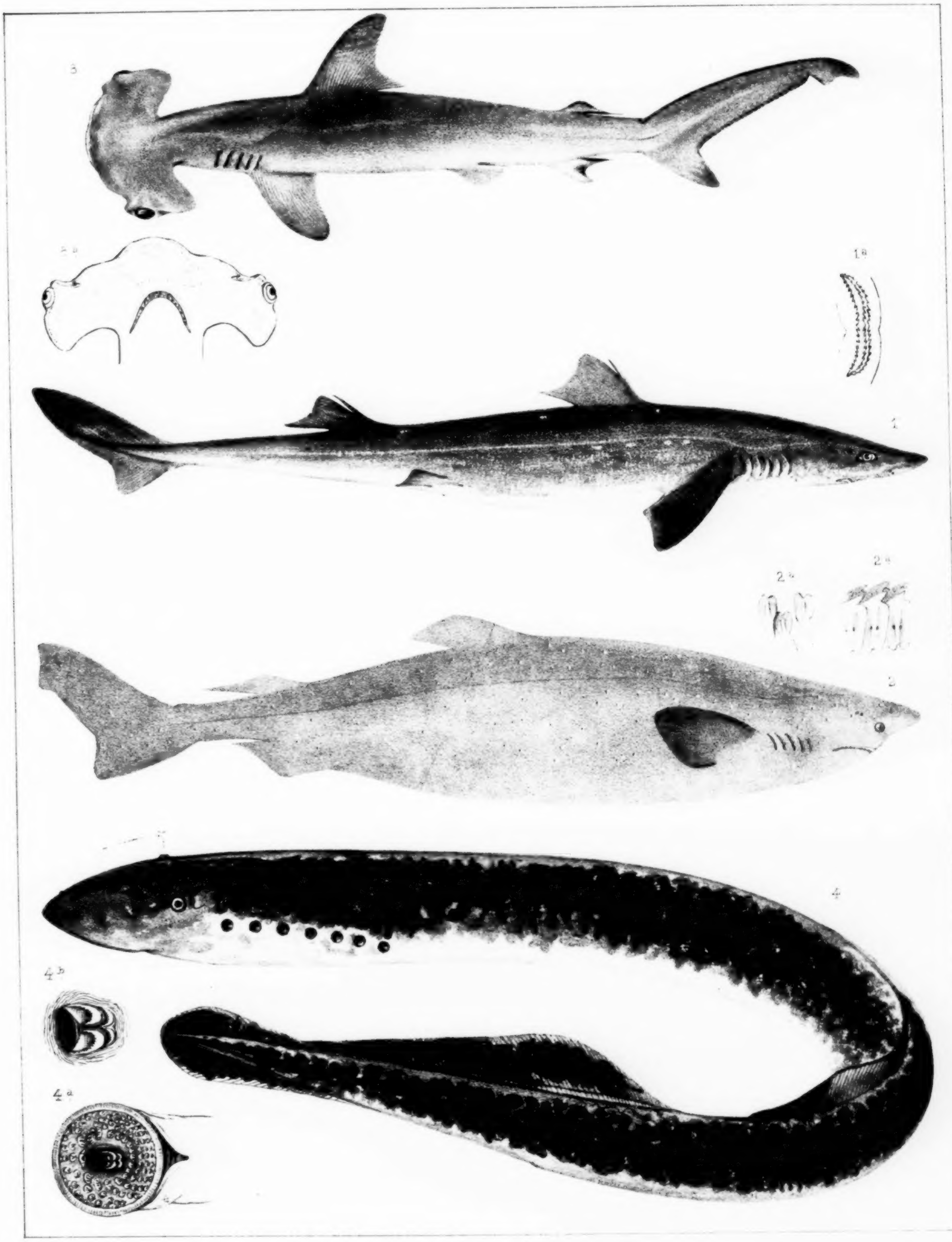
J. H. Bufford's Lith. Boston

1. Lamna punctata. *Storer*

2. Mustelus canis. *Dekay*

2ª. head, beneath

3. Selachus maximus. *Cuv.*

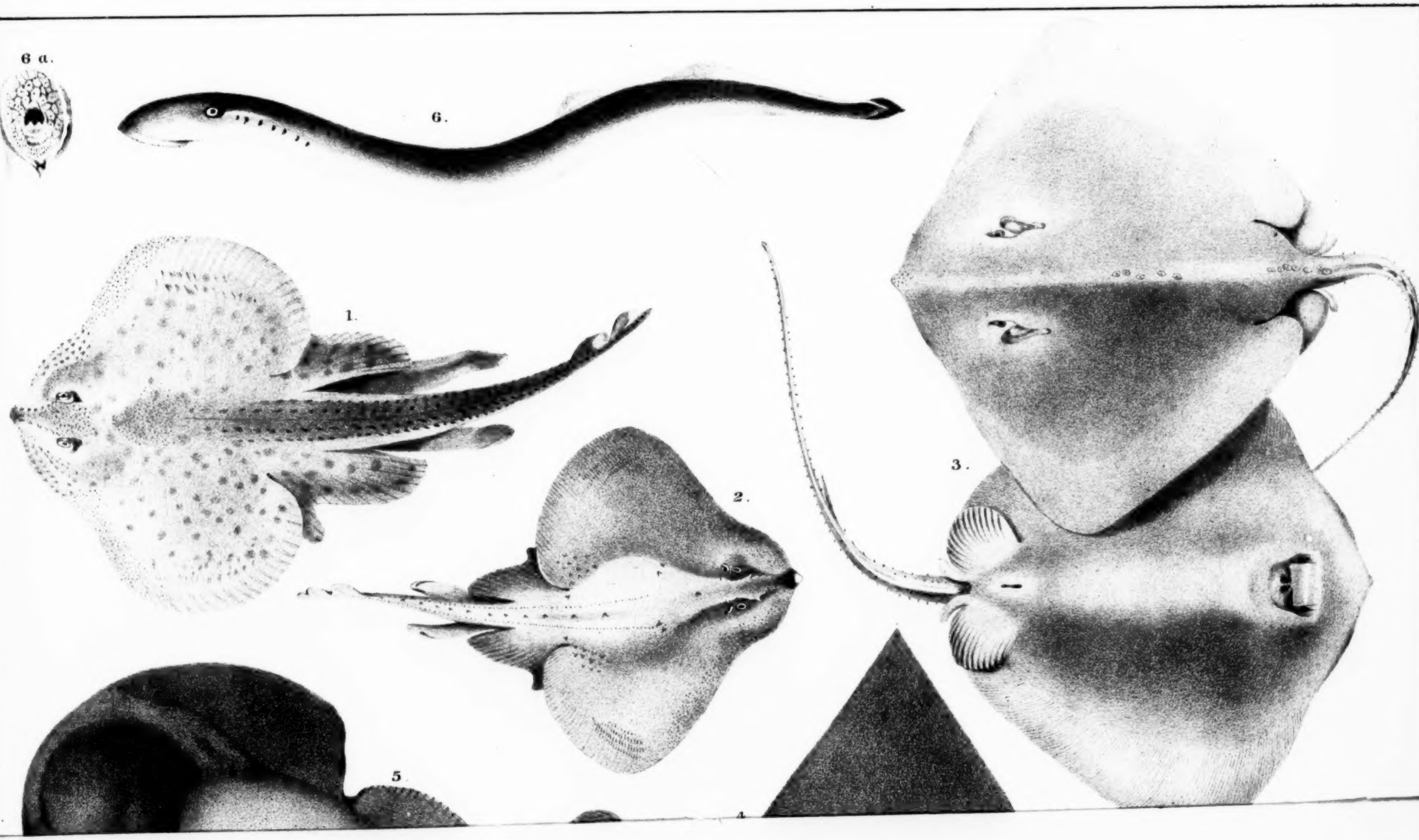


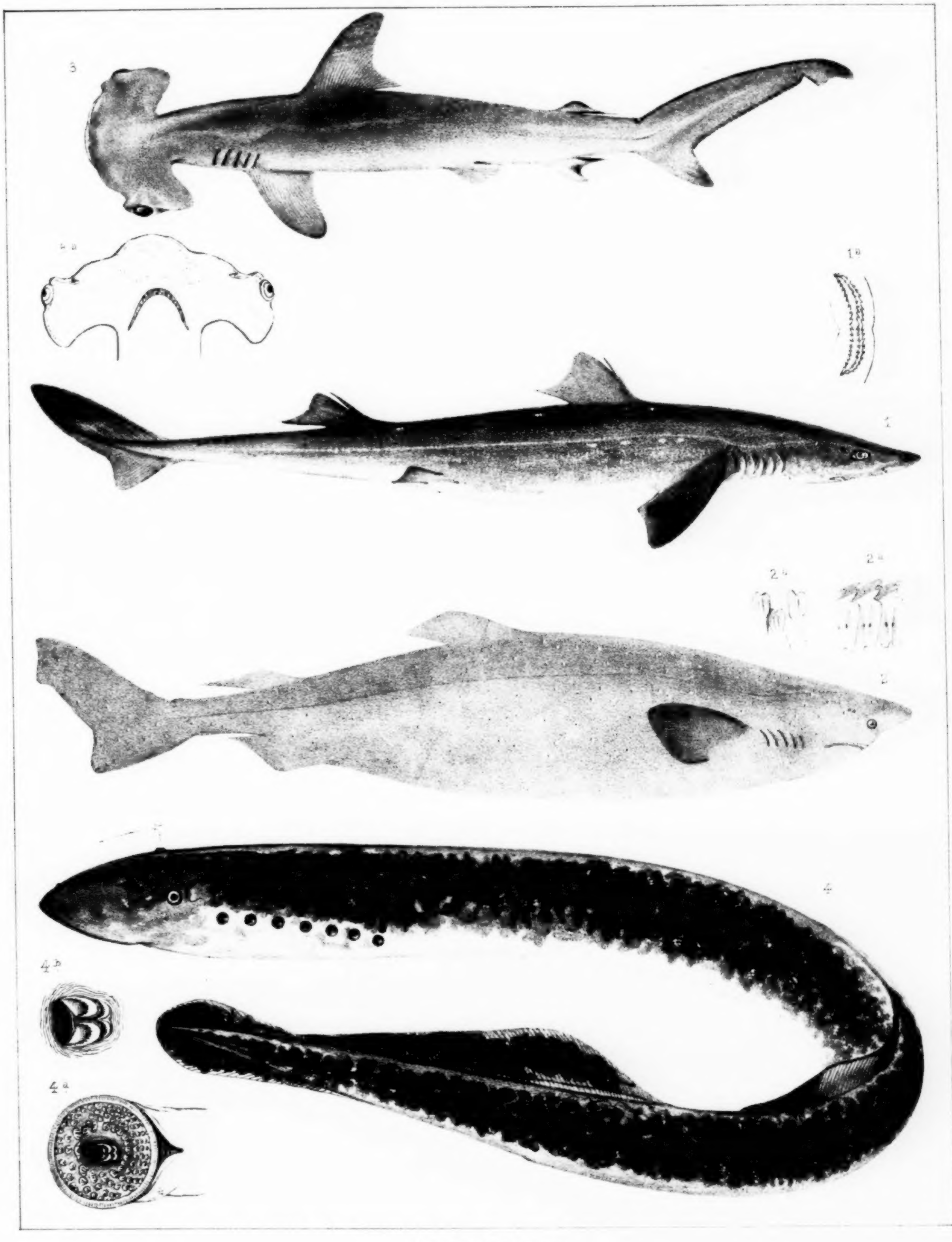
J. H. Bufford's Lith. Boston.

1 Acanthias Americanus. *Storer.*
1a teeth.
2 Scymnus brevipinna. *Dekay.*
2a teeth in both jaws.

3 Zygaena malleus. *Val.*
3a head, beneath.
4 Petromyzon Americanus. *Lesueur.*
4a mouth. 4b central teeth.

Pl. 39

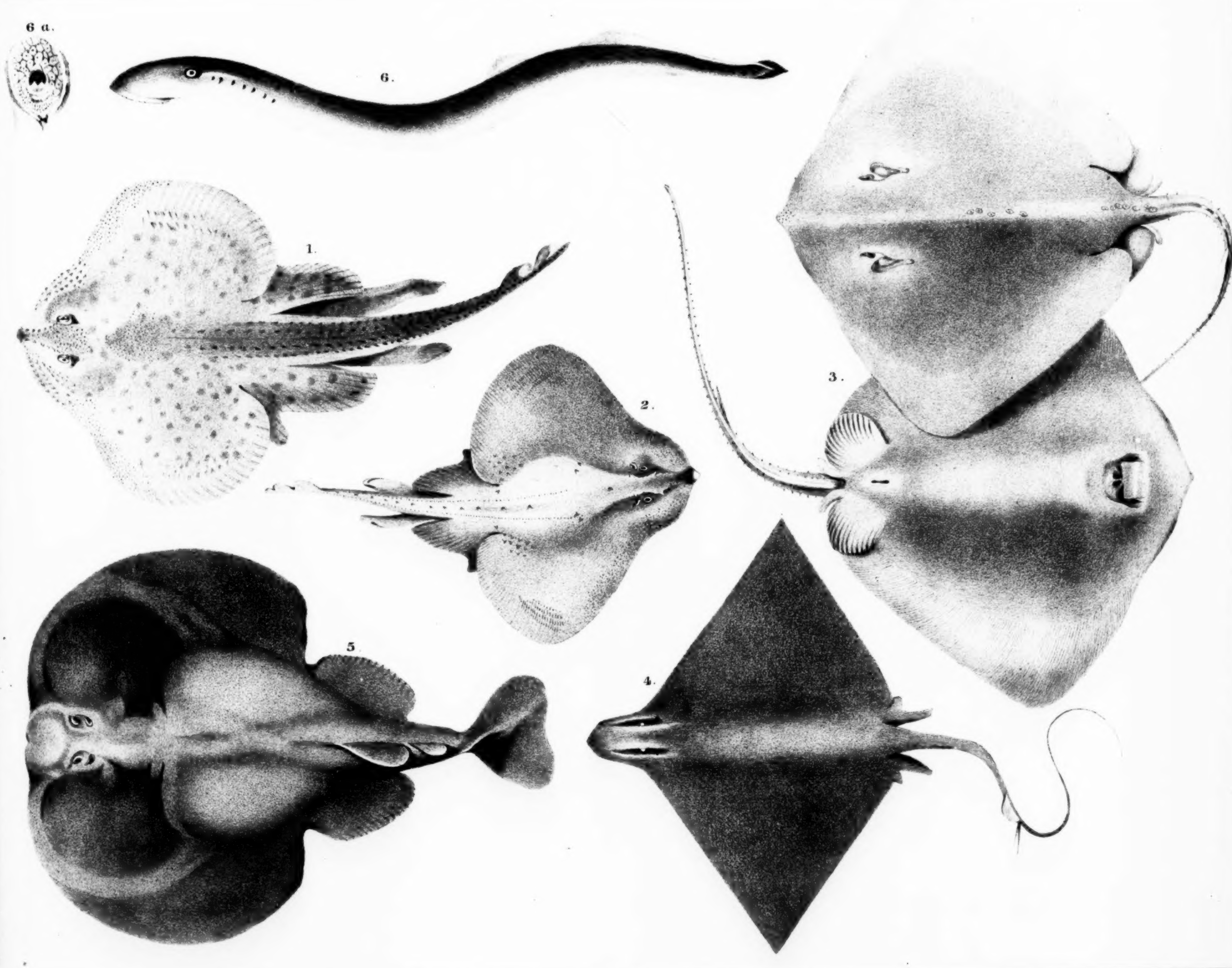

6 a.
6.
1.
2.
3.
5.
4



J. H. Bufford's Lith. Boston

1 Acanthias Americanus. *Storer*
1a teeth.
2 Scymnus brevipinna. *Dekay*
2a teeth in both jaws.
3 Zygaena malleus. *Val.*
3a head, beneath.
4 Petromyzon Americanus. *Lesueur*
4a mouth. 4b central teeth.

Pl 39



John H. Bufford's Lith. 34 Chauncy St. Boston

1 Raia diaphanes. *Mitch.*
2. Raia laevis. *Mitch.*
3 Pastinaca hastata. *Dekay.*
4 Mylicbatis acuta, *Ayres.*
5 Torpedo occidentalis. *Storer.*
6 Petromyz[illegible] nig[illegible]
6 a. Mouth.

INDEX OF SCIENTIFIC NAMES.

INDEX OF POPULAR NAMES.

XI.

Observations on the Language of Gower's Confessio Amantis.

(A Supplement to Observations on the Language of Chaucer, Memoirs of the Academy, New Series, Vol. VIII. Art. XVII.)

By F. J. CHILD,
PROFESSOR IN HARVARD COLLEGE.

(*Communicated January* 9, 1866.)

In a paper communicated to the Academy five years ago, I examined, with more thoroughness than has usually been thought necessary, certain questions relating to the Language of Chaucer. The chief point sought to be determined was the number of syllables in words. A laborious study of the Canterbury Tales, in the only text available for the purpose (Harleian MS. No. 7334), led to results on the whole unsatisfactory. The reasons why better results were not to be expected are sufficiently set forth in that paper. The difficulty arises from that "diversity in English," in Chaucer's time, which makes him put up an earnest prayer (toward the end of Troilus and Cressida) that his verses may not be spoiled by the miswriting of copyists and the mismetring of readers. It was natural that repeated transcriptions, executed while the language was changing, should greatly add to the confusion produced by the original diversity, and the unravelling of the tangled web will now require the outlay of much time and labor; and not of time and labor only, but also of money; for until several of the best texts of one of the longer poems are furnished, *in print*, for comparison, no stable conclusion can be reached. Little progress has been made in this direction during the last five years. We have indeed had a new edition* of Chaucer, with new texts of some of the poems, but this edition gives us the already printed Harleian version of the Canterbury Tales over again, so that we do not as yet possess *two* good texts of any of Chaucer's longer poems.

In the lack of any other means of pushing further the investigation into Chaucer's

* By Mr. Morris, in Bell & Daldy's Aldine Poets. Though communicated in 1866, this paper has been revised, and may be considered as dating from Nov., 1867.

language, I have thought it well to see what light could be got from Gower. Gower was a contemporary, who lived in the same part of England and in much the same society, and would therefore be likely to use nearly or quite the same dialect. The only objection to using Gower's language in illustration of Chaucer's is that he composed a good deal in French. That his language was somewhat affected by this circumstance may be seen by his un-English use of the definite article, as *the speech*, for *speech; the truth*, for *truth; the man*, for *man; the mankind*, for *mankind; the gold*, for *gold*. He even says *the God*. We observe, also, in his style a poverty of expression and a want of ease * which were probably caused in part by his using foreign languages so much as he did. Nevertheless, the practical test of a comparison shows such a correspondence in the forms and inflections of words as will remove all doubt of the utility of employing Gower to elucidate Chaucer.

The Confessio Amantis is, excepting a few trifles, Gower's only English work. It is a poem of about 34,000 lines, in the 8 (or 9) syllable metre. Many manuscripts exist, but as this poem never could have had a popularity to be compared with that of the Canterbury Tales, the presumption is that it has suffered less from repeated copying. A faithful printed copy of one of the best manuscripts would therefore be extremely desirable. Instead of this, Messrs. Bell & Daldy, the successors of Pickering, gave us ten years ago a magnificent book, edited by Dr. Reinhold Pauli, containing a text founded on that of a printed edition by Berthelette bearing the date 1532, with the orthography "restored," and the metre "regulated," both of which, we are told, "had been disturbed in innumerable places." The restoring and regulating were done under guidance of MS. Harl. 7184, the orthography of which is said by the editor to be judicious and consistent.† What constitutes a judicious orthography, we are not informed, nor does it seem to have struck the editor that all questions concerning the right form of

* Want of ease is a mild phrase. In a complex sentence he is soon out of his depth, and flounders about like a "borel clerk" indeed. For example, though the case is a strong one:—

The worthy knight Prothesalay,
on his passage where he lay,
towardes Troie thilke siege,
she which was all his owne liege
Laodomie, his lusty wife,
which for his love was pensife
as he whiche all her herte hadde,
upon a thing wherof she dradde,
a letter for to make him dwelle
fro Troie, send him thus to telle, etc., ii. 65.

† The restored orthography is at any rate not consistent.

RIGHT.	WRONG.
the more he *cam* the wellë nigh, i. 120.	that Christ that *came* this world to save, i. 180.
so *cam* she to him privëly, i. 148.	*came* for to preien Ulixes, ii. 62.

words are questions of orthography, even that essential matter of the number of syllables, without a knowledge of which we shall be rather presumptuous to pretend to regulate the metre.

I have used Pauli's text because it was the best I could get. But it must be distinctly understood that the language of Gower means in this paper the language of Pauli's Gower, as in the other the language of Chaucer means the language of Wright's edition of one of the Harleian manuscripts of the Canterbury Tales.

This article, being intended as a supplement to the paper on Chaucer, has been drawn up in the same form, so as to afford the means of ready comparison. The phraseology of the rules, the numbers of the sections, and the notation correspond. It may be observed that the subject of the article is really concluded at § 97. The miscellaneous notes which follow contain a few things noticed in passing which may on some occasion be useful; but they are purely incidental, and do not profess to be complete.

A slight inspection will show a close general agreement between Chaucer and Gower, with the difference that Gower is more regular, and on the whole uses older forms. (I am speaking, be it always remembered, only of the texts which I have used.) The number of superfluous final e's, which cannot be disposed of by rules drawn from the text itself, is immeasurably greater in Chaucer than in Gower. The frequently recurring words, *time*, *sone*, and *love*, which are in the great majority of cases monosyllables in Chaucer, are, with a single exception in the case of the first two, of two syllables in Gower. The singular of the imperfect of simple verbs loses its characteristic termination in e much less frequently in Gower, and the present participle terminates, with few exceptions, in the Saxon -endė, whereas, in Chaucer the modern ending (i. e., -yng) is decidedly the rule. The more ancient termination of the plural of the present indicative (-eth) is, however, commoner in Chaucer than in Gower.

Elda com(e) *hom* the samë night, i. 187.	at *home* with him so as he telleth, i. 207.
thus *dar* I make[n] a forward, i. 296.	as for that time I *dare* well swerë, i. 135.
that I dar speke and say all out[ë], i. 297.	for I *dare* makë this behest, i. 178.
her wombë which of childe *aros*, ii. 169.	this noise *arose*, this lorde it herdë, i. 268.
he slept and *ros* when it was timë, ii. 257.	fro deth he *rose* the thriddë day, i. 273.
and had a *wif* which Philen hight, ii. 269.	his *wife* that she him woldë saine, i. 208.
som man may like of that I writë, i. 2.	*some* goodly word that the was told(e), i. 123.
som man whan he most true appereth, i. 82.	*some* good ensample upon this lorë, i. 338.
som wildë placë that it werë, i. 290.	so that *some* beste him may devourë, i. 290.
whos rightë name Almèus was, i. 365.	*whose* reulë stant out of the wey, i. 44.
whos love his strength all overthrew(e), iii. 366.	*whose* prest I am touchènd[*e*] of lovë, i. 49.
for well I *wot* that wol nought be, ii. 276.	for whan I *wote* her good estat(e), i. 135.
I *wot* the last of my bargein, ii. 277.	God *wote* if thou it shall escapë, i. 139.

etc., etc.

It is entirely possible that further investigation may show that Chaucer is distinguished from Gower by a freer treatment of that final e, which, occurring often, is to our ears so puerile, and which coming in regularly at the end of each of a long succession of verses, produces a monotony all but intolerable. It is heartily to be hoped that this may be done, but in the mean time some signs of rebellion against the predominance of the offending letter make it necessary to repeat a suggestion already made in the paper on Chaucer (p. 449, note ‡), that we are not inquiring what sort of language and verse is most agreeable, but what was the actual rule of our language at the end of the fourteenth century.

I subjoin a few passages exemplifying the rules which follow. The last two are more than ordinarily correct, and are not fair specimens of the text.

and pridë was a vicë holdë, i. 11.
fortunë stant no whilë stillë, i. 22.
he may that werrë sorë rewë, i. 37.
I fond(e) a swotë grenë pleinë, i. 45.
and thus a mannës eyë ferst
him selfë greveth altherwerst,
and many a timë that he knoweth
unto his ownë harme it groweth.
my sonë, herken now forthy
a talë, to be war(e) therby
thin eyë for to kepe and wardë,
so that it passë nought his wardë, i. 53.
ministrës fivë ful diversë, i. 61.
of thilkë borë freë kindë, i. 68.
that sleightë shuldë helpë thannë, i. 79.
which causë was of purë dredë, i. 109.
wher(e) somë pleide and somë songë, i. 110.
he saidë nay, they saidën yis, i. 201.
this newë king of newë pridë
with strengthë shop(e) him for to ridë, i. 218.
the quickë body with the dedë
with levë takë forth they ledë, i. 253.
he hightë popë Nicholas, i. 254.
first how mankindë was forlorë,
and how the highë God therforë
his sonë sendë from abovë,
which borë was for mannes lovë,
and after of his ownë chois, i. 273.
he mustë thannë failë nedë, ii. 284.
they didë thannë suchë thingës, iii. 56.
and all bledèndë kist him oftë, iii. 60.
of thesë twelvë signës stondë, iii. 118.
he was tormented day and night,
such was the highë goddës might,
till seven yer(e) an endë tokë.
upon him self tho gan he lokë:
in stede of metë, gras and streis;
in stede of handës, longë cleis;
in stede of man, a bestës likë
he sigh, and than he gan to sikë
for cloth of golde and of perrië,
which him was wont to magnifië.
when he beheld his cote of herës
he weptë and with wofull terës
up to the heven he caste his cherë,
wepend[*e*] and thought in this manerë;
though he no wordës mightë winnë,
thus said his hert[*e*] and spak(e) withinne:
o mighty God, that all hast wrought,
and all might bring[*e*] ayein to nought,
now knowe I wel but all of the
this world hath no prosperité.
in thin aspect ben alle alichë,
the pouer man and ekë the richë;
withoutë the ther(e) may no wight,
and thou above all other might.
o mighty lord, toward my vicë
thy mercy medlë with justicë,
and I woll make a covenaunt
that of my lif(e) the remenaunt
I shall it by thy grace amendë,
and in thy lawë so dispendë, i. 143–4.

my godë derë sonë, yis.
thy shriftë for to makë plein,
there is yet morë for to sain

of lovë which is unavised.
but for thou shalt be well avised
unto thy shrifte as it belongeth,
a point which upon lovë hongeth,
and is the laste of allë tho,
I woll the telle, and thannë ho.

the mighty God, which unbegonnë
stant of him self and hath begonnë
all other thingës at his will[ë],
the heven him listë to fulfill[ë]
of allë joië, where as he
sit enthronized in his see;
and hath his aungels him to servë,
such as him liketh to preservë,
so that they mowë nought forswey;
but Lucifer he put awey,
with all the route apostazied
of hem that ben to him allied,
which out of heven into hellë
from aungels into fendes fellë,
wher(e)that there is no joy of light,
but morë derk than any night,
the peinë shall ben endëless:
and yet of firës nethëles
there is plenté, but they ben blackë,
wherof no sightë may be takë, iii. 274–6.

[NOTATION.—The verses not being numbered, references are made by volume and page. The sign f denotes that the word referred to is the *final* word of a verse. S. denotes Anglo-Saxon; rh. rhymed with.

The sign (^) is used to denote all long vowels in Saxon words, without distinction of variety; the double dot (¨) over a vowel, to denote that it makes a syllable; the italic type to denote a vowel elided; (˘) to indicate that a vowel (not elided) is silent or to be slurred (supposing the reading correct); the grave accent, to mark tone-syllables. The acute accent is used only with words which in French end in é.

Words or letters added by me to the text are enclosed in brackets, as [e]: words or letters which I should omit are enclosed in a parenthesis, as (e).]

NOUNS.

§ 1. Nouns which in Anglo-Saxon end in a vowel terminate in Gower uniformly in ĕ.

§ 2. FIRST DECLENSION OF ANGLO-SAXON NOUNS.

Neuters. (I. 1. RASK.)

ere (S. eâre), i. 59, 158, 277 : ii. 56, 271, 293 : iii. 31 (3 cases).
eye, eie (S. eâge), i. 53, 127 : ii. 41, 102, 369, etc. : eyĕ, ii. 315.

§ 3. Masculines. (I. 2.)

ape (S apa), ii. 294, 295 f, 296.
asse (S. assa), ii. 69 f, 293 (2 cases) : iii. 195, 196, 197.
bere (S. bera), ii. 338 f, 339 : iii. 263.
be-leve (S. ge-leâfa), i. 356 f: ii. 152, 172, 179, 181, 183.
bonde- man (S. bonda); iii. 320.
bowe (S. boga), ii. 101 f, 290, 339 f: iii. 357.

crede (S. crêda), i. 295 f: ii. 225 f, 372 f.
cuppe (S. cuppa), iii. 2, 3, 18.
drope, droppe (S. dropa), ii. 266, 286 f.
dwale (S. dwala), iii. 14 f.
fere (S. fèra), ii. 229 f (bed-fere): iii. 65 f (beddë-fere), 334 f (in fere).
flete (S. flota), i. 314.
fode (S. fôda), i. 142 f: iii. 25, 27, 31, 32: food[e], ii. 183.
food rh. good, ii. 362 f: iii. 26 f, 30 f.
fole (S. fola, *equuleus*), iii. 356 f.
galle (S. gealla), i. 303 f: ii. 177: iii. 13 f, 100 (2 cases).
gere (S. gearwa, geara), ii. 227 f, 233 f: iii. 295 f.
grame (S. grama), i. 281 f, 304 f.
hare (S. hara), ii. 93 f: iii. 218 f.
herre (S. heorra), "out of herre," i. 36 f, 318 f: ii. 139 f: iii. 43 f, 52 f, 138 f, 203 f, 211 f.
hewe (S. hîwa), i. 173.
hope (S. hopa), i. 227, 319: ii. 7, 29 (2 cases), 117: iii. 21.
wan-hope, ii. 115, 117. hopĕ, ii. 116, qu. hap?
husë- (housë-) bonde (S. husbonda), i. 75, 94, 101, 187 f, 247 f.
hus[ë] bonde, ii. 281 f, 329 f: iii. 253, 254, 257, 260, 261, 263, etc., etc. husbonde, ii. 282: iii. 362.
knape, knave (S. cnapa, cnafa), iii. 321 f: ii. 16, 269: iii. 147 f.
lappe (S. lappa), ii. 11 f.
like (S. lica, *corporis forma*), i. 143 f: iii. 70 f.
lippe (S. lippa), i. 283 f: ii. 314 f.
make (S. maca, *consors*), i. 367 f: ii. 5, 270 f: iii. 118 f, 276 f: (also, ii. 204 f in the modern sense of make, *formatio*.)
mone (S. môna), i. 65 f: ii. 112, 120 f, 321 f: iii. 109, 351, 362, etc.
name (S. nama), i. 123 f, 306: ii. 14, 43, 157.
necke (S. hnecca), iii. 27, 279 f.
onde (S. onda, anda), i. 75 f: ii. 260 f.
oxe (S. oxa), ii. 127 f, 128: iii. 347.
pese (S. pisa), ii. 275 f.

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cope (S. cappa), ii. 101. (?)
hunte (S. hunta), he was an hunt' upon the hilles, ii. 158.
like, liche (S. ge-lic, *adj.*), i. 118 f, 136 f, 148 f, 265 f, derived from the definite form of the adjective.

pope (S. papa), i. 203, 254, 255, 256.
pricke (S. pricca), i. 154 f, 283 f: ii. 325 f.
pride (S. prŷta, also prŷt, II. 2), i. 61, 105, 106, 109: ii. 166 f, 342 f.
see (S. sæ, m. and f., contracted), i. 35, monosyllable.
shrewe (S. screawa), i. 353: iii. 248 f.
snake (S. snaca), iii. 118 f.
sparke (S. spearca), i. 258, ii. 102 f.
spore (S. spura), i. 321.
stake (S. staca), i. 263 f: ii. 83 f: iii. 8 f.
steede (S. stêda), ii. 32 f: iii. 44 f.
stere (S. steôra), i. 60 f: ii. 201 f.
sterre (S. steorra), ii. 47, 259, 347: iii. 78, 95.
stikke (S. sticca), ii. 327 f.
swere (S. sweora), ii. 30 f.
tene (S. teôna), i. 305 f: ii. 311 f, 313 f: iii. 53 f, 80 f.
thombe (S. þuma), i. 175.
time (S. tîma), i. 1, 6, 53: ii. 2, 3, etc. etc.
time by me } i. 227, 309, 370: ii. 41, 114, 277: iii. 369: timĕ, ii. 167.
wane (S. wana), iii. 104 f.
wele (S. wela), i. 305 f: ii. 141 f, 188 f, 207 f.
welle (S. wella, etc.), i. 301: ii. 214 f, 337 f: iii. 16, 104, 260, 338.
wille (S. willa, also wille, I. 3), i. 22, 97: ii. 7, 169, 257, 300, 303, etc.
another form, will (S. will, II. 2), i. 104, 318, 319, 322, 361.
wone (S. wuna), i. 118 f: ii. 143 f, 180 f: iii. 55 f, 245 f.
wrecche (S. wrecca), i. 112 f; ii. 290.

§ 4. Feminines. (I. 3.)

almesse (S. ælmæsse), i. 64: ii. 392 f: iii. 35 f, 39 f.
arwe (S. arewe), i. 234.
belle (S. belle), ii. 13 f, 282 f, 363 f.
blase (S. blæse, also II. 2), 244 f, 264 f.
cheke (S. cêce), ii. 210 f: iii. 300 f, 339 f.
chirche (S. cyrice), ii. 187 f, 188, 362 f, 371, 375.
crowe (S. crâwe), ii. 335 f.
crumme (S. crume), iii. 35, 39.
deepe (S. dŷpe), ii. 105 f, 200 f: iii. 296 f.
erthe (S. eorðe), i. 355 (2 cases), 358: ii. 34, 77.
erth, i. 25: ii. 197: both doubtful.

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wan[e] (S. wana, a defect?), rh. Adrianë, ii. 307.
wrenne (S. wrenna), wrenne in, iii. 349.

harpe (S. hearpe), iii. 300, 301, 303 (2 cases), 325 f.
herte (S. heorte), i. 159, 319: ii. 5, 13, 18, 20, 21, 22, etc. etc.
hert, hertĕ, i. 75, 359: ii. 3, 279. (?)
hilte (S. hilte, also hilt, neut), i. 328 f.
kerse (S. cerse, cresse), i. 299 f, 334 f.
lilie (S. lilje), iii. 249.
lunge (S. lunge), iii. 100.
masse, messe (S. mæsse), ii. 39 f, 369 f.
mite (S. mîte), ii. 275 f.
molde (S. molde), i. 217 f: ii. 39 f, 63 f, 356 f: iii. 260 f.
nettle (S. netle), i. 173.
nightingale (S. nihtegale), i. 54 f: ii. 98 f, 326 f.
nonne (S. nunne), iii. 281 f.
oule (S. ûle), i. 100, 299 f: ii. 265 f.
panne (S. panne), ii. 93 f, 201 f.
pipe (S. pîpe), ii. 113.
resshe, risshe, reisshe (S. resce, risce), i. 160 f: ii. 97 f, 284 f.
se, see (S. sæ, also II. 2), always monosyllable, iii. 296, 103, 104, etc.
shete (S. scŷte), iii. 315.
side (S. sîde), 42 f, 110, 111 f: iii. 17, 58, 203, 367.
sive (S. sife), i. 294.
sonne (S. sonne), ii. 34, 84 f, 102, 183, 193, etc.
swalwe (S. swalewe), ii. 328.
throte (S. þrote), ii. 176, 206 f.
tonne (S. tunne), i. 321 (2 cases): iii. 13 f, 14, 18, 350 f.
tunge (S. tunge), i. 295, 312: ii. 22, 90: iii. 338.
wacche (S. wæcce), ii. 96, 202.
weke (S. wice, wuce), iii. 116 f.
wicche-craft (S. wicce), iii. 44.
wenche (S. wencle), i. 263 f.
wise (S. wîse), i. 3, 68, 349: ii. 34, 74.

§ 5. The following may or may not be correctly written. The combination of a liquid with **e** is unstable, the vowel easily slipping from one side to the other of the consonant.

throstel (S. þrostle, I. 3), qy. throstle? i. 54: compare nettle (S. netle), i. 173.
nedder (S. nædre), qy. neddre? iii. 118.

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lappewinke (S. hleâpwince, –winge, incert. gen., ii. 329.
more (S. môre, mulberry?), i. 98 f.
sale (S. selle? O. H. Germ. sala). ii. 29
adder, (S. nædre), qy. addre? ii. 72, 260.

§ 6. Exception to § 4.

laverock (S. lâferce, also lâferc, ii. 3), ii. 264 f.

to(e), ii. 143 f, is from the contracted S. tâ.
roo, ii. 95, monosyllable, from contracted S. râha, râ (also II. 3).

§ 7. Second Declension of Anglo-Saxon Nouns.

Masculines. (II. 2.)

bale (S. bealu), i. 329 f: iii. 295 f.
breche [spouse-breche], (S. brice), i. 351 f: ii. 328 f.
brimme (S. brymme), ii. 293 f.
chele (S. cêle, cŷle), ii. 369 f: iii. 9 f, 44 f, 371 f.
chese (S. cŷse, cêse), iii. 23 f.
ende (S. ende), ii. 61, 186, 253, 270, etc.
hate (S. hete), i. 7, 310: ii. 135: iii. 80 f, 360 f.
herde (S. hirde), i. 39 f.
ire (S. yrre, *inc. gen.*), i. 280: iii. 148 f, 202 f, 282 f.
leche (S. læce), i. 267 f: iii. 30 f, 315 f, 338 f.
lovë-drunke (S. drynce, also drinc, I. 2), iii. 12, 16.
mede (S. meadu), ii. 59 f, 285 (2 cases), 287: iii. 47 f: medow, ii. 266.
mele (S. mêle, *cyathus*), iii. 21 f.
mete (S. mete, also mett), ii. 15 f, 59 f, 137 f, 147 f, 224: iii. 25 f, 31 f, 39, 100, 299.
shipe (termination, S.- scipe):
dronkëshippe (S. druncenscipe), iii. 17.
worshipe (S. weorðscipe), ii. 65.
kindeship, felaship occur in a couplet i. 170, but doubtless should have a final e.
slitte (S. slite), i. 15 f.
stede (S. stede, styde), i. 60 f, 121 f, 197 f: ii. 10 f, 204 f, 207 f: iii. 204 f.
tete (S. tite, also tit), i. 268 f.
tie (S. tige), ii. 246 f.
whete (S. hwæte), ii. 59 f, 74 f: iii. 291, 293.

§ 8. Exceptions to § 7.

Such representatives as occur of the Saxon noun in **–erë**, denoting an agent, seem to want the final vowel. Nouns of this kind were by no means as common in the old language as in the modern. I have noticed but three fair cases in Gower:

which hath no *clapper* for to chime, ii. 13.
which was the firstë founder tho, ii. 161.
the soth(ë)-*saier* tho was lef(e), iii. 164.

There are other instances without the final **e**, but in these cases the succeeding word begins with a vowel, and it is supposable that the **e** may have been elided: thus, speker in, ii. 159; fisher in, iii. 297; furtherer of, iii. 111; maker of, iii. 135; techer of, iii. 136; keper unarraied, iii. 175. It is doubtful whether these words should be called exceptions to § 7; for, in the first place, the metre does not settle the question of their form, inasmuch as clàppërë, for instance, would suit the verse as well as clàpper; and secondly, for few, if for any of them, can we show a form in **-ere** in the Saxon dictionary.

§ 9. Third Declension of Anglo-Saxon Nouns.

Neuters. (III. 1.)

ale (S. ealu), i. 294 f, 334.
chinne (S. cinne), i. 275 f: ii. 139 f.
inne (S. inne, inn), iii. 314 f, 318 f.
-riche (S. rîce), termination:
 heven-riche (S. heofonrîce), i. 265 f.
 kinges-riche (S. cyningrîce), ii. 268 f.
 worlde-riche (weoruldrîce), i. 118 f, 366 f: worldes riche, ii. 130 f.
skille (S. scile), i. 16 f, 17 f, 329, 352 f, 358 f: ii. 117, 138 f, 312 f, 365 f: iii. 148, 153 f, 343.
 skill, found only when rh. *will*, probably should have the ë: i. 42, 49, 104: ii. 330: iii. 59, 180, 185, 227.
 skille rh. *wille*, i. 277, 292, 352: ii. 312.
spere (S. spêre), ii. 138 f, 248 f, 251 f: iii. 53, 56 f, 58 (2 cases), 380 f.
 sperĕ, i. 125.
werre (S. werre), i. 125, 245, 355: ii. 59, 68.
wile (S. wîle, n. ?), ii. 227 f, 255 f, 282: iii. 50 f.
wite (S. wîte), ii. 89 f, 351 f: iii. 23 f, 27 f.
 kne (S. cneô, cneow), i. 24; stre (S. streâ, straw), ii. 160; tre (S. treow, treô, trê), i. 137, 139, may be regarded as contracted.

§ 10. Masculines. (III. 2.)

sone (S. sunu), i. 6, 10, 11, 13, 53, 54, 57, 83, 106, 205, etc., etc.

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be-yete (S. begete (?)), i. 238 f, 369 f: ii. 59 f, 191 f, 274: iii. 324.
winge (S. winge, decl. ?), iii. 217 f.
sonĕ (now tell, my sonĕ. My fader, what ?) i. 317, should probably read, sonë. Fader, what ?
wode (S. wudu), ii. 264, 327, 336, 339 (2 cases), 358.

§ 11. Feminines. (III. 3.)

answere (S. andswaru), i. 96 f, 97, 146 f: iii. 199, 249 f, 348 f.
brede (S. brædo), iii. 66 f.
care (S. cearu), i. 339: ii. 101: iii. 254 f, 285 f.
dore (S. duru, also dyr, II. 3), i. 231: ii. 32, 102, 114, 347: iii. 203.
elde (S. yldo, also yld, II. 3), iii. 365, 377 f.
fare (S. faru), ii. 173 f, 271 f: iii 84 f, 285 f, 347 f.
heighte (S. heâhðo), i. 137.
hele (S. hælo), ii. 86, 100, 121 f, 216 f: iii. 23 f, 141, 347 f, 373.
hete (S. hæto), i. 155 f (?): ii. 264 f, 357: iii. 9 f, 44.
lawe (S. lagu, also lag, II. 3), i. 304, 354, 355: ii. 157 f, 173: iii. 2, 189 (4 cases), 381 f.
leese (S. læsu), i. 17.
lode [liv(e)lode] (S. lif-lâdu), ii. 293 f.
love (S. lufu, also lufe, I. 3), i. 39, 45, 49, 50, 52, 67, 68, 83, etc., etc., etc.
nase (S. nasu, nosu), ii. 244 f: iii. 27.
nutte-tre (S. hnutu, also hnut, II. 3), ii. 30: nutte-shale, ii. 20 f.
sake (S. sacu), i. 121 f, 276 f, 344 f: ii. 70 f, 141 f, 314 f: iii. 374.
sawe (S. sagu), ii. 91 f, 225 f, 250 f, 344 f.
schame (S. scamu), i. 150: ii. 64, 107 f, 218, 254.
shawe (S. sceadu, also II. 2), ii. 45 f, 333 f, 339 f.
scole (S. scôlu), ii. 91: iii. 46, 84, 139, 324.
spade (S. spadu, also spad, II. 3), ii. 128 f.
tale (S. talu). i. 53, 121: iii. 62, 195, 218: iii. 48.
trouthe (S. treôwðo), i. 208: ii. 27 (2 cases), 28, 30, 180, 389: iii, 137, 344. trouth, ii. 226 ?

§ 12. It will be noted that the nouns *sone* and *love* have the final e regularly in Gower, contrary to the apparent rule in Chaucer. The same is true of the important word *time*, § 3.

§ 13. Many nouns which in Anglo-Saxon end in a consonant are found in Gower with the termination **ĕ**, derived from an oblique case of the old inflection.

§ 14. Masculines and Neuters (II. 1, 2: III. 1, 2) which, though ending in Saxon in a consonant, have in Gower the termination **ĕ**.

bedde (S. bedd, *n.*), i. 24, 101 f, 102, 129 (2 cases), 343: ii. 98 f, 244: iii. 3, 307.
bede (S. bed, *n.*), i. 208 f: ii. 118 f, 363 f, 374 f, 388 f.
berde (S. beard, *m.*), iii. 319.
berne (S. bern, *n.*), i. 162 f: ii. 291.
bore (S. bâr, *m.*), iii. 268 f.
borwe (S. borg, *m.*), ii. 34 f, 241 f.
bote (S. bât, *m.*), i. 2.
botme (S. botm, *m.*), i. 108.
browe (S. breâw, *m.*), i. 95 f.
carte (S. cræt, *n.*), ii. 34, 35 (3 cases), 112, etc.
childe (S. cild, *n.*), i. 190: ii. 16: iii. 74, 75, 310, etc.
clerke (S. cleric, clerc), iii. 288.
cole (S. côl *n.*), ii. 335 f.
cope (S. cop, *m.*), iii. 102 f.
dale (S. dæl, *n.*), i. 54 f.
dawe (S. daeg, *m.*), i. 113 f: iii. 182 f, 183 f, 277 f.
dele (S. dæl, *m.*), iii. 110, "somdelë besinesse."(?)
dethe (S. deâð, *m.*), i. 202.
dome (S. dôm, *m.*), iii. 211 f.
drinke (S. drinc, *m.* also drinca, I. 2), ii. 59, 83 f, 134 f, 331 f, 362 f: iii. 14, 15.
fee (S. feoh, *n.*), monosyllable, contracted, iii. 293.
fere (S. fær, *m.*), i. 57 f, 90 f: ii. 188 f, 308 f: iii. 181 f, 321.
fire (S. fŷr, *n.*), i. 98 f, 164 f, 235 f: iii. 203.
flesshe (S. flæsc, *n.*), ii. 342: iii. 242.
flete (S. fleôt, *m.*), i. 197.
folde (S. falud, fald, *n.* ?), i. 16 f, 262 f: ii. 312 f, 347 f.
folke (S. folc, *n.*), ii. 165.
fote (S. fôt *m.*), 7 f, 27 f, 42 f, 235 f: iii. 149 f.
gate (S. geat, *n.*), i. 81, 114 f (2 cases): ii. 122.
golde (S. gold, *n.*), ii. 356 f: iii. 112.
grave (S. græf, *n.*), ii. 114 f.
grounde (S. grund, *m.*), i. 111, 116 f, 279, 289 f: iii. 306, 357 f.
-hede, -hode (termination, S. hâd):
- falshede (-hode), i. 216: ii. 158 f, 226 f: iii. 136 f.
- godhede, i. 364 f.
- hastihede, ii. 245 f.
- kinghede, iii. 144 f, 183 f.
- knighthode, i. 246 (2 cases): ii. 62 f, 67, 69: iii. 210, 220, 380.
- knightlihede, iii. 212 f.
- ladyhede, ii. 40 f.
- liklyhede, ii. 147 f.
- maidenhede, ii. 55 f, 230 f, 252 f, 335 (2 cases), 336: iii. 285 f, 322 f.
- manhede (-hode), i. 82 f, 144 f: ii. 70 f, 79: iii. 155 f, 211, 236.
- susterhede, iii. 278 f.
- wif (e) hode, iii. 51.
- womanhede, i. 333 f: ii. 145 f: iii. 25 f, 29 f. 363 f.

hewe (S. hiw, *n.* ?), i. 251: ii. 101 f, 117 f: iii. 339 f.
home* (S. hâm, *m.*), ii. 7.
horse (S. hors, *n.*), iii. 259.
house (S. hûs, *n.*), i. 294, 301: ii. 329: iii. 124.
kinge (S. cyning, cyng, *m.*), i. 117: ii. 66 (2 cases): iii. 62, 359.
kinne (S. cynn, *n.*), ii. 267 f.
leefe (S. leâf, *n.*), i. 17.
liche (S. lic, *n.*), iii. 311 f.
limme (S. lim, *n.*), ii. 10: iii. 7.
life, live (S. lîf, *n.*), i. 199, 309 f: ii. 106, 372 f.
lode (S. hlæd, *n.* ?), ii. 293 f.
londe (S. land, lond, *n.*), i. 220, 315, 340: ii. 202, 308: iii. 11, 325, 327.
lope (S. hleâp, *m.*), i. 310 f.
middle (S. middel, *n.*), iii. 120.
minde (S. mynd, *n.*, also II. 3), i. 6 f: ii. 55 f, 126 f, 148 f.
monthe (S. mônað, mônð), ii. 27: iii. 117, 119, 124, 125.
mordre (S. morðer, *m.*), i. 270, 353 (3 cases).
morwe (S. morgen, morn, *m.*), i. 186, 205: ii. 63, 99 f, 108, 122, 198.
mote (S. mot, *n.*), i. 179.
mouthe (S. mûð, *m.*), i. 149 f, 295 f, 310 f: ii. 2, 319: iii. 18, 301 f.
mule (S. mûl, *n.*), i. 210: iii. 63.
rede (S. ræd, *m.*), i. 45 f, 163 f, 346 f.
rore (S. râr, *m.* ?), iii. 74 f.
scorne (S. scearn, *n.*, Icel. skarn, Old Ger. scern), iii. 226.
sete (S. siot, set, *n.*), ii. 155 f: iii. 36 f, 99 f, 100 f, 183 f.
shape (S. sceap, *n.*), iii. 28.
shippe (S. scip, *n.*), iii. 295, 341.
shotte (S. scot, *n.*), i. 234.

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hie (as if from a S. hig, *haste*), ii. 9 f: iii. 113 f.
kepe (S.?), i. 9 f, 56, 120 f: ii. 58 f, 93 f, 130 f.
lette (S. ?), ii. 88 f, 249 f: iii. 11 f, 263 f.
leve (S. leôf, adj.), i. 343.

* The adverb should be spelt *hom* (S. hâm), as at ii. 298, and not *home*, as at i. 74, 92, 147, 256, etc. *At hom* is also the correct form (S. at hâm).

shrifte (S. scrift, *m.*), i. 66, 158, 277, 372, etc.
sithe (S. sīð, *m.*), i. 160.
slepe (S. slæp, *m.*), i. 81 f.
smoke (S. smêc, *m.*, also smoca, I. 2), i. 211 f.
sore (S. sâr, *n.*), i. 310 f.
sothe (S. sôð, *n.*), i. 31, 32, 42, 86, 91, 114, 169, etc., etc., etc.
stronde (S. strand, *m.*), i. 185, 212 f: ii. 333: iii. 321.
temple (S. tempel, *n.*), ii. 157, 171, 384, 386: iii. 16, 337.
thewe (þeâw, *m.*), iii. 5 f.
thinge (þing, *n.*), ii. 207, 251: nothingë, 337.
towne (S. tûn, *m.*), i. 205: ii. 293.
wawe (S. wæg, *m.*), ii. 105 f: iii. 230 f.
wedde (S. wedd, *n.*), i. 249: ii. 98 f.
weie, waie (S. weg, *m.*), i. 29, 59, 62, 241, 325: ii. 170, 331: iii. 57, 328.
weighte (S. wiht, *n.*), ii. 276 f.
whippe (S. hweop, *m.*), i. 283 f.
wisdome (S. wisdôm, *m.*), iii. 217.
wive (S. wîf, *n.*), ii. 217 f.
worde (S. word, *n.*), iii. 256.
worthe (S. weorð, *n.*), i. 25 f.
wronge (S. wrang, wrong, *m.* ?), ii. 324 f.
yere (S. year, *n.*), i. 53 f, 160 f, 167 f, 239 f: ii. 169 f, 303 f: (all, yerĕ to yerë).

Most, if not all, of the above, and many other nouns of the same declensions, are found in the primitive form, *without* the vowel: e. g., bed, i. 72, 74: child, i. 201, 289: deth, i. 95, 133, 150, 165: flessh, ii. 342: hew(e), i. 137: hous, ii. 312: king, i. 113, 114: ii. 71, 75, 132: lif(e), i. 143, 144, 157: lond, i. 212, 251: mouth, ii. 15, 137: ship, i. 77, 160, 314: thing, i. 62, 68, 214: town, i. 78, 209: wif, wif(e), i. 114, 128, 185, 200: ii. 269: word, i. 76, 103, 150, 296, etc., etc., etc.

In many instances the terminal **e** might be explained as the Saxon dative inflection, but it will be found on inspection that about half of the nouns in the list occur in the nominative or accusative case.

sho (S. scoh, sceo, *m.*), i. 15: iii. 236, is a contract. fo, ii. 116, 311, comes from S. *adj.* fâh, fâ.

§ 15. The following merely drop a final **n**.

swoune (as if from a S. swûn), i. 204, 268: ii. 321.
were (as if from a S. wer), iii. 253 f.
eve (S. æfen, *m.*), i. 70 f: ii. 332 f, 385 f: iii. 64 f.
game (S. gamen, *n.*), i. 94 f, 281 f: ii. 143, 249 f, 281, 338: iii. 298.
maide (S. mægden, mæden, *n.*), i. 154: ii. 113, 228 f, 234 f, 256: iii. 325.
maiden occurs, i. 154: ii. 280, 281, 334, 344.

§ 16. Feminines of the Second Declension. (II. 3.)

These nouns have in Anglo-Saxon all the oblique cases of the singular in **e**.

banke (S. banc), i. 164, 183: see bench, § 17.
bene (S. beân), ii. 275 f.
berthe, birthe (S. beorð), ii. 76, 155.
blisse (S. bliss), i. 101 f: ii. 145 f, 249 f, 328 f: iii. 276, 338 f, 349 f.
bonde (S. bend, also *m.*), i. 102 f.
bone (S. bên), i. 185 f: iii. 223 (3 cases).
bote (S. bôt), i. 228 f, 235 f, 290, 355 f: ii. 118, 386 f: iii. 11.
brigge (S. brycg), ii. 201.
cheste (S. ceâst = *lis*, f ?), i. 294, 295, 296, 299.
dede (S. dæd), i. 272, 316: ii. 129 f, 280 f: iii. 306.
drede (S. dræd), i. 139: ii. 120, 129 f, 169: iii. 213, 348 f.
egge (S. ecg), ii. 251.
fille, felle, fulle (S. fyll), i. 254 f, 367 f: ii. 98 f, 136 f, 317 f: iii. 51, 209 f.
filthe (S. fylð), i. 174.
for -gifte (see yifte), iii. 372 f.
glede (S. glêd, also *m.*), i. 280 f.
glove (S. glôf), i. 351 f: ii. 370 f.
halle (S. heall), ii. 205 f, 255 f: iii. 35 f, 74 f, 75, 299 f.
halfe, halve (S. healf), i. 8 f, 17 f, 194, 253 f: ii. 295; *on other half* occurs as an adverbial phrase, i. 77.
hede (S. hŷd), i. 82 f, 194, 364: ii. 96, 99, 193, 207: iii. 9, 323, etc., etc.
hele (S. hêl, hêla), i. 17 f: ii. 210 f: iii. 347 f ?.
helle (S. hell), ii. 119 f, 128 f, 139 (3 cases).

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arist (S. ârist), i. 320 f, should certainly be ariste.
fiste (S. fŷst), i. 175, regularly: but see *Obs. on Chaucer*, § 17.
flight (S. flyht), ii. 37 f, should doubtless be flighte.
ight (S. æht), ii. 378 f, should probably be ighte.

helpe (S. help), i. 236: ii. 36, 162 f, 208 f: iii. f, 214 f, 297 f, 350 (2 cases), 377 f.
  help (helpĕ) i. 30, 204, 331: ii. 22: iii. 215 (2 cases), 224, 367.
heste (S. hæs), i 85: ii. 111 f, 167 f, 219 f, 243: iii. 221, 302, 352 f.
hinde (S. hind), ii. 45 f, 68.
hire (S. hŷr), iii. 352 f.
keie (S. cæg), ii. 188: iii. 9 f, 11, 369 f.
kinde (S. cynd), i. 265 (2 cases): ii. 18, 229, 292: iii. 12 f, 40, etc.
kiste (S. cist), iii. 316 f: kist[e], ii. 130 f.
kithe (S. cŷð, *patria*), iii. 71.
lengthe (S. lengð), ii. 110.
leve (S. leaf), i. 244 f: ii. 54, 96, 256, 314: iii. 75, 372, etc.
linde (S. lind), ii. 46 f.
lore (S. lâr), ii. 81: iii. 302: lere (S. laer), i. 93; iii. 285 f.
marche (S. mearc), i. 245, 247, 358: ii. 160.
mede (S. mêd), i. 333 f: ii. 165 f, 187 f, 191: iii. 298 f.
merthe, mirthe (S. mehrð), i. 127: ii. 107: iii. 6, 301, 339.
mile (S. mîl), ii. 24 f, 194 f: iii. 352 f.
nede (S. neâd), i. 221 f: ii. 190 f, 292: iii. 213 f, 278, 350.
-nesse (termination, S. nes, nis):
  besinesse, ii. 11, 60.
  buxomnesse, i. 87.
  halinesse, ii. 374 f.
  idelnesse, ii. 41.
  rightwisnesse, i. 7: iii. 176, 213.
  sik(e)nesse, i. 65, 128: ii. 324: iii. 326.
  sikernesse, i. 105 f: ii. 294.
  werinesse, iii. 195 f.
  wildernesse, iii. 193.
  witnesse, ii. 223, 225, 234.
ore (S. âr, *remus*), iii. 322 f, 346 f.
quene (S. cwên), i. 46, 106 f, 343: ii. 4, 5, 26, 46, 74: iii. 62, 70, 72, 74, etc., etc. (27 cases).
  quenĕ, ii. 212: iii. 338.

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ladder (S. hlaeder), iii. 330: qy. laddre?
lefte (S. lyft), i. 276 f?
liver (S. lifer), rh. deliver, iii. 100. The *r* seems to have been transposed, *deliver* being always so spelt, though from O. French *delivre*.
nedel (S. nædl), iii. 20, 49, 293, should perhaps be nedle.

reste (S. rest), i. 75 f, 96 f, 130 f, 336: ii. 375: iii. 181 f, 184 f, 210 f, 239, 384 f: iii. rest (doubtful), 323 f, 342 f.
rewe, rowe (S. râw), i. 50, 225: ii. 2 f, 70 f, 362 f, 370 f: iii. 45 f, 117 f, 301 f, 308 f.
rinde (S. rind), i. 152 f: ii. 138 f.
rode (S. râd), i. 100 f.
roode (S. rôd), i. 198.
salve (S. sealf), i. 8 f, 253 f, 339: iii. 352.
score (S. scor), i. 176: iii. 31 f.
shelle, shale (S. scell), ii. 20 f: iii. 76, 105, 109 f.
sighte (S. sihð), i. 262 f: ii. 360: iii. 8, 108, 276, 377:
  sight, ii. 243 f, 321 f (?), 371 f (?), 373 f (?).
sinne (S. synn), i. 356, 365 (2 cases): ii. 116, 164 f, 185: iii. 345.
sleve (S. slêf), ii. 213 f, 391 f.
slouthe (S. slewð), i. 372: ii. 3, 9, 24, 30.
sonde (S. sand), i. 212 f: ii. 342 f: iii. 156 f, 185 f, 337 f.
sorwe (S. sorh), i. 99 (2 cases), 116: ii. 31, 99 f, 116: iii. 31, 311 (2 cases).
soule (S. sâwel), i. 203, 256: ii. 58, 77: iii. 34 (2 cases), 39.
spanne (S. spann), i. 79 f.
speche (S. spæc), ii. 31 f, 68 f, 167, 255 f, 299: iii. 305.
stempne (S. stemn, stefn), i. 312: (see steven, § 17.)
stounde (S. stund), i. 90 f: ii. 231 f, 345 f: iii. 26 f.
strete (S. stræt), ii. 194 f, 211 f: iii. 63 f, 162 f.
strengthe (S. strengð, also -u), i. 29, 42, 240, 291, 347: ii. 381: iii. 55.
thefte (S. þeôfð), ii. 159 f, 331, 362, 370.
throwe (S. þrag, also -u), i. 320 f: ii. 23 f, 29 f, 168 f: iii. 31 f.
tilthe (S. tilð), ii. 168, 271.

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routhe (as if from a S. hreôwð; Icel. hrygð), ii. 43 f, 308 f: iii. 295, 362 f.
sherte (as if from a S. sceort, scyrt; Icel. skyrta), i. 115 f, 234 f, 236 f: ii. 246.
slaught, in manslaught (S. man-sleaht), i. 364 f, should be slaughte.
sleighte (Icel. slœgð), i. 68, 129, 238: ii. 63, 198, 200, 255, 280: iii. 267.
stelthe (as if from a S. stelð), ii. 349, 351, 352, 353.
welthe (as if from a S. welð), i. 39 f.

tide (S. tîd), i. 326 f: ii. 3 f, 143 f, 250 f.
warde (S. weard), iii. 55 f.
wede (S. wæd), i. 221 f.
wene (S. wên, also wêna, I. 2), ii. 88 f.
while (S. hwîl), i. 282: ii. 54, 79, 104, 111: iii. 3, 348.
wombe (S. wamb, womb), ii. 94, 169, 337: iii. 123, 124.
wounde (S. wund), i. 90 f, 289 f: ii. 245 f, 328 f: iii. 352.
wrathe (S. wræð), i. 280, 290 (2 cases), 293, 307 (2 cases).
wreche (S. wræc, also -u), i. 179, 351 f: ii. 123, 140 f, 323 f, 374 f: iii. 196 f, 352 f.
wulle, wolle (S. wull), i. 17: ii. 83, 98 f, 129.
yerde (S. gerd, geard), i. 291: iii. 249 f.
yifte, yefte (S. gift), i. 276 f, 323: ii. 284, 285, 382: iii. 156 (2 cases).
youthe (S. geôgôð), i. 99: ii. 55, 267 f: iii. 303, 356, 358, 364.

sib-rede (S. sibræden), iii. 284 f, merely drops the final **n**, like the nouns in § 15. So, apparently, met-rede, iii. 68, 69.

§ 17. Exceptions to § 16.

axel (S. eaxl), i. 320. (?)
bench (S. benc), ii. 274: but see banke, § 16.
bridĕ (S. brŷd), i. 102.
flight (S. flyht), ii. 327, rh. night. (?)
flor(e) (S. flôr), ii. 326 (rh. swor(e)): iii. 337: ii. 266 (written floure).
hen (henn), ii. 264 (rh. men).
hond (S. hand, hond), i. 41, 59, 151 (rh. fond): ii. 95, 238, 333. 360.

more frequently honde: i. 5 f, 10 f, 42, 43, 94, 113, 151 f, 290: ii. 62, 154, 360 f, 372 f: iii. 276.

les (S. liss, comfort), iii. 379 f. (?)
might (S. miht, meaht), i. 56 f, 68, 210 f, 211, 291: ii. 187, 239, 253, 306.
milk (S. milc), ii. 262.
night (S. niht), i. 42: ii. 15, 97, 102 f, 143 f, 145 f, 258.

by night[e], i. 249: by nighte, ii. 9, 352: iii. 255 (?).

plitĕ (S. pliht). This word is always a monosyllable, but is continually spelt with a final e, as are also (wrongly) most of the words rhymed with it; e. g., appetite, spirite, parfite: i. 129 f, 259 f: ii. 120 f, 128 f, 136 f, 147 f, 195 f, 197 f, 265 f: iii. 172 f, 234, 296 f, 318, 362 f.
sped(ĕ) (S. spêd), i. 88, 186: ii. 395.

spedë, i. 90 f, 346 f (?): ii. 117 f.

tow (S. tow), ii. 315.
wight (S. wiht), ii. 149 f, 237 f, 309 f: iii. 63.
world (S. weorold), i. 5: ii. 20, 52, 77, 93, 116, 197: iii. 76, 305.

But, the worldë fell so thilke tide, i. 245.
into this worldë only that, iii. 286. (?)

[*hand*, *might*, *night*, *wight*, are exceptional in Anglo-Saxon, having the accusative singular like the nominative: so *world*, more commonly: *bok* (constantly misspelt boke), i. 2, 5: ii. 58: iii. 65, 133, etc.; *burgh*, ii. 232: iii. 292; *furgh*, ii. 245, all feminines, are also irregular in Saxon, and have the accusative singular like the nominative.]

Nouns derived from Saxon feminines in -ung, -ing, or formed in imitation of such, generally have in Gower the termination -ìnge, less frequently -ing: in the latter case the accent is sometimes thrown back.

axìnge, i. 171.
bakbitìnge, i. 213 f.
carolìnge, ii. 53 f.
childìnge, iii. 211.
comìnge, ii. 29 f, 53 f.
compleignìnge, i. 327 f.
grucchìnge, i. 234.
knoulechìnge, i. 123 f: ii. 25 f: iii. 34 f.
lesìnge, i. 65 f, 213 f.
likìnge, i. 58 f, 173 f.
lokìnge, i. 65 f.
mishandlìnge, ii. 189.
spekìnge, iii. 252.
tidìnge, i. 327: ii. 243 f, 385.
welwillìnge, i. 355 f.

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wierd (S. wyrd, II. 3), i. 340, should certainly be wierde, and hierd (S. hyrde, II. 2) hierde.
wente (S. ?), iii. 161 f, and elsewhere.

steven (S. stefn), i. 144, 195; ii. 30, 253, 326; iii. 30; in all these cases rhymed with heven, and therefore uncertain.

wepìnge, ii. 122.
writìnge, i. 4 : iii. 110.

beginnìng (rh. spring), iii. 104.
knoulechìng, i. 32 f.
techìng, i. 95 f.

hùnting, i. 53.
lìking, iii. 319.
wèning, i. 107, 108.
wrìting(*e*), i. 5.

excusìng (of), i. 107,
huntìng (as), i. 53,
shedìng (of), i. 316, 364,
are apparently cases of elision.

§ 18. The following nouns, of etymons more or less uncertain, but mostly of undoubted Gothic origin, are found in Gower terminating in ë.

babe (*O. Swed.* babe, *Ger.* bube ?), i. 344.
bothe (*Ger.* bude, *Dut.* boede), iii. 281 f.
brinke (*Icel.* bringr, *colliculus*), i. 119 f, 142 f, 326 f: ii. 309 f.
bulle, bolle (*Icel.* boli, bauli, *Ger.* bulle, *Sax.* bulluca), iii. 118 : ii. 72 f. (?)
cake (*Swed.* kaka, *Dan.* kage), iii. 216 f.
chaffare (S. ceâp-fær, -faru ?), ii. 278 f, 332 f.
clowde, iii. 310 f.
creple (*Icel.* kryppill, *Dutch* krepel, *Ger.* krüppel), iii. 147.
deìnte, ii. 255 : iii. 27.
felàwe (*Icel.* fèlagi), 30 f, 84 f, 144 f, 239 f, 279 f: ii. 175 f, 393 f.
  felàw, i. 170, 171 : felèw, i. 225.
  fèlaw, i. 171 (3 cases) : ii. 208 (2 cases) : fèlowe, i. 227.
funke (*Ger.* funke), iii. 18 f.
gesse, guesse (*Dutch* gissen, *Swed.* gissa, to guess), iii. 211 f : i. 105 f.
mone (*O. Fris.* mêne, *O. H. Ger.* meina), i. 97 f : iii. 285 f, 333 f.
packe (*Dan.* pakke, *Swed.* packa, *Ger.* pack), ii. 312 f, 393 f.
rote (*Icel.* rôt, *Goth.* vaurts, *Sax.* wrôt, inc. gen.), i. 7 f, 46 f, 138, 140 : ii. 261 f.
sculle (*O. H. Ger.* sciulla, *Sax.* scell ?), i. 128 f: but scull i. 127.
snowte (*Dan.* snude, *Swed.* snyte, *L. Ger.* snûte), i. 283 f.
tacle (*Ger.* takel, *Dan.* takkel, *Swed.* tackel), i. 312.
were (S. worry), i. 107 f, 318 f.
wicke, — "pride is the worste of all[e] wicke," i. 154, 176.
  wìndow occurs, ii. 347.

§ 19. The final e of nouns (and other words) of French origin forms a syllable in Gower, as in French verse. Exceptions are by no means so common as in (Wright's text of) the Canterbury Tales.

abbesse, iii. 337.
adventure, ii. 236.
avarice, ii. 127, 131, 284, 289.
baptisme, i. 276.
beste, i. 280, 316, 326 : iii. 93, 206.
borde (*O. Fr.* bourde), i. 304 f.
bounde, bonde (*M. Lat.* bunda, *O. Fr.* bonde), iii. 102 f, 104 f, 110 f.
bowele (*O. Fr.* boele), ii. 265 f.
chere, i. 55, 66.
Constance, i. 185, 186, 187, 188, 200.
defaulte, ii. 206.
deserte, ii. 391.
egle, iii. 105.
entente, i. 101.
envie, i. 223, 224.
feste, i. 182 : iii. 30, 327.
fortune, i. 22 (4 cases), 87, 88, 90, 247 : ii. 187, 234 : iii. 55, 201.
grace, i. 9, 208 : ii. 25, 302, 303.
haste, i. 252, 273, 335, 337.
homicide, i. 316 (2 cases).
houre, ii. 9 f, 10, 23 f, 34 f.
joie, i. 208.
justice, iii. 201.
madame, iii. 300, 301, 315.
magique, iii. 128.
manere, iii. 5, 141.
mappëmounde, iii. 102 f.
marriage, i. 101 f : ii. 222 : iii. 265.
matere, i. 43 f, 146 f, 343, 365 : ii. 207, 383 : iii. 157.
medicine, ii. 265.
merveille, i. 327 : ii. 236.
message, i. 288.
mewe (*Fr.* mue), i. 326 f.
mule, i. 210.
multitude, ii. 201.
nature, ii. 17.
navie, i. 197.
offrende, i. 73 f.
oile, iii. 168.

pacience, i. 302, 316.
passàge, i. 223 : ii. 65.
persòne, ii. 202.
pestilence, ii. 346.
phisique, i. 265.
place, i. 93, 151, 154, 164, 331, 332, 351.
pompe, iii. 382.
Rome, i. 282 f : ii. 195 (2 cases), 196 (3 cases), etc.
spume, ii. 265.
vice, i. 157, 167 : ii. 87.
virgìne, ii. 186.
ymage, i. 34 : ii. 178, 179.

A few exceptions, after the sounds **r** and **s**, are cited under § 84 and § 91, *f*.

So in adjectives : —

chaste, iii. 234, 244 f.
double, i. 181 : iii. 187.
hughe (*Fr.* ahuge), i. 236.
invisìble, ii. 247.
nice (= foolish), ii. 22, 285 : iii. 180, 222 f.
riche, iii. 228, 249, 327.
solempne, iii. 327, etc., etc., etc.

§ 20. THE GENITIVE CASE, Singular, ends in **ës**.

lovës, iii. 85, 98.	nightës, iii. 96, 217, 290.
mannës, iii. 86, 93, 147.	daiës, iii. 111, 113.
goddës, iii. 88, 156, 175.	bullës, iii. 119.
worldes, iii. 90, 144, 149.	kingës, iii. 146, 147, 151.
wivës, iii. 73.	

The following have, at least sometimes, no termination.

Dec. I. *the chirche keie, i. 10.
*mone light, iii. 109. But, the mones cercle, iii. 109.

my lady side, i. 160.	ladies lovers, i. 228.
this lady name, ii. 157.	" hest, i. 84.
my lady chere, ii. 213.	" selve, i. 228.
my lady kith[e], iii. 5.	" doughter, ii. 227.
my lady good, iii. 30.	" mercy, ii. 118.

So, the horse (II. 1) side, i. 40, 119.
*horse halters, ii. 47.
*horse knave, ii. 48.
horse hed, ii. 49.
heven (II. 2) mede, ii. 187 : heven cope, iii. 102.
heven king, iii. 228, etc., etc.
the helle (II. 3) king, ii. 97, 165.
his soule (II. 3) helthe, i. 39.
soule hele, i. 29, 371.

So, fader, brother, mother, doughter (as in Saxon).

fader halve, i. 209.	But, faders lifĕ, i. 157.
	faders day, i. 214.
brother doughter, i. 199.	brothers herte, i. 214.
the mother barm(ĕ), i. 289.	moders discipline, ii. 354.
his moder breast, i. 348.	
moder side, i. 352.	
his moder sake, ii. 229.	
doughter sake, i. 208.	doughters speche, i. 150.

So, many proper nouns in -s, as in Anglo-Saxon and Modern English.
Polyphemus love, i. 166.
Bachus wode, ii. 358.
Phebus temple, iii. 250, etc.

§ 21. THE PLURAL OF NOUNS. Nominative.

The Nominative Plural is formed for the most part in **ës**.

to berës, tigrës, apës, oulës, iii. 50.

weiës, iii. 102.	droppes, iii. 94.	notes, iii. 90.
tiranniës, iii. 207.	herbes, iii. 161.	frostes, iii. 94.
thewës, iii. 85.	leves,† i. 53 : iii. 120.	bestes, iii. 101.
soules, iii. 87.	lives,† iii. 278.	flodes, iii. 104.
hilles, iii. 93, 102.	wives,† iii. 278 : i. 209.	cloudes, iii. 93, 94.
formes, iii. 87.	turves, ii. 262, 283.	hevedes (= heads), iii. 249.
philosòphres, iii. 124, 135.	bokes, iii. 87, 91, 160.	monthes, iii. 118.
fires, iii. 96.	clerkes, iii. 88, 96.	mouthes, iii. 174.
lores, iii. 85.	beinges, iii. 87.	
sterres, iii. 91, 106.	thinges, iii. 88, 91, 146.	

* Some of these, perhaps, should be regarded as compound nouns.

† *Sing.* lef, i. 186 : lif(e), i. 143 : wif, ii. 289. The v in the plural may be a case of Pauli's "restoration of the orthography."

-s only is frequently added, especially to nouns terminating in a liquid or in -t: sometimes when **-es** is added (rightly or wrongly), only -s is pronounced.

aungels, iii. 275, 276.	Sarazins, iii. 216.	lovers, i. 88: iii. 110, 36.
cardinals, i. 254.	complexions, iii. 97.	flatrours, iii. 159.
nations, iii. 278.	masons, iii. 167.	fethers, ii. 335.
courts, iii. 354.	saints, iii. 188.	words, } i. 176.
points, } i. 149, 157, 372 (3 cases).	estat(e)s, iii. 381.	wordës, } i. 151.
pointës, } i. 151.	craftës, iii. 142.	Grekës, ii. 171 (2 cases), 172 (2 cases).
elements, iii. 92 (3 cases), 97.	clïmats, iii. 106.	Grekës, ii. 165, 171.
jugements, iii. 106.	herts, } i. 325.	knes, kneës, iii. 59: i. 49.
arguments, iii. 146.	hertës, }	tres, treës, i. 35.
tiraunts, iii. 206.		

§ 22. The following have **-en**, **-n**, derived from the Saxon plural in **-an** of the First Declension.

eyen, eien (S. eâgan), i. 68: ii. 29, 113, 114: iii. 370, etc.
oxen (S. oxan), ii. 244, 245, 248: but oxes (rh. foxes), ii. 63.
ton (S. tân), i. 33.

§ 23. The following, which have the termination **-u** in Saxon, have superadded the **-en** of the First Declension to a weakened form of the Saxon plural.

{ brethren (S. brôthru), ii. 154: iii. 60, 247.
{ bretheren, iii. 38.
{ brethern, ii. 154: iii. 102, 360.
bretherne, iii. 37, should be brethern.
children (S. cildru), i. 148: ii. 155.
doughteren, sistren, do not occur.
doughteres, i. 148: ii. 378: doughter (?), ii. 172.
susters, ii. 324, 329.

§ 24. The following have *no* termination in the plural, according to the rule of Saxon neuters of the Second Declension.

hors (S. hors), i. 347, 349 f: ii. 35 (2 cases), 45, 110, 161, 199: iii. 113, 162, 204.
thing (S. þing), i. 27 f, 224: thinge, iii. 333 f, should be thing.
yer, wrongly spelt yere (S. gear, gêr), i. 148, 183: iii. 326.

and whan thre yer(e) ben full despended, i. 197.
yerës five, thre yer(e) are found together, i. 183.
full many yerës, iii. 356.

[good is wrongly classed with plurals in *Obs. on Chaucer*, § 25. Cf. Gower, i. 183 (2 cases), 251: ii. 78 f, 287 f: goodës, i. 147: goodës, ii. 332.

for other mennes good *is* sweete, ii. 332.
the worldes good *was* first comune, iii. 152.
We find, to be sure, i. 155,
what worldes *good* that thou wolt crave
are of my yift, and thou shalt have:
but *are* should probably be *ask*.]

For "a thousand winter," etc. (i. 80), see further on, § 100 *c*.

§ 25. The plurals formed by change of vowel are the same in Gower as in Modern English.

[among the *menne* shoulde dwelle, i. 366, should read among the *men ne* shulde dwelle.]

§ 26. The Genitive Plural in Gower is the same as in Modern English, saving, of course, the use of **ës** instead of **s**.

the Grekes lawe, ii. 173.	the Grekes feith, ii. 176.
alle mennes speche, ii. 173.	alle mennes lok(e), iii. 179.
mennes goodes, ii. 332.	princes hevedes, iii. 249.
out of all other briddes sighte, i. 100.	of the goddes purveiaunce, i. 119.

ADJECTIVES.

§ 27. Adjectives which end in a vowel in Saxon, end in ĕ in Gower.

blithe (S. bliþe), ii. 333 f: iii. 306 f.
a-cale (S. â-câle), iii. 296 f, 303 f.
clene (S. clæne), ii. 126, 187, 336 (2 cases), 354.
dere (S. deôre), ii. 249 f: iii. 359 f.
derne (S. derne), i. 107 f.
drie (S. dryge, dry), i. 234: ii. 266: iii. 3, 10.
fre (S. freo, fri), i. 107 f.
grene (S. grêne), ii. 30, 82 f: iii. 131 f, 158 f, 352 f, 357.
kinde, unkinde, } (S. cynde), ii. 85, 145 f, 149, 291, 292.
mete, unmete, } (S. mæte), i. 163: ii. 166 f, 199 f, 332 f: iii. 42 f, 260 f.
milde, unmilde, } (S. milde, mild), i. 84 f, 195: iii. 65 f, 309 f.
neisshe (S. hnesce, nesc), ii. 284 f.
newe (S. niwe), i. 3: ii. 20, 188, 270: iii. 9.
softe (S. sôfte, sôft), i. 71: ii. 40 f, 47 f, 328 f: iii. 351 f, 365 (2 cases).
sterne (S. sterne), i. 113, 361 f: iii. 289.
stille (S. stille), i. 321 f: ii. 6, 102, 133: iii. 28, 350 f, 352 f.
swete (S. swête), ii. 107 f, 332 f, 353 f: iii. 8, 110 f, 366 f.
thicke (S. þicce), iii. 90.
thinne (S. þynne), i. 102 f.
trewe, untrewe, } (S. treôwe), ii. 88 f, 223, 224 f, 281 f, 282, 332.
un-wylde (S. un-vylde = impotens), i. 312 f: iii. 147 f.
yare (S. gearu), ii. 237 f, 315 f: iii. 335 f.
fore (i. 312, the fore stempne) is the prefix fore.

all- (al-) onë, is from the S. definite form âna = solus: ii. 45, 99, 111, 293 (and thus *alonë* there he lay), 360: iii. 72 f, 141 f, 231, 329.

So onë, in at least one case: and rather shall an *onë* man, iii. 231: so perhaps, iii. 213, for he stant onë for hem alle.

§ 28. The following adjectives and adjective pronouns, though ending in a consonant in Saxon, have sometimes, or always, the termination ĕ in Gower, resembling the nouns in § 13.

a. alle (S. eall, all), i. 6, 8 (for *allë* reson woldë this), 36, 38, 84: ii. 17, 88, 288 (2 cases), etc.
bare (S. bær), ii. 12 f, 40 f, 181 f, 260 f, 286 (than wolde a *barë* straw amount): iii. 371 f.
bleche (S. blâc), ii. 210 (some on(e) for she is pale and *blechë*).
blinde (S. blind), i. 8, 185 (a *blindë* man, which cam ther lad(de).
brode (S. brâd), ii. 107 (ful oft whan it is *brodë* day).
faire (S. fæger), ii. 253 (that Jason was a *fairë* knight).
false (S. fals), ii. 329 (and that was *falsë* Tereus).
gladde (S. glæd), i. 211 (he goth a *gladdë* lif(e) to livë), 47 f.
grete (S. greât), i. 251: ii. 345 (so *greatë* lust in her he had).
leve (S. leôf), ii. 324 (do Tereus no morë grevë than slee his child, which was so *levë*).
lewde (S. læwed), iii. 2 (of him that is a *lewdë* man), 6, 147, 343.
like, liche (S. -lîc), i. 25 f, 261 f, 268 f: ii. 124 f, 379 (so that he was of children richë, so therof was no man him *lichë*).
longe (S. lang, long), ii. 251 (for *longë* time it so befell).
lowe (S. lah), i. 84 f, 124 f, 299 f: ii. 294 (an apë which at thilkë throwë, whan that the cordë cam down *lowë*).
olde (S. eald, ald), iii. 356 (that *oldë* grisel is no folë), 368.
one* (S. ân), ii. 255 (som(e) man said *onë* som(e) said other), iii. 213. (?)
everych-one, ii. 45 (and *everychonë* rid(e) on sidë).
righte (S. riht), iii. 129 (by *rightë* namë men it callë).

* The common forms are *on*, *o*. The misspelling *one* continually occurs in Pauli's text, as rh. John, i. 64: rh. Lamedon, ii. 375.
of *on(e)* semblaunce and of *o* make, ii. 204.
of *o* nature, of *on(e)* accord(e), ii. 77.
they knelen alle and with *on(e)* vois, ii. 206.
was *non* but Nestor hem answerde, i. 340.
non such time, ii. 10.
o occurs, ii. 202, 204, 209, 283, etc.

sharpe (S. scearp), ii. 82 (in mannes voise or softe or *sharpë* that found(e) Jubàl. And of the harpë).
stronge (S. strang), iii. 4 (and *strongë* Sampson overcome), 207.
suche (S. swylc), i. 319 (let hopë serve at *suchë* nedë), ii. 15, 94, 280.
tame, } (S. tam), i. 144 (anon(e) as he was
un-tame, } humble and *tamë* he found toward his god the samë), 287 f: ii. 350.
thilke (S. þylc), i. 2 (for thilkë cause if that ye rede), 9, etc.
whiche (S. hwylc), ii. 177, 395 (of timë *whichë* thou hast spended), i. 135.
suche, thilke, whiche, might all have been placed under *like* above.
wilde (S. wild), i. 236, 290 (som *wildë* placë that it werë), ii. 180, 200, 264: iii. 256, etc.
wise (S. wis), i. 156 (his daughter *wisë* Petronelle).

So moste: (wher(e) thou hast *mostë* knowleching), i. 92, 112.

[But most or all of the above are found also in the older form, without the **-e**: *e. g.*, al, i. 35, 42; fals, i. 70, 223, 231: iii. 23, 159; fair, i. 345; glad, i. 70, 87, 134, 167; great, i. 70, 229, 237, 360; lich, i. 33: ii. 21, 37, 149; strong, i. 28, 221; wis(e), iii. 216 f, 226 f; unwis, ii. 174 f, etc., etc.

b. It will be observed that the foregoing adjectives are all from monosyllabic Saxon stems, or from contracted dissyllables. A few *polysyllabic* adjectives are also found in Gower with the termination ē. See, also, § 36, *c, d;* § 42.

in womannishë vois they singë, i. 58.
wherof in womannishë dredë, i. 72.
for fere of womanisshë shamë, iii. 304.
whan bodelichë thurst him hent, iii. 14.
of allë womannischë gracë, iii. 338.

So, diverse (Fr. divers):
they worchen by diversë way, ii. 85, 77, 125.
but that there is diversë kindë, iii. 12.
and sheweth, as I shall rehercë,
how she was to this lord diversë, iii. 295.
divers, i. 356 f: iii. 3 f, 384 f.
comune (Fr. commun):
the worldës good was first comùnë,
but afterward upon fortunë, iii. 152, 159:
but còmun, i. 216, 284: ii. 76, 143, 156, 202, etc.
devoute (Fr. dévot):
among his bedes most devoutë
goth in the worldës cause aboutë, i. 64.

So, secoundë, i. 159: ii. 84 f, 198, but the form *seconde* is found in Old French.

So, as if by dropping the final consonant,
golde (S. golden), ii. 356 f.
lite (S. lytel), i. 18: ii. 89 f, 391 f: iii. 23 f, 221 f, etc.
moche (S. micel, mucel), iii. 81.
mochel occurs, ii. 384, 386: iii. 25, 81.

§ 31. The following adjectives, of uncertain derivation, are found terminating in ē.

badde, ii. 47 f, 48 f.
dronkelewe, iii. 5 f.
meke (Goth. muks, Icel. mjúkr), ii. 210 f.
wikke, i. 295, 306.

§ 32. The Definite Form of *monosyllabic* Adjectives, including Participles and Adjective Pronouns (i. e., the Adjective when preceded by the Definite Article, by any other demonstrative, or by a Possessive Pronoun), ends in Gower in ē.

the wise man, i. 5.
the blinde world, i. 67.
thilkë borë freë kindë, i. 68.
this foule greate coise, i. 100.
my faire maide, i. 154.
the ferre leve, i. 343.
this ilke point(e), iii. 101.
thilke same ring, i. 247: ii. 14.
the dede lady, i. 248.
the whiche, i. 318.
the longe day, ii. 14.
her dreinte lord(e), ii. 105.
thy fulle mind, ii. 126.
the sharpe swerd, ii. 128.
the leude folk, ii. 158.
the righte feith, ii. 173.
min holë herte, ii. 277.
the brente wall, ii. 375.
the highe god, ii. 262, 384: iii. 61, 148, 168, etc.
that highe tour(e), that stronge place, ii. 376.

the smale path, the large strete, ii. 194.
her yelwe hair, iii. 255.
this yongë, fairë, freshë may, iii. 302.
the thridde, ii. 85: iii. 119, 129.
the fourthe, ferthe, } ii. 182 f: iii. 120, 130, 364.
the fifte, i. 237: iii. 120, 130.
the sixte, sexte, } iii. 121 f, 130.
the ninthe, iii. 131.
the tenthe, iii. 124, 131.
the twelfthe, the twelfte, } iii. 126, 132.
his ownë lif(e), i. 9, 52 (but, uncontracted, his owen, iii. 317 f.)

So, in the derke, i. 190.
in the depe, i. 194: ii. 200.

§ 33. So, for the most part, the Definite Form of *monosyllabic* Superlatives.

her beste, i. 51, 151.
the beste, iii. 24.
the leste, ii. 151: iii. 229.
the firste, i. 23, 146: iii. 127, 240.
the laste, i. 23: iii. 133.
{ at the laste, i. 151.
{ ate laste, i. 16: ii. 345, 377.
the nexte, iii. 121 f.
the worste, i. 174.
my moste, ii. 34.
the moste, ii. 379.

§ 34. Among Definite Forms of the Adjective are to be reckoned adjectives occurring in forms of address (as in Anglo-Saxon, leôfa fader, etc.)

false (S. fals), "false cherl!" i. 321: ii. 317.
gode, goode (S. gôd), "ha, gode suster," ii. 48, 93, 320: i. 148.
foule (S. fûl), "thou foule beste," ii. 337.
leve (S. leôf), "leve sir," ii. 58: iii. 301, 317.
wise (S. wîs), "O wise Diogene," i. 323.
proude (S. prŷte, prût), "thou proude clerk(e)," i. 261.
highe (S. heâh), "O hihe fader," iii. 15, 196.
blinde (S. blind), "O blinde," iii. 168 f.

§ 35. The Definite Form of Adjectives of more than one syllable has not (generally) the final ĕ There are, however, more exceptions to this rule in Gower than in Chaucer.

a. Comparatives: (those of the "Irregular" form, § 38 *b*, are, of course excepted.)

the higher, iii. 15.
the better, i. 213: ii. 193: iii. 224.

As to Superlatives, we find, to be sure, the christenest, i. 213: but, on the other hand,

of women is the unsemliestë, i. 96.
the lothliestë what, i. 98.
the mightiest[ë] was to fightë, ii. 176.
or the fairèst[ë] love of alle, iii. 24 (?)

b. Past Participles.

like to the chaced wildë bore, iii. 268.
the descoloured palë hewë, iii. 339.
tho wide furred hodes, i. 63.

c. Adjectives with derived endings, as -full, -ly, -y.

this wofull, i. 195, 272: ii. 322: iii. 15, 354.
our ferful, ii. 189: iii. 94.
the rightfull, iii. 183.
the wrongfull, iii. 226, 268.
the mighty, iii. 275.
the lusty, iii. 242.
this worthy, iii. 259.
his hevy, i. 264.
the proudë tirannish Romain, iii. 246.

So, the bitter, iii. 350.
the siker, ii. 208, 392.
that other, ii. 198: iii. 36: [in the sense of *the second*, i. 220.]
this litel, i. 36, 219: iii. 217, 349.
her evil, ii. 207.
thy wicked, i. 179.

d. Various adjectives of Latin derivation.

his moral, iii. 140.
his real, iii. 167, 168, 338.
my finall, iii. 384.
the cruel, ii. 52, 305.
the subtil, iii. 92, 140.

the gentil, ii. 50.
the vertuous, iii. 129.
the pietous, iii. 201.
this feverous, ii. 147.
this curteis, ii. 132.
the pouer, ii. 392.
the jolif, iii. 4.
the comun,* i. 7, 9, 263: ii. 44, 387: iii. 142, 152, 178, 186, 213, 380.

§ 36. The following exceptions to § 32, § 33, § 35 occur.

a. To § 32.

his fals, i. 65, 309.	his high lignage, ii. 71. (?)
her wrong, i. 169.	the high prowesse, ii. 342. (?)
her glad, iii. 25.	his high suffraunce, iii. 376. (?)
the bright, iii. 113 f.	his sligh compas, i. 238. (?)
the ninth, iii. 123.	But, the highë god, ii. 127, 155, 215, etc.
the seventh, iii. 130(?)	his highë worthinesse, iii. 377.
	his slië caste, i, 239.

b. To 33.

the best, i. 67, 90.

c. To 35 *c*: but see § 30, *b*.

this tiranisshe knight, iii. 256.
her wommanishe drede, ii. 66, 229, 335: iii. 28.
thy bodeliche kinde, i. 271: (the bodeliche thinges, iii. 89).
the hevenliche might, i. 138.

d. To 35 *d*.

the covetouse flatery, iii. 158.
and thus this lecherous[ë] pride, iii. 259.
of the parfite medicine, ii. 89.
the secounde, seconde, } ii. 84 f: iii. 21, 23, 85 f, 89 f, 140, 199, etc.: but see § 30, *b*.

§ 37. The distinction of the French masculine and feminine adjective may *perhaps* be noticed in one or two cases.

which is the vertue *sovereinë*, my fader, I shall do my peinë, } i. 277.
wher(e) Paris stood with faire Heleinë, which was his joië *soverainë*, } iii. 360.
(cf. god soverein, ii. 52.)
O thou *gentilë* Venus, loves quene, iii. 352.
(cf. gentil love, iii. 354: gentil folk(e), iii. 358): but gentilë may be only a definite form; see § 34.

§ 38, *a.* The Comparative Degree of the Adjective is generally formed in Gower, as in modern English, in -er (S. -re); the Superlative in -est (S. -ost, -est).

fairer, i. 201.	gladder, ii. 307, iii. 51.
hoter, iii. 8.	lever, i. 242: ii. 145
higher, iii. 93.	(lef. ii. 153: levest,
stronger, iii. 229.	ii. 133).

lowest, swiftest, i. 65, etc.

b. A few Comparatives of "Irregular" Adjectives retain the Saxon ĕ.

worse, werse (S. wyrse), i. 5, 332, 366: ii. 202: iii. 159.
the worse, werse, i. 299 f, 334: ii. 380 f.
the werre, i. 9 f, 334 f: iii. 238.
lasse, lesse (S. læsse), i. 107 f: ii. 331 f: iii. 215.
the lasse, ii. 76 f, 120.
more (S. mâre), i. 76, 113, etc.
mo (plur.), ii. 70, 180: iii. 279, 364.

c. The vowel change of the "ancient" comparison is found in the following instance:—

strengest, iii. 55, 147, 151.

d. Some analytic forms of comparison are found.

morë fastë, i. 86: morë vertuos, i. 159: well morë whit(e), i. 306: morë low[e], ii. 156.

(So with the Adverb: wel morë hardë shaken, ii. 146.)

most pleasaunt, i. 92: most worth, i. 153.
most worthy, ii. 379: most derk(e), i. 224.
which was one of the most[ë] wise, ii. 231.

* Although the Indefinite form of this adjective, like the Definite, is còmun (Fr. *commun*), we find "the worldes good was first comùne" (*rh.* fortune), iii. 152, and "comùn of propreté," i. 239, which point to a full form comùnë. *The comùnë*, i. 20, 64, 221: iii. 187, 341, is *the commonalty*, Fr. *commune*: so possibly at ii. 127, where, however, the more likely meaning is *for common use* (Lat. *in commune*).

that he the most[ë] riche array, ii. 231.
most devoute, i. 64.
the most[ë] principall of alle, i. 52.
the mostë chef(e), iii. 11.

§ 39. The Plural of Monosyllabic Adjectives and Adjective Pronouns ends in ē. So, also, bothe, fele, fewe, and many of the Cardinal Numbers.

* *a.* now sharpe notes and now softe, now highe notes and now lowe, } iii. 90.

blinde, i. 73.
colde, ii. 237.
gladde, ii. 45.
grete, i. 53, 64, 140, 214.
harde, ii. 72.
i-nowe, i. 80 f, 268 f: iii. 198 f, 353: i-nough, sing, i. 80: iii. 156.
loude, i. 137, 170.
olde, i. 5.
save, ii. 50: iii. 175 f, 190 f.
shorte, i. 308.
smale, i. 80.
softe, i. 170 f.
sothe, ii. 99.
swifte, i. 119.
all the thoughtes, i. 66, al the waters, i. 38, should probably read *alle* thoughtes, etc.

b. bothe, i. 70, 170: ii. 17, 78, 271: bothe two, ii. 124.
fele, ii. 37 f, 207 f, 362 f: iii. 371 f.
fewe, i. 2: ii. 292: iii. 380: a fewe men, ii. 376.
some, i. 15, 84, 110 (5 cases): ii. 7 (3 cases): iii. 13 (3 cases).
som men, i. 21 (2 cases).
suche, i. 70, 73, 77, 100, 176, 365 (2 cases).
whiche, i. 55, 137, 362: ii. 103, 188: iii. 135.

c. tweine, tweie, i. 276 f: ii. 392 f.
two, iii. 195, 231.
thre, iii. 140, 145.
foure, i. 194: ii. 82 (2 cases): iii. 116, 120, 362.
five, i. 52 f, 61, 280 f: ii. 88 f: iii. 119.
eighte, iii. 122.
nine, i. 330: ii. 261 f, 265, 305 f.
twelve, i. 21 f, 177 f: ii. 25 f, 209: iii. 108 f, 118 (3 cases): twelvĕ (twelf?), ii. 68.
thrittene, iii. 132 f.
fourtene, i. 148: iii. 326.
fiftene, iii. 128 f.
sixtene, iii. 120.
eightëne, iii. 119 f: i. 102.
seven, iii. 116, 117 f, 121 f, 123 f, ten, i. 21, 177: ii. 349: iii. 120, elleven, iii. 124 f, } are undeclined.
twenty, iii. 126, 215: thritty, iii. 214, 290, etc.

§ 40. The plural of Adjectives and Participles of more than one syllable has no -e.

a. furred hodes, i. 63.
lered men, iii. 283. (No other cases observed.)

b. no dedly werres, iii. 222.
thes(e) dredfull, i. 56, 247: thes(e) wofull, ii. 323: wofull teres, iii. 260.
dolefull clothes, iii. 291.
other,* i. 106, 159, 223, 310: ii. 352, 391: iii. 156, 362, 363.
these other, i. 20, 63: al other, i. 64, 69.

c. hastif rodes, ii. 56.
certein sterres, iii. 128.
gentil hondes, ii. 281.

§ 41. Adjectives and Participles *standing in the predicate* sometimes take ë in the plural, sometimes are unvaried.

a. that be greatë, i. 5 f, 18 f, 30 f.
ben to smalë, i. 6 f.
ben un-warë, i. 17.
wittes be so blindë, i. 49 f.
to him were allë thinges couthë, i. 138.
whiche are derkë, i. 63.
they werë gladë, i. 79, 80 f, 182, 208.

---

* Many of these adjectives have, however, *sometimes* a *singular* form in -e. See § 30.

* We find another care = another's care, i. 167. Other is sometimes undeclined in A. S.

weren dedë, i. 76 f, 111: ii. 173 f, 201: iii. 198 f.
the gates werë shettë, i. 348.
we be saufë bothë two, i. 198.
hem that were him levë, i. 273 f.
briddes been made, ii. 80.
that him thoughte allë women lothë, i. 118.
havĕ be full oftë sithes wrothë, i. 52, 280.
they shull of reson ben answerdë, i. 51.

We have even: whan that thesĕ herbës ben *holsomë*, iii. 161.
in thingës that ben *naturelë*, iii. 133.
of hem that weren so *discretë*, iii. 167.

b. hem that ben so derk, i. 78.
we ben set, i. 317.
they be shet, ii. 10.
so ben my wittes overlad, ii. 21.
all men be left, i. 119.
hem that thanne weren good, i. 11, 4.
which only weren sauf by ship, i. 38.
the thre were eth to reule, i. 60.
they were cleped, ii. 165.
they ben laid, ii. 245.
they ben corrupt, ii. 153: so, 185, 192, etc.

§ 42. Exceptions.

To 40 *c.* of golde and *preciousë* stones, ii. 47.
his bedes most *devoutë*, i. 64 f.
divèrsë occurs, i. 56, 252, 256: ii. 154, 325: iii. 26, but is found also in the singular: see § 30, *b.*

§ 43. At least one adjective of French origin exhibits the French plural in s.

long time amongës the Romains,
til they becomë so *vilains*, i. 28: so, perhaps, iii. 188.

§ 44. The following combinations containing the Genitive Plural of *all* occur:—

altherwerst, i. 53: ii. 224: iii. 9: allthermest, i. 147, 224.
altherbest, i. 106: ii. 20: althertrewest, i. 176.

PRONOUNS.

(See, also, §§ 30; 32, *a*; 35, *c*; 39, *b*; 44.)

§ 45. Personal Pronouns and their Possessives.

I (S. ic), i. 2, etc.

min (S. mîn), min herte, i. 3, 215: ii. 277: iii. 5: min hond, i. 41: ii. 341: min holy, i. 66: ii. 212: min hole, ii. 277: min houndes, ii. 382: min hevinesse, iii. 351, etc.
min honour, ii. 310, 334.
min owne, ii. 323: iii. 5: min one, i. 45: min opinion, ii. 214: min entent, ii. 222: min ere, min eye, iii. 31, etc.
sonë min, i. 363: ii. 139.

my: my king, i. 3: my wit, my boke, i. 4: my minde, i. 6: my writinges, i. 41: my fortune, i. 45: my spekinges, i. 49: my trust, my lust, my lif(e), min helë, ii. 249.

absolute forms: half so well as min (rh. engin, Fr. engin), i. 225.
mo of thy servauntes than of min[ë] (rh. covine, Fr. covine), iii. 163.
for certes if she werë min[ë], I had her lever than a minë (Fr. mine), ii. 130.
malgre min, ii. 3, 374, Old French malgré mien = malgré moi.

me (S. me), dat., i. 45, 177: ii. 61, 368: iii. 268, 377.
me (S. me), acc., i. 2, etc.

thou (S. þû), i. 45, etc.
thin (S. þîn), thin herte, i. 60, 159, 335: ii. 279: thin happes, iii. 10: thin hering, i. 57: thin help, iii. 215, etc.
thin eye, i. 56, 61: thin ere, i. 61: thin ende, ii. 280: thin owne, ii. 323: thin informacion, iii. 145: thin asse, iii. 196.
thy: thy shrifte, i. 49: thy conscience, i. 50: thy wittes, i. 60: thy lust, i. 67, etc., etc.: thy hole, ii. 279 (?).

the (S. þe), dat., i. 50, 98: iii. 174, etc.
the (S. þe), acc., i. 49, 51, 83: iii. 36, 354, 357, etc.

he (S. he), i. 2, etc.; it (S. hit), i. 1, etc.; she (S. seo), i. 46, etc.
his (S. his), gen. masc., i. 2, etc. (a knight of his, i. 244: a castell in Provence of his, i. 260.)
his (S. his), gen. neut., i. 19 (whom *it* hath elles of *his* wille), 23, 33, 53: ii. 124.
her (S. hire), gen. fem., i. 48, 68, 69, 71, etc.

absolute forms: it shall be *hers*, ii. 287.
his clothës of and *her*[*ë*]*s* bothë, ii. 358.

him (S. him), dat. masc., i. 97, 133, 142: iii. 284, 329.
herë, her (S. hire), dat. fem., ii. 341: that well was *her*[*ë*] which him might, ii. 73: *her* thought he hath her well deserved, iii. 258: so as *her* thought it was to done, i. 47: she bad me tell and say *her* trouthe.
him, accus. masc., (from S. dat. him) i. 6, etc.
hirë, herë, her, accus. fem. (from S. dat. or gen.):
at the bordel because of *hirë* (rh. spire), iii. 324: i. 198, 226.
as I towardës *herë* wenë, i. 122.
and axen *herë* for to weddë, iii. 270.
that it of *herë* so befall[ë], iii. 312.
and after *herë* with his hond(e), iii. 332.
with *herë* which his lemman was, iii. 149.
of *herë* whom mercy preserveth, iii. 340.
that she to *her*[*ë*] mightë go, ii. 314.
he tok(e) of *her*[*ë*] what him list[ë], ii. 344.
to *her*(*ë*) did I never nought, ii. 349: so, i. 54, 71: ii. 39, 166, 341, 344: iii. 18, 261, etc.
commonly *her*.
and bad that she *her* shuldë mekë, i. 71, 72, 73, 74, 100, 102, 103, etc., etc.

we (S. we), i. 1, etc.
ourë (S. ûre), i. 1: in *ourë* time amonge us here, i. 262: ii. 90, 192, 215.
our, ii. 191: touchend *our* curë spirituall, which is *our* charge in speciall, iii. 379, 380, 385.
us, dat. (S. ûs), that thoughte *us*, iii. 309.
us, acc. (S. ús), i. 1, 4, etc.

ye (S. ge), i. 61, 62, 87, etc.
yourë (S. eôwer):
but upon *yourë* consciëncë, ii. 108.
as ye by *your*[*ë*] bokës knowë, i. 316.
in *yourë* dome I put it all, ii. 226.
your, ii. 344: iii. 372.
absolute form: thus am I yourës ever mo, i. 104.

you, dat. (S. eôw): *you* semeth, ii. 61.
wel *you* be, i. 210, 135: ii. 205: iii. 351, 360.
you, acc. (S. eôw), i. 62, 87, etc.

her (S. hira), = their, i. 9, 19, 58, 69, 81, 87, 111, 227, etc. (sometimes wrongfully written *here*).

absolute form: it shall be *her*[*ë*]*s* while I live, ii. 287.

hem (S. hem), dat., i. 12, 14, 18, etc.
hem thoughte it souned in her ere.
hem, accus. (from S. dat.), i. 15, 16, 17, etc.

they (S. þa), plural of third person (instead of S. hi): i. 10, etc.

their but seldom occurs, and wherever it is found, we should doubtless read *her*.
they saiden of *their* ownë pride, i. 111.
fro Romë lond they went *their* waie, i. 245 (read *wente her*).
there as they had *their* hertes sette, ii. 48 (read *hadde her*).
and whan they wiste *their* avauntage, iii. 219 (read *wiste her*).
So, i. 55, 59, 76, 115.
them is not found.

§ 46. The following are the combinations of the Personal Pronouns with *self* in Gower:—

myself (myselfë), i. 160, 228: ii. 30.
myselfe, myselve, i. 49, 103: ii. 30.
myselven, i. 161: ii. 380.
thyself, i. 278.
thyselven, i. 178.
himself (himselfë), i. 107, 109, 143, 163, 165, 215.
himselfe, himselve, i. 4, 53, 105, 109, 239: ii. 284.
himselven, i. 106, 162, 222, 266.
herself, i. 129, 165: iii. 300.
herselve, ii. 5, 337.
herselven, iii. 30, 316.
pl., usself, i. 256.
hemself, themselves, i. 13, 17, 26, 30, 230, 304: ii. 171, etc., etc.

my ladies selve, i. 228, should doubtless be my ladie, the s being caught from selve.

selfe, preceded by the article, means *the same*, as in Saxon: the selfe prest, i. 48.

§ 47. Demonstrative, Relative, etc.

Demonstrative: that (S. neut. þæt), definite article, in the phrase that on (misspelt one) — that other, i. 25: ii. 193, 204, 208: that o, ii. 376.
the, definite article, i. 1, etc.
that, demonstrative sing., i. 5, 13, "and that is shewed."

(In one case *that* seems to be used as the plural of the demonstrative (perhaps an error for *they*): —

the saints that weren us tofore,
by whom the feith was first up-bore,
that oughten better be beleved
than these, etc., ii. 188.)

tho (S. þâ) = those, i. 52 (tho be the gates); tho two, i. 85; tho wordes, ii. 188; tho men, ii. 209; tho children, ii. 271; alle tho that, iii. 146.
this (S. þis, þes), i. 1, etc.
thesĕ, plural of this (should be thes, S. þâs), i. 34, 56, 309: ii. 17, 56, 147, 240, 329, 332, 355: iii. 253, 278.
theseë = these, ii. 85, 154, 165, 188, 275 f: iii. 11, 45, 278, 279, 363, 364.
thilkë (S. þylc), = that, i. 105, 123, 216, etc.
so (S. swâ) = such, ii. 33 (if there were *such* a way: there is none so).

Relative: —

as she *that* was through out untrewe, i. 191.
the cardinals *that* wolde save, i. 254.
all *that* ever he herde,
all the prestes *that* there are, } ii. 271.
hem *that* ben nought grete, ii. 275.
Cesar Julius *which* tho was kinge, i. 28.
Adrian *which* popë was, i. 29.
she *which* shall be thy norice, i. 195.
in *Cristes* feith *which* died upon the rodë tre, i. 198.
thilke lord(e) *whiche* al may kepe,
to *whom* no counseil may be hid, i. 9.
an emperour *whos* name was Othes, i. 31.
that ymage on *whom* he foundeth, i. 31.
tho *whos(e)* herte stood upon knighthode, i. 244.
whos love his strength all overthrew(e), iii. 366.

that = that which: eschewe *that* wicked is, i. 19.
man is cause of *that* shal falle, i. 21, etc.

what = that which: shal ordeine *what* he wille i. 16.
what thou hast felt, i. 48, etc.
what man wolde him self avise, he, etc. i. 21: ii. 141.
what man on hem his cherë caste, he, etc. i. 55.
the which, which that, etc. = simple which, etc.
the which was cleped Clemene, ii. 34, 168, 281, 366, 383: iii. 61, 105, 130, 294.
(pl.) among *the whichë* ther(e) was on(e), ii. 375: iii. 19, 73, 203.
thing *which that* is to lovë due, ii. 18, 77.
which timë that I am her fro, iii. 7.
man, *the which that* wit and reson can, i. 34.
the which that this science all one, iii. 141.
thing *which as* may nought ben acheved, ii. 380: i. 298: iii. 134.
for that thing may nought be refused
what that a king him selfë bit, i. 4.
the whos power as now is falle, ii. 187, 87, 103, 354: iii. 41, 116, 130, 140, 205.
the whom no pité might arest[e], iii. 203.
to Venus *whos(e)* prest *that* I am, ii. 61.
who that, what that, etc. = *quisquis, quicumque*:
who that desireth, he shall, i. 13.
he is not wis(e) *who that* ne troweth, i. 21, 22, 45, etc.
what thing *that* the body hateth, i. 37.
what man *that* his lust desireth, he shall, etc., i. 82.
what knight *that* all other passeth, i. 104.
what that other sain, i. 223.
upon *what* side *as ever* it falle, i. 264.
what as ever cometh, ii. 139, 144.
what so falle, ii. 23.
who so these oldë bokes troweth, ii. 146.
whom as ever that they serven, ii.
(So, *how as ever that* it be, ii. 7: *how that ever* he ete, iii. 39: *how so that ever* it fall[ë], iii. 43: *when so as ever*, ii. 143.)
what, in the same sense: *what* thou herë, yef no credence, i. 59.

Interrogative: —

who, which, what, as in English.
whether = which of two, i. 217, 337: ii. 60: iii. 17, 355.

Indefinite: —

somwho = *aliquis* (once only):
but if *somwho* the flamme staunche, i. 15.

For some peculiar cases of pronouns, see further on, §§ 103, 104, 105.

VERBS.

§ 48. Present Indicative. — The First Person Singular of the Present Indicative terminates in **-ë**.

I thenkë, i. 2 : ii. 353.	I trowë, i. 107 : ii. 79, 190, 213.
I make, ii. 21.	I live, ii. 211.
I finde, ii. 6, 79.	I telle, ii. 7, 203.
I seche, ii. 11.	I hope, ii. 317.
I come, ii. 25.	I bidde, ii. 276.
I take, ii. 70.	I wisshe, ii. 352.
I redë, i. 311, 358 : ii. 93, 300, 302 : iii. 16.	I pleie, ii. 41.

Exceptions: hast ben er this, I *redë* the leve, iii. 47 : also, i. 117.
though I *tell* that I werë ded(e), (probably incorrect), i. 299.

§ 49. The Second Person Singular of the Present Indicative ends in **-st** as in modern English.

thou hast, ii. 209, 211.	thou accusest, ii. 279.
tellest, ii. 279	seest, iii. 15.
saist, ii. 279 : iii. 85.	feignest, i. 47 f, 67 f.

§ 50. The Third Person ends in **-eth**, **-th**.

groweth, bestoweth, ii. 84.	maketh (one syllable), iii. 46 (6 cases).
falleth, calleth, ii. 84.	taketh (one syllable), iii. 46.
hath, ii. 129, 187.	berth, i. 135 : ii. 129.
geth (goth), iii. 247.	tath (taketh), ii. 129 : iii. 120 f, 217 f.
comth, iii. 370.	wroth (S. wrâðjan), iii. 331.
lefth, ii 391.	

has occurs on the same page with hath, ii. 187.

§ 51. But Saxon verbs which have t or d for the last consonant of the root, and one or two which have s, form the Third Person Singular in **-t**, as in Saxon.

writ, i. 1.	put, ii. 135 : iii. 355.
smit, i. 40.	set, ii. 135 : iii. 42.
let, i. 21.	holt, ii. 144, 391.
betit, i. 226.	get, ii. 331 : iii. 16.
shet (= shoots), i. 258.	byt, ii. 40.
spret (= spreads), i. 173.	fret, iii. 98.
beholt, i. 132.	sit, iii. 357.
hit, i. 294.	bint, iii. 4.
abit, i. 286 f.	blent (= blinds), ii. 210 f, 348 f.
fint, ii. 116, 145.	

(In a few cases we find d instead of t: stond, ii. 84 : send, iii. 221 : held, iii. 328.)

arist, i. 20, 175 : ii. 151 f : iii. 200, 313.
lost, lest (= loses), i. 168 : ii. 34, 147 : iii. 346.
wext, i. 133.

he *let* it never out of his honde,
but *get* him more and *halt* it fast[e], ii. 128.
he *taketh*, he *kepeth*, he *halt*, he *bint*, ii. 284.

The longer form in -eth, sometimes found in Saxon, hardly occurs in Gower. We find lasteth, overcasteth, i. 317 : but should probably read *arist* in the mede *ariseth* of the service, iii. 342.

§ 52. The Plural of the Present Indicative ends, —

a. rarely in **-eth** (S. að).
ye thenketh, i. 135 : they saileth, iii. 292.
men calleth (for rhyme's sake), iii. 103.
and they also him *hath* behight, iii. 248, very doubtful.

b. generally in **-en**.

bridlen, i. 110.	worchen, ii. 85.
receiven, weiven, i. 180.	waxen, ii. 85.
axen, i. 184.	ben, i. 85 : ii. 189.
knowen, ii. 25.	aren, iii. 135.
fallen, ii. 31.	sain, i. 1.

saine, iii. 107, *if right*, is probably a transposition of saien.

c. sometimes in **-e**.

clepe, i. 7.	ride, i. 110.

they *setten* up[on] thilkë dedë,
and *spillen* morë than they *spedë*, ii. 88.
that *knowen* litel what they *menë ;*
it is nought on(e) to wite and wenë, ii. 89.

§ 53. Imperfect Indicative.

Simple (or "Regular") Verbs.

a. The Imperfect of Simple Verbs is generally formed in **-de** or **-te**, with occasional change of vowel.

didde, i. 112 : ii. 123.
made, ii. 27, 29, 83.
hadde, i. 55, 101 : ii. 26 : iii. 364.
saide, i. 47, 49.
answerde, iii. 36, 38.
betidde, iii. 374.
filde, ii. 204.
wolde, i. 2, 101.
shulde, i. 199 : ii. 27, 28.
lefte, ii. 28, 309.
laste, i. 342 : ii. 49.
highte, ii. 161, 236 : hatte, ii. 163.
mighte, iii. 74, 76.
loste, i. 207 : ii. 150, 324.
thoughte, ii. 36 : iii. 366.
wroughte, ii. 188.
roughte (recked), i. 240.
smette, for smot, i. 234 f.

b. The Imperfect Indicative sometimes drops the ë of the above-mentioned terminations.

madĕ, i. 199.
had, i. 77, 128.
said, i. 276 : iii. 315.
wold, i. 321, 338 : iii. 325.
shuld, i. 360 : iii. 235, 322.
thought, i. 302 : ii. 24.
liked, i. 326.
herdĕ, iii. 69.
happed, i. 199.
trusted, iii. 70.
feigned, i. 170 ?
lacked, iii. 256 ?

ete of and *died*, god wote how, i. 261, should read, *et* of and *diedë*, god wot how.

c. The Second Person Singular of the Imperfect Indicative of Simple Verbs is formed in **-est**, as in Saxon and modern English.

woldest, couthest, haddest, i. 60.
behightest, ii. 294.
mightest, iii. 83.

§ 54. Imperfect of Strong, Complex, or "Irregular" Verbs.

a. Several Strong or Complex Verbs have in Gower the Imperfect Tense in ë, contrary both to ancient and present rule.

but how as ever it *fellë* so, ii. 67 : (but befell, i. 214 : ii. 68 f : fel, i. 27, 34, 67 : ii. 72.)
with him he tokë manifold(e), ii. 231.
she drovë forth by chare and wheel, ii. 261.
with whiche he *bondë* both her armes, ii. 318.
with her where as I *camë* fro, ii. 98.
upon my self this ilkë talë *comë*, iii. 350.
till that he *founde* out at laste, ii. 231 (very suspicious).
til it befellë that at laste, i. 27, should probably read, til it befell that *attë* laste.

b. The Second Person Singular of the Imperfect Indicative of Strong Verbs (which in Saxon ends in **e**), in the few cases which occur, either has **e**, or is the same as the First Person.

thou *sighë* never thilkë placë, i. 367.
whan thou thy worthy lady *sigh*[*ë*],
werë thou afered of her eyë ? ii. 21.
whan thou in thy batailë *were*, iii. 155.
that thou amonges hem *were* one, iii. 354.
thou *knewe* all other mennes chaunce, iii. 79.
first whan thou *come* under my shrift, iii. 344.

Even "my sone if that thou well *bethought*" (rh. nought), occurs, iii. 357.

c. Examples of the Complex Imperfect in Gower.

brak, ii. 30.
bar, i. 153, 187.
sat, ii. 47.
spak, ii. 272.
stal, ii. 335, 355.
cam, i. 50, 120, 148, 156.
bad, i. 47, 127, 140, 170, 186.
yaf, i. 111 : ii. 120.
quod, i. 47, 321.
fell, i. 95 : ii. 72.
wax, i. 217, 312 : ii. 337.
sew (= sowed), ii. 190.
hew (= hewed), ii. 263.
heng(e), i. 263.
wissh, ii. 137 : iii. 162.
fleigh, ii. 335.
drough (drew), i. 182, 212.
slough, i. 165.
lough, i. 101, 104, 171.
shop, i. 257.
wan, i. 157.
halp, iii. 209, 322.
starf, i. 189.
malt, ii. 37.
bark(e) (= barked), i. 221.
drof, i. 314 : ii. 64.
aros, ros, i. 142 : ii. 160, 169, 257, 327.
shof, i. 165 : ii. 5.
ches (= chose), i. 242, 368 : ii. 237.
lees, les(e) (= lost), iii. 61, 215.
sigh (A. S. seah = saw), i. 24, 52, 93, 137 : iii. 360, 361.

§ 55. The Plural of the Imperfect Indicative (both of Simple and Complex Verbs) ends, —

a. in **-en**.

weren, i. 4 : ii. 36, 188, 190 : iii. 251.
diden, i. 4.
fellen, i. 34 : ii. 184.
sawen, i. 115.
sighen, i. 194 : ii. 113 : iii. 251.
comen, i. 368 : iii. 251.
kisten, ii. 27.
maden, ii. 85.
mighten, ii, 36.
shulden, ii. 85.
wroughten, ii. 55.
hadden, ii. 173.
translateden, ii. 90.
foundeden, ii. 157.
foughten, ii. 72.
knewen, ii. 127.
loveden, iii. 31.
herden, iii. 251.
founden, iii. 253.
contreveden, ii. 84.

b. in -e.

werë, i. 4, 10, 33, 75, 78, 101, 110, 111, 127, 242, etc., etc.
founde, ii. 83.
dorste, ii. 237.
berefte, i. 28.
begonne, begunne, ii. 36, 326.
shulde, ii. 69 f.

Or, *c*, has no termination.

the Grekes let, i. 80.
they which understood, i. 80 f.
they both[e] stood, i. 232 f.
festes stood, iii. 257 f.
men shuld, iii. 144.
teres ran, iii. 300.
men wold, iii. 355.
(they) had, i. 101 (?)
(they) wist, ii. 163 (?)
the peril er we *fall* therinnë, ii. 380 (?)

§ 56. Subjunctive.

The Singular of the Subjunctive, both Present and Imperfect, ends in ë through all the Persons, as in Saxon.

a. Present.

forthy, my sone, if that thou *felë*
that lovë *wringë* the to sorë, ii. 91.
for who that *lese* or *findë* grace, i. 160
God it the *foryivë*, i. 123.
who so *takë* kepe, ii. 93.
but what man that this work *beginnë*, i. 85.
for what man thilkë vicë *suë*, ii. 141.
who that *redë* right[e], iii. 102: i. 231.
me recheth nought who *overthrowë*, i. 168.
(axeth) that she him *holdë* covenaunt, i. 250.
but er the timë that he *spedë*, i. 65.
what thou *herë*, i. 59.
and namëly that thou ne *chidë*, i. 307.
I wolde that thou *arise*, iii. 346.
but though that holy chirche it *bidde*, iii. 280.
praiden her that she ne *sparë*, iii. 261.
that non(e) it *knowë*, ii. 272.
God grant it *motë* be redressed, iii. 380.
he *kepë* Simon fro the foldë! i. 162.
but God *forbedë* that it *werë*, i. 162, 295 f.
O Diogenë, God the *spedë!* iii. 162.
up pein[*e*] of deth that no man *weivë*
that he baptismë ne *receivë*, i. 276.
of eightë sterres where (wherever) he *wendë*, iii. 122.

upon thy fomen alle,
sir king, thy sweven *motë* falle! i. 139.

b. Imperfect.

as though she verrilichë *seie*, i. 72.
though I her *kepte* and *heldë* fast[ë], ii. 130.
who that it *understodë*,* i. 162: ii. 33.
who that it *knewë*, ii. 88 f.
for who that *woldë* taken hedë, iii. 271.
to hope it *werë* than anon(e), i. 8.
it *werë* better dike and delvë, i. 15.
so faine he wolde it *werë* she, i. 207.

So, i. 39, 156, 211, 233, 243, 359, 365, etc.: ii. 33, etc.

for that my selfe have ofte sithe
desired thou *woltë*, as men saith, i. 181.
I not how Jason that night *slepe*, ii. 241.

Exceptions: *werë* not infrequently. (See §§ 84, 86.)

for me werë lever to lackë breth, i. 177.
but he werë lustles in his herte, i. 127.

So, i. 265, 309, 333 (?): ii. 93 (?), 276: iii. 212.

for if thou *herë* my talë wel(e), ii. 340.

c. The Plural of the Present Subjunctive is in **-en** and e, like the usual forms of the Indicative: se we, torne we, iii. 292.

§ 57. Imperative.

In those forms of the Singular of the Imperative which in Saxon end in a vowel, the vowel is not very well preserved in Gower. In Pauli's text an **e** is generally appended to the forms which in Saxon end in a consonant; erroneously, as the slightest inspection will show.

§ 58. Singular.

a. Forms which in Saxon end in a vowel.

medlë (S. midla), i. 143.
lokë (S. lôca). ii. 140: iii. 290. But lokë, i. 83.
tellë (S. tele), ii. 39. But tel, i. 49, 60, 61, 88: tellë, i. 47.

herken (S. hercna), i. 53, 95, 135, 158, 243,* should very likely be herkne: "now *herke* hereafter" occurs, iii. 325.
shewë (S. sceawa), i. 48.

b. Forms which in Saxon end in a consonant.

list (S. hlyst), i. 69, 105, 234.
let (S. læt), i. 49.
yif, yef (S. gif), i. 299, 46, 59.

* who so that reson *understood*, ii. 129.

shrif, shrivĕ (S. scrîf), i. 133, 175, 280: ii. 13, 57.
drynk (S. drinc), i. 128.
kepĕ (S. cêp), i. 83, 178: ii. 268.
redĕ (S. ræd), iii. 138.
levĕ (S. læf), i. 179.
spekĕ (S. spec), ii. 283.
takĕ (S. tac), i. 86, 144, 303, 335, 353.
farĕ (S. far), i. 289.
comĕ (S. cum), i. 48.
abidĕ (S. abid), ii. 280.
beholdĕ (S. beheald), i. 46.

c. In the following verses, if the reading be correct, the final *e* is difficult to account for, unless an abridged plural form is confounded with the singular: —

behold(e) and *demë* (demeth?) my querele, iii. 196.
for *witë* (witeth?) well that never man, ii. 242.

§ 59. Plural, generally in -eth (S. að).

axeth, i. 162, 166: ii. 115: iii. 274.	warneth, ii. 49.
telleth, i. 89, 150, 169.	witeth, ii. 205.
leveth, ii. 33, 129, 395.	aviseth, ii. 246.
demeth, i. 61: ii. 395.	helpeth, ii. 260.
putteth, i. 194.	comforteth, iii. 316.
betaketh, i. 194.	tristeth, iii. 316.
taketh, ii. 205.	goth, ii. 205.
doth, i. 46.	

b. In the following cases it is more probable that the -th has been wrongly dropped.

so *help[th]* me now, I you beseche, ii. 260.
now *shape[th]* ye the bestë way, ii. 381.
yef[th] what ye list to my penaunce,
and *axeth* . . . i. 371.

§ 60. INFINITIVE.

The Infinitive in Gower ends **-en** (S. **-an**); or, more frequently, the n is dropped.

worchen, i. 166.	gon(e), i. 147: ii. 240.
ben, i. 5.	loven, i. 87, 106, 107.
tellen, i. 6, 139.	sain, i. 95.
yeven, i. 9.	seen, i. 101.
comen, i. 147.	knowen, i. 140.
answeren, i. 149.	slain(e), sleen, i. 219, 330.
write, i. 1, 2, 5.	sende, i. 9 f.
beleve, i. 1.	seche, i. 9 f.
make, i. 2, 12	stande, i. 10.
befalle, i. 3.	beare, i. 13.
beholde, i. 4, 10.	clothe, i. 14 f.
drawe, i. 5, 6.	ete, i. 14.
hope, i. 8.	drinke, i. 14.
here, i. 229.	save, ii. 30, 334.

I mighte amendĕ that is amis, iii. 274, an apparent case of the termination being wholly dropped, should perhaps be read, I might amendë that's amis (= what's amis, in English).
to till the londes and set the vines, i. 83, should probably read, to tillë londĕs and settë vines.
to done (best to done, etc.), i. 185 f, 191 f, 204 f, 233 f, etc., is the Saxon gerund to-donne.

The Infinitive sign *to* is often said to be omitted in such phrases as *I see you stand*, the fact being that those phrases preserve the original Saxon Infinitive, and have not exchanged it for the form with *to*, derived from the Saxon gerund. We find in Gower the Infinitive without *to* after several verbs which now require that sign, thus: —

I thenke assaie, telle, etc., i. 157, 158, 162: ii. 21, 45, 52, 100, 215, 361, 371: iii. 155, 288, 327, 374, 378.
I wende have said, ii. 21: iii. 258.
he himself assay desireth, i. 30.
they crie begunne, ii. 326.
they gonnen say, ii. 253.
a man is free defende, i. 354.
it oughte put, i. 57, 117.
her ought[e] nought be wroth, i. 297, 303: ii. 151.
that other were lever have had, i. 305.

We also find the Infinitive with *to* or *for to* in the same connections, and *to* and *for to* indifferently used: —

I thenke to do, ii. 50, 72.	I thenke for to flee, ii. 121, 20: iii. 165.
began to say, ii. 64: gan to clepe, ii. 150.	oughte for to be, iii. 154, 157.
ought wel to be, iii. 376: i. 294.	a king behoveth for to have iii. 231.
	me belongeth for to say, ii. 381, etc.

§ 61. PARTICIPLES.

Complex Perfect. — The Perfect Participle of Complex ("Irregular") Verbs terminates in **-en**: as printed by Pauli, more frequently in **e**.

comen, i. 126, 150, 182: iii. 51.
forlain, ii. 234 f.
wasshen, i. 138.
stonden, i. 141.
throwen, i. 167.
storven, i. 330.
stoken, ii. 21.
forlorn, ii. 190.
sen, ii. 394: sain, iii. 224.
besein, i. 168.
oversein, ii. 234 f.
beflain, iii. 183.

come, iii. 51.
stole, ii. 32.
founde, ii 83.

holde, iii. 130.
agrise, i. 24 f.
forfare, i. 45.
befrose, i. 220.
hote, i. 280.
slawe, i. 185.
begonne, i. 80, 258.
knowe, i. 82.
shape, i. 82.
shore, i. 101 f.
yive, yove, i. 114, 127.
bounde, i. 127.
lore, i. 298.
bore, i. 136 f.
lette, ii. 3.
say, ii. 206 f, oversay, i. 193, do, i. 47, 111, } contracted.

The contracted Participle seems, in a few instances, to be protracted. (?)

sene, i. 42 f, 82 f, 120 f, 254 f, 301 f: ii. 234, 235 f: iii. 342 f.
be-seine, i. 54 f.

§ 62. Simple Perfect.

The Perfect Participle of the Simple Conjugation ends in -ed, -d, or -t, and requires but slight notice.

beloved, i. 106, 107.
shewed (S. sceâwod), i. 7, 13, 76, 201: ii. 29: iii. 39, 195.
cleped, iii. 57.
londed, iii. 49.
proved, iii. 52, 61.
feigned, i. 226.

herd, i. 46, 115, 127: iii. 49.
hid (S. hŷded), i. 70: iii. 52, 326 f.
kid (S. cŷðed, cŷd), i. 286: iii. 51. f.
miswent, i. 55 f.
sent, i. 70.
lost, i. 155, 243, 356: ii. 326.

We find, for I havĕ *herdĕ* tell also, ii. 58.

astonë occurs iii. 54 f, a complex form where a-stoned (S. stunod) would be expected.

beleftë (rh. efte, S. *eft*), i. 246, 275, is doubtless a mistake.

§ 63. The prefix i- (S. ge-) occurs, but not frequently.

i- write, i. 169.
i- bore, i. 176.
i- lore, ii. 51 f.
i- wounde, iii. 314 f.

§ 64. Present Participle.

The Present Participle terminates, with few exceptions, in **-ènde** (S. **-ende**). Many words of French origin adopt this termination.

accordènde, i. 213 f.
comènde, i. 88, 133 f, 220 f.
touchènde, i. 243.
wepènde, i. 74.
criènde, i. 137.
knelènde, i. 155.
praiènde, i. 345.
suènde, i. 278, 213 f.
spekènde, ii. 6 f.
thenkènde, ii. 369.
thonkènde, ii. 297.
ridènde, i. 191: ii. 46.
amblènde, ii. 45.
winkènde, ii. 189.
boilènde, ii. 201.
swounènde, i. 188.
sailènde, i. 200.
bledènde, iii. 60.
unsittènde, iii. 143.
continuènde, ii. 18 f.
to me spekènd[ë] thus began, i. 48.
thenkènd[ë] what was best(e) to done, i. 185.
ridènd[ë] was, stood that time out[e], i. 214, etc., etc.

In innumerable cases the elided **e** is not printed in Pauli's Gower: as touchènd of, i. 49: feignènd of, i. 63: feignènd an, i. 70: stalkènd he, i. 72: glistrènd ayein, i. 80: hangènd about, i. 115: thenkènd he, i. 144: knelènd he, i. 151: slepènd appere, i. 271, etc., etc., etc.: brennènd hast, iii. 352.

Much less frequently the accent is thrown back.

cómend after, i. 1.
toùchende of, i. 52: toùchend, i. 53, 67: ii. 3.
belòngend unto, i. 12.
waìlend in, i. 144.
wàlkend on, i. 185: ii. 45.
wèpend eye, i. 236.
knèlende on, ii. 96.
slòmbrend eyen, ii. 103.
suènd after, ii. 104.
drèdend alle, ii. 334, etc.

The final **e** is most frequently not printed when the accent is thrown back, but this is probably an error.

The only cases observed of Present Participles in **-end**, where no elision could take place, are the following suspicious and probably incorrect verses.

by sleightë feignend than he wrought, i. 68.
of goldë glistrend spoke and whele, iii. 112.

A very few cases occur of the later form of the participle in **-inge**, **-ing**:

sailinge (rh. singe), i. 59.
wisshinge and wèpinge, i. 45.
the whel meving of love, i. 213.
brenninge (rh. comminge, noun), ii. 29.
sitting (rh. dwelling, noun), iii. 253.

§ 65. Anomalous Verbs.

can = know, be able.

Pres. Indic. Sing. 1, 3.	can, i. 174.
2.	canst, i. 117: iii. 38.
Plur.	connen, i. 356: ii. 313: iii. 158. conne, i. 92: ii. 32, 81 f.
Imper. Indic. Sing. 1, 3.	couthe, ii. 33, 44, 378.
Plur.	couthen, i. 26: iii. 362.
Infin.	conne, i. 200 (know): ii. 158 (be able).
Partic.	couth, i. 138, 174.

dar = dare.

Pres. Sing. 1, 3.	dar, dar(e) i. 296, 297: iii. 44(2), 257.
2.	darst, ii. 22.
Plur.	darë, ii. 13.
Imper. 1, 3.	dorste, ii. 118: iii. 381.
2.	dursest, ii. 2, should doubtless be durstest.
Plur.	durste, i. 148.

We find, and that I *durst* right wel avouche, i. 295, where durst is either Conditional or Present Indicative. if that I *dore* (rh. dore, noun), ii. 96, is probably a Present Subjunctive.

may = may.

Pres. Indic. Sing. 1, 3.	may, iii. 6, 7. 47, etc.
2.	might, i. 50, 59, 66, 96, 141, 143, 178, etc.
Plur.	mowe, iii. 275. mow, i. 44: ii. 42, 95, 355. may, ii. 286.
Imperf. Sing. 1, 3.	mightë, i. 28, 30, 176: ii. 19, 28, 44. might, i. 212.
Infin.	it shall nought wel *mow* be forsake, i. 215.

mot = must, may.

Indic. Pres. Sing. 1, 3.	mot, i. 93, 114, 155, 266, 274: ii. 245 (5 times), etc.
2.	must, i. 118, 145: ii. 25.
Plur.	moten, mote, i. 268: ii. 386 f.: iii. 286 f.
Sub. Pres. Sing.	mote, i. 228, 139.
Indic. Imperf.	muste, ii. 242.
Imperf. (sense of present),	muste (= Eng. must), iii. 10. must (should probably be muste), ii. 245: *must* three times, *mot* five times in the page. musten, i. 179.
In the sense of *may*. Pres Subj.	mote: thy sweven *mote* falle, i. 139. God leve (read lene) it *motë* well ben(e) holde, iii. 188.

mot: my lege lord, god *mot* (?) you quite, i. 155.
Inf. to mote, iii. 296. ?

owe = *debeo*.

Pres. oweth = *debet*, every lif(e) *oweth* wel to knowe, i. 362.
Imperf. { oughte = *debet*, i. 57: ii. 64, 150, 369.
ought, i. 117, 291 (?).
oughten, iii. 66, 188.
ought(e) = owned, ii. 294.

shal = Eng. shall.

Pres. Sing. 1, 3. shal, shall, i. 49, 50, 51, etc.
2. shalt, i. 48, 49, 51, etc.
Plur. shullen, i. 117: ii. 76.
shulle, i. 128 f.
shull, iii. 70, 305.
shall, i. 84, 117.
Imper. Sing. shulde, i. 79.
Plur. shulden, shulde, i. 76, 77, 79.

N. B. — I shall, ii. 96, = am destined to, may: also iii. 7 = *je dois*.
sholde, ii. 96, = were destined to, might.

thar = need.

Pres. Indic. Sing. 1, 3. ther, iii. 164.
2. tharst, ii. 61.

wot = wot, *scio*.

Indic. Pres. Sing. 1, 3. wot. ii. 276, 277.
2. wost, ii. 121, 123, 275: iii, 352, 356.
Plur. witen, weten, ii. 322: iii. 372: witĕ ye, i. 175.
woten, iii. 185.
Imperf. wiste, ii. 197, 204.
Subj. Pres. Sing. (er he it) wite, i. 303, 304.
Imper. wite (qy. witeth ?) ii. 242.
Infin. witen, wite, i. 134, 310 f, 320: ii. 4 f, 95, 377.
Past Part. wist, i. 339: ii. 46: iii. 276.
I *wit*, ii. 3, should doubtless be I *wot*.

§ 66. The verb *will*.

Pres. Indic. Sing. 1. wille, ii. 279, wol, i. 6, 48, 50.
2. { wilt, ii. 388.
wolt, i. 49, 169, 277, etc. (wold 310).
wol, ii. 122: perhaps also, if thou *wol* here, ii. 73.
3. wille, i. 352: woll (God woll it shall ben overthrowe), iii. 201.
as he that fighte *wolle*, ii. 72, maybe Subj. Mood.
Plur. wolle, i. 103.
woll, wol, i. 43, 88, 129, 177, etc.
will, i. 177: ii. 370.

Imperf. wolde, woldest, etc., i. 176: ii. 44: iii. 311, etc.
I wold, i. 311: thou wold, i. 310: (he) wold, i. 321: ye wold, i. 338.
Subj. Pres. what so she *wollë*, so woll I, ii. 41.
Infin. desired thou wolte, i. 181.
Partic., (made from the Perfect Tense), if that he had *wold(e)* his timë *kept* (!), ii. 9.

§ 67. Some Impersonal Verbs.

him hungreth, iii. 29.
him lacketh, ii. 12: iii. 52, 286, 357.
him liketh, ii. 280; misliketh, ii. 370.
him, her, etc., list, lust, Pres. i. 42, 228, 316: ii. 16, 32, 40, 42, 107, 110, 167, 169, 189 f, 282, 356, etc., etc.
Imperf., liste, i. 66, 73, 123, 225: ii. 48, 226, 264, 278, 366: iii. 275, 281, 323.
But, if thou list, i. 279.
say what ye liste, i. 103.
what so ye liste, i. 371.
what ye list, i. 371: ii. 25.
as ye list, ii. 25.
she liste, i. 134, 283.
the women listen, ii. 107.
me longeth, iii. 82.
him nedeth, ii. 281, 282, 330: iii. 341: whom nedeth, iii. 350.
him, her, oughte (*oportet*), i. 231, 297, 303: ii. 96, 278: iii. 157, 345.
him oughte (*oportuit*), ii. 323.
me quemeth (S. cwêmð, *placet*), ii. 34: her quemeth of, ii. 26.
him reccheth, ii. 284: iii. 169.
the, him, me, thenketh, thinketh, ii. 61, 135, 368: it thought us, iii. 358.
you, me, semeth, ii, 61.
him ther, the ther(e) (= thar), iii. 164: i. 241.
him thursteth, ii. 135.

§ 68. Negative Verbs.

am. Indic. Pres. Sing. 1, nam, i. 67, 135: iii. 273.
3, nis, i. 161, 299.
Subj. Imp. nere, ii. 160.
will. Indic. Pres. Sing. 3, nill, i. 311. Imperf. nolde, iii. 47, 329.
wot. Indic. Pres. Sing. 1, 3, not, i. 230, 308: ii. 3, 33: iii. 347, 350. Imperf. S. niste i. 72: ii. 382: iii. 79.
2, nost, i. 61. Pl. nisten, i. 313.

ADVERBS.

§ 69. Adverbs of Saxon derivation have commonly, in the positive degree, the termination ë, as in Saxon.

clene, ii. 24 f: iii. 5.
depe (S. deôpe), i. 98.
dimme (S. dimme), ii. 293 f.
un-ethe, iii. 195: i. 364. (See, also, § 73.)
faire, i. 24: ii. 204, 307, 382: iii. 56.
faste, i. 55: ii. 24, 29, 198, 219: iii. 369.
harde, i. 220: ii. 146.
highe, ii. 35 f, 36 f.
hote, ii. 28 f, 301 f: iii. 113 f, 363 f.
inne, ii. 85 f, 170 f: iii. 203 f, 286.
ther-inne, i. 224, 275 f: ii. 272.
with-inne, i. 30: ii. 17, 204: iii. 252.
late, i. 211 f: ii. 370: iii. 55, 329.
-liche: a-liche, i. 268: ii. 76, 82, 113.
besi-liche, ii. 3, 43.
comun-liche, ii. 226.
duë-liche, iii. 245.
even-liche, ii. 179 f.
open-liche, ii. 328: iii. 219.
parfit-liche, ii. 185.
privë-liche, ii. 336, 365: privelich, iii. 252.
un-proper-liche, ii. 129.
sodein-liche, ii. 336: iii. 9.
solempnë-liche, iii. 329.

verri-liche, i. 72 : iii. 5.
longe, i. 342: ii. 28, 184 f, 215, 302 f: iii. 285.
loude, ii. 308 : iii. 307 f, 358 : a-loude, iii. 268.
oute, i. 16 : ii. 143 f: iii. 294 f.
same (S. same, *pariter*), ii. 240 f. (?)
smale (S. smale), ii. 279 f.
softe, ii. 45, 240, 309.
sone, ii. 250 : iii. 51.
sore, ii. 185, 215, 280, 308.
stille, ii. 121 : iii. 285 f.
swithe, iii. 57 f, 306 f.
uppe, i. 15 f, 128 f: ii. 249.
wide, iii, 208, 324, 326.
high (rh. eyë), should be highë, ii. 35 f.
So, alofte, ii. 103 f, 109 f, 195 f: iii. 136 f, 195 f, 351 f.
blive, i. 314 f: ii. 238 f: iii. 49 f, 58 f, 292 f, 313 f.
lowe, ii. 35, 101 f.
smarte, iii. 113 f.
straite (= L. *stricte*), ii. 354 f: iii. 47 f, 227.
wēle (S. wela, wel), iii. 149 f.
We have halfing, *halving* (S. healfunga), ii. 65: iii. 206, 353, 356.

§ 70. Comparatives and Superlatives of the Ancient ("Irregular") Form.

compar. bet, i. 126 : "for bet for wers," ii. 24, 325.
better, i. 66, 88.
the bet, i. 108 : ii. 285.
the better, ii. 285 : iii. 213.
compar. ner (= nearer), i. 314 ("ner and ner") : nerë, i. 318 f.
the ner, i. 236 f: (wrongly spelt nerĕ), i. 120 : iii. 212.
super. neste = next is found, iii. 121 f (rh breste).
compar. ferre, ii. 19.
the ferrë, i. 281 f.
compar. lassë, i. 25, 366 f: ii. 138.
the lassë, i. 303 : ii. 212.
super. lest, i. 121.
compar. lenger, ii. 257, 297 : iii. 54, 185, 249.
compar. morë, ii. 283 : morë hard, i. 25 : morë strong, ii. 381 : morĕ hardë, ii. 146; the morĕ, i. 3.
super. mostë (glade), i. 182.
compar. wers, "for bet, for wers," ii. 24.
worsë, iii. 209, 269.
the wersë, worsë, i. 334 : ii. 364 : the wors, iii. 153.
the werrë, i. 334 f: iii. 238 f.
no woman mightë worsë farë
ne sorwë morë than she did[ë], ii. 383.

§ 71. The following Adverbs have an internal ĕ which is not found Saxon.

truëly (treôwlice), i. 227 : ii. 57, 362.
nedëly (nŷdlice), ii. 41.
redëly (S. hraedlice), ii. 198, 371.

§ 72. The following Particles, of various terminations in Saxon, have ĕ more or less frequently. Those marked * have also a form in **-s** : see § 73.

a. From Saxon forms in **-an**.

{ a-boven, ii. 185 : iii. 12, 81.
{ a-bove, ii. 16, 274.
abovĕ, iii. 236. (?)
a-boute,* i. 127 : ii. 50, 102 f, 188 f: iii. 36, 76.
a-twinne (S. on-tweônan ?), i. 371 f: ii. 244 f, 326 f.
be-hinde, ii. 31 f, 217 f, 380 f: iii. 44 f, 97, 313.
be-twene, ii. 327 : iii. 53 f, 284 f.
{ betwenĕ, i. 6, 9, 20, 286, 309 : ii. 22 : iii. 139, 232.
{ betwen, i. 12, 15, 332 : iii. 350.
-forn, -fore : a-forn (S. onforan), iii. 32 f.
a-fore i. 364 : ii. 267 f: iii. 227.
to-fore (S. tôforan), i. 204, 209 : ii. 31 f, 47, 77 f, 154 : iii. 40, 273, 342.
toforĕ, i. 59, 117, 155, 201, 205, 321, 348 : iii. 61.
beforĕ, i. 138.
-nethe : be-nethe (S. be-niþan), i. 35.
under-nethe (S. under-niþan), i. 258.
-side : a-side (S. on-sidan ?), ii. 85 f, 143 f, 278 : iii. 166 f.
be-siden * (S. be-sidan), ii. 379.
be-side, iii. 82 : i. 289.
{ sithen, i. 42.
{ sithe, i. 31, 43, 210 : iii. 71 f.
{ withouten, ii. 70, 118, 282, 333.
{ withoute, i. 7, 30 f: ii. 118, 143, 187 : iii. 379.
without, i. 8 (?)

b. al-gate,* i. 25, 33, 85, 114, 248: ii. 89, 96, 242, 334 f.
a-longe, ii. 22 f, 310 f.
a-midde,* ii. 58 f, 119 f: iii. 31 f, 44 f.
a-monge,* ii. 22 f, 310 f.
among, iii. 236, 379; ever among, ii. 320: iii. 360: among(ĕ), i. 34.
bothe, ii. 229: iii. 143, 327.
efte (S. eft), i. 171.

ekĕ (*i. e.* ek, S. eac), ii. 161 (4 cases), 167, 171 (3 cases): iii. 89, 379.
eke, i. 71 f, 190 f, 258 f: ii. 73 f, 158, 166 f, 172, 228 f: iii. 139 f, 273 f.

ferre (S. feor), ii. 19.
fore (S. for), ii. 59, to rhyme with bore: ii. 239, to rhyme with forlore: iii. 308, rh. more.

her (S. her): herĕ, i. 23, 124.
here, i. 47 f, 70 f: iii. 316, 325 f.
ther (S. þaer, þere, þara), i. 80, 87, 96, 151, 290: ii. 57, 331, 367: iii. 215.
therĕ (*i. e.* ther), i. 23, 106, 111, 113, 114, 118: ii. 372.
there, i. 56 f, 60 f, 112 f, 130, 135 f, 188 f, 191, 212, 223 f, 265, 321, 338, 342 f, 346: ii. 241, 263, 360: iii. 359.
wher (S. hwar, hwær), i. 26, 54, 273: ii. 125: iii. 93, 319, 357.
wherĕ (i. e., wher), i. 28, 93, 95, 113, 142, 160.
where, i. 93 (?), 349 f: ii. 199 f, 218 (2 cases).

nede,* i. 147: ii. 96, 281 f, 284 f: iii. 309: nedĕ, i. 155.
ofte (S. oft), i. 14, 43, 75, 148: ii. 25, 78: iii. 157.
ofte-time, i. 86, 87, 229: ii. 142: iii. 24: often-time, ii. 287. (?)
selde (S. seld), selde wherĕ, iii. 234 f: selde whanne, ii. 93: iii. 353.
selden, ii. 96, 113, 275, 354.
sone, ii. 135, 250, 255, 335.
eft-sone, i. 203: ii. 295.

thanne, i. 11, 49, 62, 69, 101, 142, 160, 220, 226, 256, 302: ii. 2, 31 f, 62, 185, 232, 355, 363: iii. 39, 371.
thenne (rh. brenne), iii. 36.
than, i. 6, 7, 224: ii. 283: iii. 348, 366.
then (?), i. 17.
whanne, i. 212, 226: ii. 77, 337.
whan, i. 1, 34, 98, 102, 116, 142, 182, 221, 224, 230, 254, 255, 352: ii. 18, 22, 77, 331, 348: iii. 39.

thenne (= inde), ii. 185: iii. 28, fro thenne: i. 130.
whenne (= unde), i. 198: ii. 21: iii. 64, 300 (the *e* elided in the three last cases).
whennĕ, ii. 46: iii. 308; with from, fro, ii. 46: iii. 64.

therefore, i. 109 f, 208 f, 273 f: ii. 335 f, 339 f, 364: iii. 381 f.
to-warde,* i. 315: ii. 357.
towàrd, ii. 13, 374, 387: tòward, i. 122, 356: ii. 247.
wele (S. wela, wel), iii. 149 f.
while,* i. 268: ii. 380.
whilĕ, whil, i. 12, 17, 67, 74, 81, 103: ii. 370.

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a-lofte (S. ?), i. 234 f, 235 f: iii. 136 f, 351 f.

§ 73. The following Particles, of various terminations in Anglo-Saxon, have the termination -es, -s. Many of these have also a form in e: see § 72.

aboutes, iii. 162 f, 358 f.
algates, i. 102.
amiddes, i. 45, 349: ii. 98: iii. 67, 92.
amonges, i. 4: ii. 16, 168, 283: iii. 51, 137, 330.
besides, ii. 359.
elles, i. 1: ii. 203, 205, 211, 336 f.
nedes, i. 108, 228: ii. 48, 249, 346: iii. 251.
ones, i. 106: ii. 130, 333: at ones, ii. 72, 357 f.
thries, ii. 260 (3 cases), 264, 333.
twies, ii. 264.
un-ethes, ii. 20, 118, 318: iii. 6.
up-rihtes, i. 35 f, 140, 367.
-wardes: to-wardes, i. 5, 122, 159: ii. 282, 289: iii. 40.
after-wards, ii. 356: afterward, iii. 37, 39.
whiles, i. 26: iii. 51, 186: whilĕs, ii. 345.
So, for-the-nones, ii. 72 f, 124 f: iii. 144 f, 216 f, 318 f, 357 f.
now-on-daies, now-a-daies, ii. 292, 362: iii. 234, 236, 356.
his thankes, ii. 211 (S. his thances = of his own will).

ELISION OF FINAL VOWELS.

§ 74. Unaccented **e** final may be elided (slurred),

I. before a vowel following:
II. before a few words beginning with **h**:
1. before the pronoun **he** (**his, him, her, hem**):
2. before **hath** (has) and **hast**; before **have**, except perhaps the Infinitive Mood; sometimes before **hadde** (**had**).
3. before the adverbs **how** and **here** (**her**).
4. before two or three words of French origin, in which h is silent.

When one of these words beginning with h ends the verse, no elision takes place before it.

The **e** final of a monosyllable generally does not suffer elision.

Elision seems frequently to be prevented by the cæsural pause.

§ 75. Unaccented **e** final is commonly elided before a vowel.

in ourë tim*e* among*e* us herë, i. 1.
in sted*e* of lov*e* is hatë guided, i. 7.
ensampl*e* and reul*e* of allë tho, i. 10.
and sobr*e* and chast*e* and larg*e* and wisë, i. 11.
and et*e* and drink*e* and housë bothë, i. 14.
wher(e) that he lov*e* or lovë nought, i. 226.
to lov*e* a newe and levë me, ii. 7.
of such*e* as mak*e* a man mishappë, ii. 11.
his lov*e* upon this fair*e* ymagë, ii. 14.
to serv*e* accidie in his officë, ii. 19.
somtim*e* of her is sore adrad, ii. 24.
I serv*e*, I bow*e*, I lok*e*, Iloutë, ii. 41.
in cold(*e*) I brenn*e* and fres*e* in hetë, iii. 9.
this find*e* I writ*e* in poesy, iii. 17.
as ye spek*e* of, what shuld*e* I more, iii. 24.
in latin tung*e* it red*e* and singë, iii. 34.
the form*e* of that figur*e* embracë, iii. 53.

Exception: for whannë I my lady herë, i. 60: qy. whan that?

§ 76. Unaccented **e** final is elided before a few words beginning with **h**:—

1. Before the pronoun **he** (**his, him, her, hem**).

they wold*e* him save, he mad*e* a vow, iii. 182.
ful oft*e* he shuldë chaung*e* his cherë, iii. 212.
and he may help*e* himself also, iii. 173.
but if a princ*e* him woldë reulë, iii. 164.
and in this wis*e* he told his talë, iii. 185.
and there upon they swor*e* here oth(e), iii. 186.
thus held the law*e* his rightë way, iii. 179.
so wold*e* he set him in office, iii. 179.
with lov*e* her hertes to him drawe, iii. 193.
but ned*e* he mot that nedë shall, iii. 309.
and as the tigr*e* his tim*e* awaiteth, iii. 258.
in hopë for to cacch*e* his pray, iii. 258.
but tendrë sham*e* her word delaieth, iii. 261.
and he which wold*e* her wo restreigne, iii. 261.
and Brutus told*e* hem all the tale, iii. 263.
wher(e) dedly prid*e* hem hath conveied, iii. 276.
etc., etc.

But not when these pronouns stand at the end of a verse.

wenendë that it *werë* he, i. 243.
and in this wisë *speddë* he, ii. 74.
but non(e) amendes *haddë* he, ii. 150.
and every man tho *saidë* his, ii. 383.
now tolden they, now *toldë* he, iii. 139.

2. Before **hath** (has?), and **hast**: before **have**, except perhaps the Infinitive Mood; sometimes before **hadde** (**had**).

a. how Mid*e* hath don(e) his curtesy, ii. 133.
which(e) oftë tim*e* hath shent the hous, ii. 151.
Ovidë the poet*e* hath write, ii. 156.
the hert to kep*e* hath for his part, ii. 176.
the mannes soul*e* has reconciled, ii. 187.
the worldes waw*e* hath welnigh dreint, ii. 189.
which(e) holy chirch*e* hath undertake, ii. 190.
his conscienc*e* hath all foryete, ii. 194.
and law*e* hath tak*e* her doublë face, i. 7, 353.
Timon the caus*e* hath undertake, i. 11, 32, 29[illegible].
which every kind*e* hath upon honde, i. 42, 57.

So, i. 31, 32, 43, 52, 57, 63, 82, 123, 128, 130, 153, 164, 169, 183, 188, 189, 205, 211, 218, 243, 264, 265, 268, 276, 292, 299, 355, 359, 363.
ii. 8, 18, 20, 26, 28, 31, 41, 54, 74, 79, 92, 101, 109, 126, 190, 329, 339, etc., etc.

Exceptions:

som(e) causë hath wherof it groweth, i. 264.
a sonë hath which as his lif(e), ii. 324.
men sain that nedë hath no lawe, iii. 277.

of lovë hath within her warde, ii. 354,
(but, in the next verse, Phebus to love hath so constreigned).
which kindë hath and reson can, i. 366,
(but, just before, and either kind[e] hath suche a way).

b. if thou, my sone, hast joië had, i. 167.
of kinde hast formed to be liche, i. 268.
that thou at any time hast chid, i. 295.
which loves cause hast for to guide, i. 328.
now thou, my sone, hast herd this tale, i. 329.
whan thou the tale hast understonde, i. 342.
that thou to love hast don(e) er this, ii. 2.
which thou to love hast so dispended, ii. 62.
as thou, my sone, hast herd above, ii. 111.
my sone, hast thou such covetise, ii. 211.
how thou of love hast wonnë smale, ii. 279, 325, 394.
if thou, my sone, hast wone or lore, ii. 396.
So, that with thy dart brennènd[*e*] hast set a fire, iii. 352.

c. though I siknesse havĕ upon honde,
and longe havĕ had, yet wol I fonde, i. 5.
but I his gracĕ havĕ so pursued, i. 73.
that I misspoke havĕ ought behinde, i. 178.
if I for love havĕ ought don so, i. 224.
till they such love have underfongë, iii. 160.
and that the soule havĕ holy nomë, iii. 101.
as I now havĕ and longe havĕ had, i. 46.
for I no such brocage havĕ used, ii. 283.

Exceptions:
and thannë havĕ they lovë non(e), ii. 394.
qy. they havĕ?
though thou victoirë havĕ on honde, iii. 166.
qy. victoire havĕ *upon* honde?
and thus full oftë havĕ I bought, i. 309.
qy. I havĕ full oftë?

The cæsural pause may prevent elision in the last two cases. See § 82.

d. Not often before **have** in the Infinitive.
e. g. his ownë reule havĕ upon honde, i. 318.
of love havĕ any proprеté, ii. 394.
But, thou might the morë havĕ thy will[e], i. 178.
he thoughtë havĕ and thus aboutes, iii. 162.
his lorë havĕ and in such wisë, iii. 302.
No elision at the end of the verse.
she said him that she woldë *have*, ii. 358.
shold every gentil hertë *have*, ii. 50.
he saide how treson shuldë *have*, iii. 139 : i. 127.
and doth a man great medë *have*, iii. 88.
and he it shall of yiftë *have*, i. 170.
for thou shalt of my yiftë *have*, i. 323.
that he the surplus mightë *have*, iii. 24.
such on(e) there is I woldë *have*, ii. 211.
besidë thilke ymagë *have*, ii. 124.

e. Elision sometimes takes place before **hadde** (**had**): more frequently not.
e. g. for he his love had so beguiled, i. 77.
thus he which love had in dis-deigne, i. 121.
and of the sculle had made a cuppe, i. 128.
wherof the sone had also fere, i. 285.
the god an eye had unto this, ii. 149.
this Adriagne had mochel fere, ii. 308.
But, was hotë, haddë doughters thre, i. 55.
the sceptrë haddë for to right[e], i. 179.
wher(e) they the quenë hadden do, i. 201.
that Romë haddë werres stronge, ii. 196.
a werrë had in jeopartie, ii. 200.
so as the quenë had hem preid(e), ii. 271.
a sonë had, and Androchee, ii. 302.
victoirë had upon his foe, iii. 165.
which lovë haddë for to gie, iii. 364.

Hadde often stands at the end of the verse, and then there is no elision.
that he that time a werrë had, i. 125.
if thou, my sone, hast joië had, i. 167.
wher(e) they the quenë hadden do, i. 201.
and the Romains that timë hadde, i. 219.
was hote, and he a sonë hadde, i. 313.
whom Anchises to sonë hadde, ii. 4.
for I whilom no lovë hadde, ii. 48.
as he whiche all her hertë hadde, ii. 65.
his lust, that he his willë hadde, ii. 169.
So, ii. 35, 69, 169, 232, 236, 303, 342: iii. 194, 226, 238, 271, 277, 283, 293.

3. *a.* Before **how**.
how trewe, how large, how juste, how chaste, iii. 272.
he saide how that betoken sholde, i. 25.
of my fortune how that it ferde, i. 45.
and of old time how it hath be, i. 78.
to teche how that obedience, i. 104.
I rede how that this proudë vice, i. 123.
they shape how they to-gider mighte, i. 129.
and thenke how that I am nought mete, i. 175.
of Rome how that the gentil blood, i. 199.
the waie knowe he shall go, i. 233.

at Troie how that Agàmemnon, i. 242.
and saide how that abed all warm(e), i. 243.
and thenke how she woll me refuse, i. 282.
forthy, my sone, how so it stonde, i. 290.
of pacience how that it stood, i. 303.
maketh knowe how that the Gregois were, i. 313.
to knowe how that the sothe was, i. 351.
ther(e) shall be knowe how that it is, i. 365.

So, i. 52, 69, 78, 183, 228, 308, 330, 352.
ii. 4, 8, 9, 31, 36, 84, 101, 112, 116, 148, 173, 179, 207, 208, 235, 242, 292, 299, 313, 314, 317, 333, 340, 358, 361.
iii. 49, 61, 90, 138, 139, 141, 151, 154, 160, 234, etc., etc.

Exceptions (?)

if no man writë how it stood, i. 4.
and thoughtë how(e) it was not good, i. 269.
and all the causë how it went, ii. 122.

In these cases it is probable that we should read *how that*, which phrase actually occurs in twelve out of the eighteen verses cited above.

b. Before the adverb **here** (**her**) (when not at the end of the verse).

what shall befalle here afterward, i. 3.
and for to beare herof record(e), i. 70.
lo, sone, her(e) might thou taken hede, ii. 50.
I not what falle herafter shall, ii. 278.
of dedely peine here afterwarde, iii. 37.
my sone, herafter thou shalt here, iii. 145.

[It is to be observed that falle[n], beare[n], may be read as monosyllables. The other three cases cannot be explained away, if the readings are correct.]

Examples of here at the end of the verse.

thou shalt havë no penauncë here, ii. 43.
it thoughte her faire, and saidë, here, ii. 45.
whil(e) that they were alivë here, ii. 171.
well mo than I the tellë here, ii. 175.
endure upon this erthë here, ii. 269: i. 37.
among us upon erthë here, iii. 94, 38.
benethe upon this erthë here, iii. 106.
upon this wofull erthë here, iii. 277.
among us on this erthë here, iii. 379.
above all other on erthë here, iii. 381.
that Crist(e) in erthë taught[ë] here, i. 15.
but this which(e) I you tellë here, i. 168.
a man to livë chastë here, ii. 342.

4. Before two or three words of French origin.

a. the vein[*e*] honour was nought desired, i. 11.
for thilke honour whiche Aaron tok(e), i. 261.
and hindreth many a cause honest, ii. 9.
of armes thilke honour forsake, ii. 64.
that love honest in sondry wey, ii. 78.
of treble honour he was certain, iii. 165.
of pees, richesse, honour, and welthe, iii. 273.
may never be to lovës lawe honeste, iii. 352.

But: which techeth thilkë honesté, iii. 141,
but, upon allë honesté, iii. 272,
(where elision is prevented by the ictus.)

b. to feigne humilité withoute, i. 66.
and with low(e) herte humblessë sue, i. 118.

c. they havĕ of thilke horrible sinne, i. 77, 76.
that thilke horrible sinfull dede, i. 365.

d. and of his quenë dame Heleine, ii. 230.
tho was in thilë quene Heleine, ii. 384.
of that Paris had wonne Heleine, ii. 387.
(*cf.* after his moder quene Eleine, i. 276.)

We find,
and saide Ha, suster, if ye knewe, ii. 320.
and whan he wok(e) he saide, Ha, wif(e), iii. 310.

But saide should perhaps be printed said, as
and *said* Ha, now thou art atake, ii. 338,
or Ha should perhaps be Ah.

We find,
receivë till he saidë ho, ii. 201.
I woll the telle and thannë ho, iii. 274.

§ 77. Except in the cases mentioned above, **final e** is not elided before **h**.

e.g. which hath the proudë herte fired, i. 11, 189.
unto his ownë harme it groweth, i. 53, 172.
that sleighte shuldë helpe thanne, i. 79.
for I havĕ oftë herd you saide, i. 148.
in allë haste his cause spedde, i. 180, 193: iii. 258.
his wif(e) Heleinë hight also, i. 199.
but he that thannë herd hem two, ii. 323.
for lovë hateth nothing more, ii. 18.
and shamë hindereth every wight, iii. 151.
the gretë hete in which I brenne, iii. 36.
upon his gretë holinessë, i. 257.
etc., etc., etc.

§ 80. The **e** of monosyllables is rarely elided, except in the case of the Definite Article.

wherof I holdë *the* excused, iii. 354.
but upon *the* it shall be bought, ii. 282.
nought all perchaunce as *ye* it wolde, iii. 354.
and *he* it haddë when I went, ii. 298.
of him, or *be* it evil or good, ii. 285.
that he *be* of his entré let, ii. 347.
misteppë, but he *se* it all, ii. 143, 374: iii. 5.
into the deepë *see* he caste, ii. 156, 334.

So even with ne: —

none other office I *ne* have, ii. 48, 131: i. 134.
thus hath he that he nought *ne* hath, ii. 129.
that *ne* ought to desirë pes, iii. 379.

But,

tho spak(e) he to *me* in such a wise, i. 51: (qy. such wise?)
my godë sonë, god *the* amende, i. 61.
whil(e) that ther(e) lasteth *me* any breth, i. 289: (qy. last?)
that I *n*approche her ladyhede, ii. 40.

§ 81. The **e** of the Definite Article is generally elided (especially before e), as in modern English verse. The th is frequently united to the following word.

of rhetoriquë th*e* eloquence, iii. 40.
and thus th'unkind*e* unkindë fond(e), iii. 282.
and tho ben as th'apostel telleth, i. 18.
wher(e) th'emperour him self shall be, i. 208.
and who that taketh (takth) away th'onour, ii. 153.
it environneth th'erth*e* about[e], iii. 92.
a mannes ey*e* is th'erë nerre, iii. 95.
among th'ebrews was none in sight, iii. 240.

§ 82. The cæsural pause seems sometimes to prevent the elision of final **e**, but an incorrect text forbids the forming of a decided opinion.

he weptë — and with woful teres, i. 143.
with strengthë — of his ownë might, i. 236.
supplant of lovë — in our waies, i. 241.
in the croniquë — as I finde, ii. 82.
kisse her eftsonë — if I sholde, ii. 96.
with all min hertë — I woll serve, ii. 110.
though he ne woldë — it allowe, ii. 146.
and in worshippë — of her name, ii. 171.
and with spellingë — and her charmes, ii. 263.
Jason bar(e) crounë — on his hed(e), ii. 267.
her lov*e* is sonë — after (aft'r) ago, ii. 300.
with shamë — and the nimphes fledde, ii. 337.
which kindë — in her law*e* hath set(te), i. 268.
by whom that timë — eke had he, ii. 378.
and doth grevauncë — on som sidë, i. 304.
and set his welthë — out of herrë, iii. 52.
besought and toldë — hem of this, iii. 57.
of heven, of erthë — and of helle, iii. 87.
lo thus, my sonë — it hath ferd(e), iii. 97.
after the lawë — of justice, iii. 155.
for we be bothë — of o kinde, iii. 168.
which of long timë — he hath hid, iii. 205.
the wey on(e) fotë — in despeir(e), iii. 217.
to her which mentë — alle good, iii. 257.
that he his sonë — Isaac, iii. 279.
unto the gamë — all[*e*] and some, iii. 298.

graunt mercy, lordë, he answerde, i. 272, should read *lordës*.

§ 83. Other vowels are sometimes elided, as in modern verse.

to whom god gaf so worth*y* a yifte, iii. 136.
her stud*y* at thilkë timë so, ii. 91.
and of that thing right merr*y* hem thought, ii. 253.
for ther(e) be man*y* untrewe of tho, ii. 329.
to Jun*o* it was don(e) understonde, ii. 281.

SILENT FINAL e.

§ 84. Were we to follow Pauli's capricious spelling, we should have as many cases of silent final **e** as we find in Wright's text of the Canterbury Tales, or more. But if we follow the general rules deduced from the text, guided by the metre and assisted by the history of the language, we shall find very few such cases. The only cases, indeed, which are supported by instances enough to make them of consequence are the words **have**, **here** (= their), **were**, **more**, and the termination **-fore**, (to-fore, be-fore). The pronoun **here** is often written **her**, and there seems to be no reason why the e should be preserved in this form unless the same is done in the case of the pronouns **our**, **your**, **her**

(fem.), which are written with an **e** only when that vowel is pronounced or elided. The pronominal forms, the comparative **more**, and the verbs **have** and **were**, were necessarily very much used, and were very likely therefore to be shortened. All of these, excepting the pronoun here (their), are found both in the shortened and the full form.

Several words besides the pronouns just mentioned are found written both with and without a final **e**. Thus, —

and my manèrë so mistorned, iii. 5.]
in such manèrë for to ledë, iii. 141.
in what manèr she shuld him save, ii. 306.
and write in such a màner wise, i. 4.
and right of such a màner kindë, i. 88.
So, i. 107, 123, 184, 206, 342, etc.

(It will be observed that when the e is dropped, the accent is generally thrown forward.) We find also màter written twice without a final e, i. 146, 180, but as in both cases the following word begins with a vowel, we cannot be sure that the **e** is not lost by elision: matèrë is found, i. 343, 365: ii. 207, 383: iii. 157, 176, etc. We have also the double forms còmun, comùnë; divèrs, divèrsë; here the longer form seems to be a license for the sake of rhyme. See § 30, *b*; § 35, *d*.

The Comparative of Adjectives is always written in Pauli's text, as it generally is in Wright's text of the Canterbury Tales, with -er, instead of the Saxon -re. French words are written indifferently with both terminations, though the French **re** is more commonly preserved than not: thus, letter, i. 10, 201, 305: iii. 314; tender, i. 115, 266; fever, ii. 146; chamber, i. 102; ponder, i. 24: and tendre, i. 115; chambre, i. 112; chartre, i. 155; ordre, i. 50; monstres, i. 56; suffre, i. 334; attempre, i. 333, 335; sobre, i. 11: iii. 365. We even find philosophre, in imitation of the French spelling, iii. 234, 272. Slight reliance, however, is to be placed upon the editor's spelling, and from this alone we could not be justified in assuming that -re was already pronounced as -er.

§ 85. The e final of **here**, their, is silent;* *e. g.*, i. 5, 8, 9, 10, 17, 52, 59, 63, 80, 81, 113, 114, 115, etc. The form **her** is perhaps equally common: i. 10, 17, 19, 62, 69, 87, 88, etc.

In a few cases **oure**, **youre**, have the e sounded, as, ourë, i. 262; ii. 90, 192, 215: yourë, ii. 108, 226: i. 316. (See § 45.) The forms **our**, **your**, are more common: our, i. 191, 192, 329, 359: ii. 191: iii. 379, 380, 385; your, i. 150, 153, 155, 166, 178, 210, 272, 300, 328, 371, etc.

The dative and accusative of the feminine personal pronoun often preserves the Saxon e: see the forms **hirĕ**, **herĕ**, § 45: but **her** is much more common for the personal pronoun, and is the only form for the possessive: see, i. 7, 47, 54, 56, 68, 69, 70, 71, 72, 73, 74, 75, etc. Accepting the spelling of Pauli's text, we can only say that here = their has the e silent. (No account can be taken of a few cases in which, for example, youre is found before a word beginning with a vowel.)

§ 86. **were**: e final is sometimes "silent" in **were**.

werĕ thou afered of her eye, ii. 21.
she bar(e) and they werĕ cleped tho, ii. 165.
six gatës werĕ there of the town, ii. 376.
and was long time or we werĕ bore, i. 231.
for me werĕ lever (levr) to lackë breth, i. 177.
but he werĕ lustles in his herte, i. 227.
So, i. 265, 309, 333 (?): ii. 93, 143, 212, 276, 296: iii. 212, 265, 365.

On the other hand, werë, pl. Imperf. Ind., i, 4, 10, 33, 55, 75, 78, 79, 101, 110, 111, 115, 127, 170, 189, 212, 242: ii. 74, 80, 88, 148, 166, 173, etc., etc.

werë, Subj., i. 8, 15, 39, 96, 156, 197, 207, 210, 211, 233, 241, 243, 283, 305, 333, 356, 359, 365: ii. 33, 62, 80, 88, 114, 141, 154, 173, 190, 226, etc., etc.

§ 87. The foregoing are the only cases of **e** silent after **er**, except a few isolated ones: *e. g.*

ther halp(e) him nouther *sperĕ* ne shelde, i. 125.
for if thou *herĕ* my talë wel(e), ii. 340.
he yav(e) hem *answere* (answre?) by and by, iii. 305.

It has been observed already that such representatives as occur of the Saxon noun in -ere, denoting an agent, want the final vowel, but none

* I mean by a "silent" e (ĕ) one that does not form a full syllable. Whether the letter was absolutely mute, or slurred, or, in words ending in -re, pronounced *before* the r, I do not pretend to say.

of the few cases that occur are worth much: see § 8.

§ 90. E final is sometimes silent in **-fore** and **more**.

-forĕ (to-fore, be-fore), as, i. 59, 117, 138, 155, 201, 205, 321, 348: ii. 120: iii. 61.

On the other hand, -forë (to-fore, be-fore, a-fore), i. 32, 204, 209, 364: ii. 47, 154: iii. 40, 44, 227, 273, 342.

morĕ, i. 158, 168, 180, 198, 199, 203, 210, 263, 265, 337, 349: ii. 2, 3, 57, 76, 136, 146, 149, 157, 171, 213, 226, 276, 335, 389: iii. 44, 54, 214.

But oftener morë: i. 60, 61, 129, 168, 198, 208, 237, 294, 296, 297, 299, 306, 319, 336: ii. 2, 7, 13, 15, 38, 60, 76, 78, 88, 98, 117, 135, 156, 186, 207, 209, 232, 255, 279, 309, 310, 311, 324, 345, 348, 381, 383: iii. 19, 20, 24, 25, 44, 88, 106, 202, 213, 216, 231, 234, 275, 295.

Differently in successive lines:—

the wo no *morĕ* than the welë,
no *morĕ* the hetë than the chelë, ii. 44.

We find two forms, sirë and sirĕ = sir; corresponding to French sire, sieur, Italian ser, sere: e. g.

ha, levë sirë, tho quod she, iii. 301.
but, sirë, for it is nigh day, ii. 246.
and saide, sirë, graunt mercy, ii. 314.
Antiochus, the gretë sirë,
which full of rancour and of irë, iii. 292.

But, sirĕ king, quod he, and that I can, i. 322.
sir king, if that it werë so, iii. 226.
she shall be rendred forth with her.
she saith, graunt mercy, levë sir, iii. 317.

§ 91. Disregarding the numerous demonstrably misspelt words in Pauli's text, the only other important instances of silent **e** final are the word **have** and some forms in **-ce** (se).

e is generally silent in **have**, except at the end of a line.

now havĕ ye herd(e) and I havĕ said(e), i. 178.
as ye havĕ told(e) I havĕ well herd(e), ii. 235.

So, I havĕ, i. 47, 57, 60, 134, 161, 162, 177, 224, 227, 228, 229: ii. 13, 25, 31, 39, 61, 75, 107, 119, 124, 278: iii. 21, 31.

ye, they, havĕ, i. 27, 59, 63, 76, 103, 104, 124, 127, 131, 138, 157, 170, 177, 222, 228, 274.

subjunctive, i. 105, 161, 175, 329: ii. 31: iii. 346.
imperative, i. 287: ii. 96.
infinitive, i. 85, 91, 93, 94, 102, 103, 124, 236, 240: ii. 6, 9, 16, 20, 21, 46, 102, 103, 107.

But, (I) ne havë whan I spak(e) most softe, i. 296.
ye havë thilkë vice of slouthë, ii. 55.
all though him selvë havë non(e), i. 295.
be so they havë swerd(e) or knif(e), i. 316.
madame, if ye wolde havë routhë, i. 47,
and (infinitive), i. 94, 170: iii. 222, 302.

The infinitives and the plural forms of the indicative and subjunctive may have originally been written haven; so written, the word might perhaps have been contracted at pleasure into a monosyllable.

e is in a few words of Latin origin silent, or absent where it might be expected, after **c**, **s**.

gracĕ rh. encres (O. Fr. a-crois), ii. 392: gracë, i. 9, 106, 115, 122, 208: ii. 25, 302, 303.
Bonefacĕ, i. 258, 261: but rh. grace, i. 258.
Moricĕ, Moris, = Maurice, i. 206, 211, 213, 191.
forcĕ, rh. hors, ii. 392.
fallas (Fr. fallace), iii. 158, rh. was: fallas inne, ii. 85.

We find use (usĕ) written in several cases where we should undoubtedly read us (Fr. us): as

to mannes us(e) wherof I rede, ii. 132.
rh. vertus(e), i. 15, 50: iii. 19, 136.
rh. jus(e) (Fr. jus), ii. 266.
rh. refus(e) (Fr. refus), iii. 298.

We find, "of avaricĕ the progenie," ii. 290, but avaricë, ii. 127, 131, 284, 289, which makes it probable that the line should be read "of avaricë the prog'nie."

We find pursĕ, iii. 155, purs, ii. 298. This word derived from Middle Latin bursa, probably does not come to us through the Fr. *bourse.* It has dropped the e, like Swed. and Dan. *börs,* and Germ. *bors,* (which is found as well as *börse*).

lacĕ, iii. 237, from O. Fr. lac, las, has no right to the e.

purchas, noun, ii. 331, 351 (O. Fr. pourchas), is not to be confounded with the verb.

Noteworthy instances of e final silent after other consonants than those already mentioned are very

few. By noteworthy instances is meant cases in which a final e, that by general laws should be sounded, is required by the metre to be silent. Some of the apparent exceptions can be easily explained away. A few cannot. Thus, we find, helpĕ, help, i. 30, 204, 331: ii. 22: iii. 215 (twice), 224, 267, eight cases, to nine of helpë: quenĕ, ii. 212: iii. 338, but quenë twenty-seven times: sight, ii. 243, 321 (rh. night), but sightë six times: food, ii. 362; iii. 26, 30 (rh. good), but fodë five or six times: timĕ, ii. 167, but timë everywhere else: nedĕ, i. 155, but nedë elsewhere: spedĕ, i. 88, 186: ii. 395, and spedë about as often: I redĕ, iii. 47: i. 117, elsewhere redë. We also find "now tell, my sonĕ: my fader, what," i. 317, but elsewhere sonë always; "for than he woll his hopĕ reherse," ii. 216, elsewhere hopë; "for erth which meinëd is with steel," i. 25: "her gold under the erth begrave," ii. 197, elsewhere erthë; "I havĕ an hertĕ liche unto thine," i. 359: "his hert stod(e) in a sory plit(e)," i. 75: "thy holë hert fro the she tok(e)," ii: 279: "min hert stant ever in o stedë," ii. 3, hertë in innumerable cases; "for I dar(e) unto this trouth dwelle," ii. 226, elsewhere trouthë; "that his eyĕ might he nought witholde," ii. 315, elsewhere eyë.

In these last instances we should perhaps read,—

now tell, my sonë: fader, what.
for than he woll his hap reherse.
for erthë which is meind with steel.
her gold undèr the erthë grave,
(*or*, under the erth*e* her gold begrave.)
I havĕ an hertë lich to thine.
his hertë stod in sory plit.
thy holë hertë fro the tok.
min hertë stant ever (ev'r) in o stede.
for I dar to this trouthë dwelle.
that his ey*e* he might nought witholde.

skill occurs, but only as a rhyme to will, i. 42, 49, 104: ii. 330: iii. 59, 180, 185, 227. Elsewhere we find skille (11 cases), and even rh. wille, i. 277, 292, 352: ii. 312: so that we should probably read skille, wille, in all the above instances.

bridĕ (i. 102), florĕ, iii. 337, probably belong with the exceptional feminine nouns noted under § 17, and if so should have no e. Hond, night, world, might have been expected from the peculiarity of those nouns in Saxon, but hondë, nightë, worldë also occur. See § 17.

CONTRACTIONS.

§ 93. Final **er** is very frequently contracted especially under circumstances in which final **e** would suffer elision: *e. g.*

adder (= addr): in likenesse of an *adder* he slipte, ii. 72.
after (= aftr): that nevër *after* that ilke day, ii. 23.
better (= bettr): that but thou be the *better* avised, i. 94.
that *better* him is to flee than save, iii. 220.
chambre (= chambr): somtime in *chambre*, somtime in halle, ii. 39.
ever (= evr): and *ever* hath do, sith it began, i. 36.
that *never* hereafter shall be lassed, i. 103.
that he *never* aftër getë may, ii. 2.,
that I *never* yet to chirchë went[e], ii. 371.
yet might I *never* have o repaste, iii. 32.
fader (= &c.): iwis my *fader* no more I shall, i. 83, 237.
hinder: to *hinder* and shove another out[ë], i. 238.
and *hinder* hem all that *ever* I may, i. 228, 177.
lenger: me thenketh (thenkth) is ay the *lenger* (lengr) the ferrë, i. 281.
letter: this *letter* as thou hast herd devisë, i. 192.
lever: for him was *lever* in the bataile, ii. 66.
monster: a cruel *monster*, as saith the gest, ii. 304.
other: to warn*e* all *other* of such deceipt, i. 237.
if o woman an *other* beguile, ii. 234.
abov*e* all *other* the moste chef(e), iii. 11.
suster: that to my *suster* ne to my brother, i. 148.
tender: was faire and fressh and *tender* of age, i. 68.
thunder: with *thunder* and lightning is forsmite, iii. 308.
weder: the *weder* was merie and fair inough, i. 112.
whether (= whe'r): *wher(e)* that he love or lovë nought, i. 226.
wher(e) she be clothed or be naked, ii. 350.
se *wher(e)* this were a noble wif(e), iii. 364.
So, iii. 5, 110, 231, 303, 322, etc.

written *wheder*, ii. 121, though contracted.
and out of Romë *forever* (= frevr) exiled, iii. 247, seems to be a case of two contractions.
and *tarie* (= tar') thou in my court no more, iii. 373, is essentially like the above cases.

Letter (i. 192, 193, 194, 201, twice: iii. 313, 314), *monster*, *tender*, if spelt in the French way, could be treated as cases of simple elision, and *adder* (derived like *laddre*, iii. 330, from a Saxon feminine) might also be written with *-re*. *Chambre*, it will be observed, though written in the French way, is contracted to a monosyllable, as if spelt with *er;* or pronounced, as in French prose, with the *e* mute.

§ 94. The vowel is elided under similar circumstances in the syllable **-en**.

men *shulden* it in the prestes finde, i. 76.
as they that *wolden* his thank deserve, i. 79.
they *treten* and axen of her wille, i. 250.
to *geten* hem helpe, and atë laste, i. 339.
to *vengen* him at his ownë will[ë], i. 345.
unnethë *stonden* in any doubt[ë], i. 364.
a *woman* upon an hors behindë, ii. 46.
such *wepon* also for him she dight[ë], ii. 306.
I shall *rehercen* unto thin ere, iii. 19.

§ 95. The third person singular of the Present Indicative is frequently found with an unpronounced **e** in the termination, and perhaps should be written without this letter, as commonly in Saxon. Following the printed copy, we may describe the **e** as elided.

which *speketh* of Peter and of John, i. 64.
which *maketh* the hertes eyen blinde, i. 68, 156.
he *wepeth*, he *crieth*, he axeth grace, i. 120.
but she, that *kepeth* the blindë whel(e), i. 126.
and *leseth* (qy. lest?) full many timë grace, i. 305.
he *eteth* and *drinketh* the bestë drinke, iii. 39.
but *taketh* what thing *cometh* next to honde, iii. 280.
the mede *ariseth* (arist?) of the service, iii. 342.

§ 96. Miscellaneous Contractions.

yet in the *bible* this name is bore, 136.
ther(e) may no trewë *quarel* arise, ii. 223.
the *devil* that lieth in hellë fast, iii. 203.
and so *distempred* and so esmaied, i. 281.
the wombe hath two, the *heved* hath thre, iii. 117, 119.
the *heved* planete is nought to wite, iii. 376.
till *augst* be passëd and septembre, iii. 370.
unto this signe is *augst* applied, iii. 121.
Sortes [Socrates] and Plato with him come, iii. 366.

Benedicite is not contracted, as in Chaucer:—
and saidë, *benedicite*,
my sone, of the felicite, i. 48.

§ 97. Contiguous words are not often blended, but some cases occur.

fall it (fall't) to the beste or to the worst[ë], ii. 380.
it is (it's) unto lovë no disport(e), iii. 348.
my sone, I havĕ (I've) herd(e) of thy matere, ii. 61.
I might amendë that is (that's) amis, iii. 247.

§ 97*, *a*. Accent.—Many words of French origin have a variable accent: the same is occasionally true of native words. The eliding of final e often causes the accent to be thrown back.

resòn, i. 333, 360: ii. 131: iii. 272.	rèson, i. 352, 357: ii. 120.
purpòs, i. 134, 144, 162.	pùrpos, i. 238, 250, 325.
labòur, ii. 82.	làbour, ii. 80.
colòur, i. 225.	còlour, i. 133.
revèrs, i. 230.	rèvers, i. 167.
powèr, i. 345, 347: ii. 187: iii. 353.	pòwer, i. 341, 359: ii. 106.
daungèr, i. 331, 332.	daùnger, i. 331.
forèst, ii. 68.	fòrest, i. 119, 142, 326.
povèrtë, i. 357.	pòverte, i. 355.
vertù, ii. 187.	vèrtu, ii. 38.
supplànt, i. 239, 253.	sùpplant, i. 239, 253.
passàge, i. 223, 237: ii. 65.	pàssage, i. 237.
visàge, i. 237 f.	vìsage, i. 227: ii. 9.
manère, i. 96 f: iii. 5, 341.	màner, i. 4, 107, 184, 206, etc.
matère, i. 343: ii. 207, 383: iii. 157, 176.	màter, i. 146, 180 (e elided).
fortùne, i. 20 f, 22, 88, 130, 337.	fòrtune, i. 5, 6, 8, 105, 210.

* This section corresponds to § 99 of the paper on Chaucer, there wrongly put among the Miscellaneous Notes.

comùne, i. 20 f: iii. 152 f, 159 f.	còmun, i. 7, 9, 16, 55, etc.
envioùs, i. 171, 174 f, 175.	envìous, i. 172, 173, 217.
lòvers, i. 175, 228.	lovèrs, i. 64, 227, 238.
ànswere, i. 331: iii. 305.	answère, i. 96 f, 97, 146 f, 331, etc.
wòrthy, i. 226: ii. 290.	worthỳ, i, 107 f: ii. 224.
làdy, i. 332: ii. 225.	ladỳ, i. 332: ii. 227, 279.
ayeìn, i. 81, 88.	àyein, iii. 61, 237.
felàw, i. 170, 171.	fèlaw, i. 171: ii. 208.
Achillès, ii. 62, 127, 232.	Achìlles, ii. 58, 227, 229.
Apollò, ii. 366.	Apòllo, ii. 367.
Noè, ii. 181 f: iii. 133 f, 278.	Nòe, iii, 102, 278.
Jàsòn, ii. 251, 253.	Jàson, ii. 250, 253.
Leò, iii. 121, 127 f.	Lèo, iii. 120.
Venùs iii. 122, 131 f.	Vènus, iii. 119, 123, 126.
Echatès, 260.	Echàtes, ii. 262.

Proper names of Latin origin have generally the French, or foreign, accent: Cesàr, ii. 366: Medeà, ii. 212, 251: Gowèr, iii. 373: Eneàs, Anchisès, ii. 4: Aprìlle, ii. 327.

We find "was inhabited here and there," i. 324.

b. At this point it is proper to say that in al likelihood some troublesome forms in Gower are to be explained as simple licenses. Such, very probably, are the cases of the singular of the Imperfect of Complex Verbs which have an *e* (§ 54, *a*). So when *vertu*, ii. 38, 187, is stretched to *vertuë*, i. 7, 18; when the proposition *for* is made to rhyme with *borë*, ii. 59, the pronoun *min* with *minë*, ii. 130, the noun *men(e)* (Fr. *moyen*) with *lenë*, ii. 351, (if thou well) *bethought* with *nought*, iii. 357, (I) *sigh* with *eyë*, iii. 370, *oxes* (elsewhere oxen) with *foxes*, ii. 63, perhaps all that it is necessary to say is that a clumsy poet has taken an extraordinary liberty. Such shortening of words as *pusillamité* for pusillanimité, ii. 12, 25: iii. 210, Climestre for Clytemnestre, Methamor for Metamorphoses, is rather to be attributed to ignorance; so Agamenon for Agamemnon, Nanplus for Nauplius, etc. The vowels are not infrequently freely treated in the rhymes: *e. g.*, minde, ende, ii. 23, 67; ende, kende (i. e. kinde), iii. 120; nine, peine, ii. 261; seen, eyen, iii. 18; say, see, iii. 31; wit, yet; fell, hill; men, kin, ii. 158: iii. 211, 280; kenne, senne (i. e. sinne), ii. 309; spedde, hadde, ii. 191; deth, geth (i. e. goth, i. 345, Sax. gæð), ii. 303: i. 220, 247; piche, suche, iii. 312, etc.

MISCELLANEOUS NOTES.

§ 98. Letters.

a. **ch**, for Saxon **c**, **(k)**, before or after e, i, in cases where modern English has the primitive sound.

seche (= seek), i. 290: ii. 190, 193.
reccheth (= recketh), i. 168: ii. 284.
worchen (= work), i. 166: ii. 142.
schenche (= skink), i. 263.
bishopriche (= bishopric), i. 10.
lich, liche (= like), i. 118, 136, 258, 265.
So besi-liche, ii. 3, 43; even-liche, ii. 179, etc., now shortened to -ly.

Saxon **c** (**k**) not changed to **t** as in modern English: make (= mate), i. 45, 112, 367. **cc** changed to **tt**, when changed to **ch** in modern English, fette (S. feccan = fetch), ii. 233, 237.

We find chever = shiver, iii. 9.

b. Saxon **g** changed to **y**, where we have retained **g**.

ey (S. æg, E. egg), iii. 76, 105.
yaf, yive, yove (gave, &c.), i. 111, 114, 127.
foryete (forgotten), ii. 129, foryetelnesse, ii. 19.
ayein (again), i. 83, 88.

On the other hand,

ligge (S. licgan, E. lie), ii. 73, 218.
bigge (S. bycge, E. buy), ii. 187: a-begge, i. 340, a-bey, ii. 40.

gg (S. cg) where we have the sound **j** (dg), brigge (S. brycg), ii. 201: iii. 160.
gh (S. h), where we spell with **w**: slough (slew), i. 165: fleigh (flew), ii. 335: iii. 96: withdrough (drew), iii. 103 (drowe, iii. 198).

Saxon **g** changed to **w**, where we have y.

dawes (S. dagas, days), i. 136: ii. 113, 176: iii. 182, 183.

morwe (S. morgen, E. morn), i. 186, 205.
wowe (S. wag, E. wall).
wawes (S. wægas, E. waves), i. 141, 223, 312.
gerarchie = hierarchy. iii. 145, is Old French *gieraucie*, Ital. *gerarchia*.

c. Saxon **d** retained, where we have changed to the aspirate **dh** (spelt **th**).

fader, i. 49, 60, 61 : iii. 260, 332 : father, ii. 174, is undoubtedly wrong.
moder, i. 104, 181, 191 : iii. 251, 368.
weder, i. 112, 140, 223 : iii. 351 : wether, iii. 295, is wrong.
hider, i. 70 : ii. 61 : iii. 37.
thider, i. 186, 209 : iii. 335, 336.
whider. ii. 21, 117, 144.
gader, ii. 293, 296 : togider, i. 324, 340

On the other hand we find rother (S. rôðer), = rudder.

th dropped after **t**, in contracted forms:

ate (at the) bord, iii. 299.
ate laste, i. 16 : ii. 345, 377.

d. Metathesis of **r** and of **s**:

brid, *bird*, i. 112, 133, 206, etc.: bird, i. 206.
hunderd, *hundred*, ii. 92, 249, 381.
thrid, *third*, i. 55.
thritty, *thirty*, iii. 214.
brenne, *burn*, i. 334 : brent, i. 109.
kerse, *cress*, i. 299 f, 334 f.
Adriane, *Ariadne*, ii. 307, 306, 308, 309 : iii. 361.
axe, *ask*, i. 334 : ii. 222 : iii. 308.

e. **m** reinforced by **b** or **p**:

thombe (S. þuma), i. 175.
stempne (S. stemn), i. 312.

n changed to **m** before **p**:

wimpel (S. winpel), i. 326, 327.

n not yet reinforced by **d**, as in English:

kinled = kindled, iii. 96. Cf. kin-d-red, and kind, *genus*, which is apparently from Saxon *cynn*, not *cynd*.

s reinforced by **t**:

lost (for *loss*, S. los), i. 147 f, 238 f: ii. 186, 277. We find loss, i. 270.

§ 99. See § 97.*

§ 100. Syntax for KIND, QUANTITY.

b. Maner (= kind) followed not by a noun with *of*, but by a noun in apposition (like Germ. *art*).

a maner kinde, i. 88, 123.
what maner name, i. 206.
such a maner wise, i. 342.
what manner thing, ii. 142, etc.

c. Things numbered put in the singular after numerals (as in Germ. and A. S.)

twenty *winter* age, ii. 266.
of eigh[te]tenë winter age, i. 102.
withinne seven winter age, i. 267 : ii. 266.
of nine hundred winter old(e), ii. 265.
of thre yer(ė) age, ii. 22.
of twelv(e) yer(e) age, ii. 68.

So after numerals preceded by **a**:

of an hundred winter age, ii. 343 : of a ten yer(e) age, ii. 17.
a thousand winter (tofore, after), i. 267 : ii. 266.
a thousand yer(e), ii. 9 : a ten mile, i. 209.
a thousand sithe, i. 160 : a thousand score, i. 176.
a thousand del(e), i. 295.

The Saxon use of winter for year is to be noticed, and also the *of*, supplying the place of the Saxon genitive, in *old of* nine hundred winter.

d. Besides the foregoing, we find these modern phrases:—

a thousand times, i. 330.
a fewë yeres, iii. 246.
seven yeres, ii. 9.

§ 101. GENITIVE CASE.

b. Genitive sign not annexed to a compound phrase:

in Vestes temple the goddesse, ii. 157.
the kinges daughter of Cecile, i. 104, 235.

c. The Genitive Case of classical proper names is frequently used as a Nominative.

so is Sibeles of goddèsses the moder, ii. 265: Sibele, ii. 166.
Cereres, ii. 168, 170, 177: Ceres, ii. 168.
Circes, iii. 49, 50, 56, 362: Echates, ii. 260, 262.
So Spercheidos, ii. 261.

Sometimes classical proper names are declined (a custom still with some old-fashioned Germans).

unto the temple Apollinis, ii. 366.
that he wolde
upon knighthode *Achillem* sue, iii. 212. Achilles, nom., same page.
and Delboram hath Abel take, iii. 277. Delbora, nom, same page.
till they Pentapolim have take, } iii. 341.
by shippe for Pentapolim, }
at domesday shall with him bring Judeam, ii. 191.

Ephesim (i. e. Ephesum), iii. 335, 336, 338.
Thelmachum his sone he shette, iii. 54. Thelemachus, iii. 60.
(On i. 55, we find Methamor for Metamorphoses.)

§ 102. Dative Case.

a. After *to be.*

be him lief, or be him loth, i, 97: ii. 27, 65.
she, which was him nothing loth, ii. 240.
me is lever, iii. 268: i. 177: you is lever, ii. 205.
him were better, iii. 241: him werë best, ii. 306.
er him be woo, i. 78: wel you be, i. 210.
what may you be? iii. 260.
wo the be, i. 98 (wo *thou* be, i. 295): wo worth false envy, iii. 320: wo worth alle slowe, iii. 362.
So, but yet *that other* werë lever
havĕ had the loking of his eye, i. 305.

b. After verbs of motion, as in Saxon: —

goth him, i. 96, 256: iii. 257, 292, 294, 318, 319, etc.
goth her, ii. 258: iii. 331.
gon(e) hem, i. 115: ii. 387: iii. 253.
comth him, iii. 50.
rod(e) him forth, iii. 57.
him hasteth, i. 119.

c. After other verbs: —

he drad him of his owne sone, iii. 54: ii. 239.
as he which drad him of vengeaunce, iii. 321 (as in Saxon).
him thenketh, i. 133: iii. 284, 329.
thoughte him, i. 142.
her thoughte, ii. 73.
that thoughte us, iii. 309.
if you thenketh, i. 135.
So, for though it thenke a *man* first swete, iii. 281,
it thought a kinges *doughter* straunge, ii. 319.

§ 103. Pronouns. — Personal Pronouns.

it am I, ii. 123.
it am nought I, iii. 6, as in Saxon and German.
So, it bin delites = *es sind*, &c., iii. 33.

We find her lord(e) his herte, for her lords heart, i. 235; but as this is the only instance of this analytical form which I have noticed, I suppose we should read her lordës or lordïs.

§ 104. Demonstrative and Relative Pronouns.

These used somewhat like Latin *ille:*

as tellen us *these* oldë wise, i. 300, 62, 63.
whan that *thes(e)* herbes ben holsome, iii. 161.
so as *these* oldë gestes sain, iii. 246.

a. **Which** with a personal pronoun, expressing only a single relative.

that I may se min hus[e] bonde,
which whilom he and I were one, iii. 337.
let in the strem(e), *which*, with gret paine,
if ever, man *it* shal restreigne, i. 21.
her worthy fader, *which* men saide
that *he* betwene her armës deide, i. 212.

b. **Which** has frequently the signification of *what, what sort of* (like *welch* in German), as:

which a sorwe! iii. 3: o *which* a sinnë violent! iii. 244.
i. 69: ii. 42, 140, 309, 388: iii. 202, 262.
So *what* = what sort of: he axeth hem *what* child that were, iii. 325.

c. **What** = thing:

which was the lothliest[ë] *what*, i. 98.

as he which couthë mochel *what*, i. 320 (cf. some what).
love is bought for litel *what*, ii. 275.

f. **Who so, who that** = siquis (with the Subjunctive).

and, *who so thenke* therupon,
his namë was king Pandion, ii. 313.
the boke of Troië *who so redë*,
ther(e) may he finde, etc., i. 312.
for oftë, *who that* hedë *tokë*,
better is to winkë than to lokë, i. 54, 206.
her names, *who that redë* right[ë],
Sem, Cham, Japhèt, the brethern hight[ë], iii. 102.
So, *that man that wolde* him well avise,
delicacy is to despise, iii. 40.

g. **As who saith** = as one might say, so to speak:

he sigh, and *as who saith*, abraide, i. 268.
and am, *as who saith*, lovës knavë, ii. 131.
So, i. 4: ii. 57, 258, etc.

§ 105. Indefinite Pronouns.

a. Redundant use of **on** (= one).

lawe *is on(e) the best* . . . to make, etc., iii. 189.
a fairer child than it was *on(e)*, i. 201.
so fair a wight as she was *on(e)*, ii. 70.
an other such as he was *on(e)*, ii. 159, 259: iii. 327.

We also find,
in all this world ne mightë be
a gladder woman than *was she*, iii. 51.

b. **One** = only, single (Sax. âna).

and rather shall an *one* man, iii. 231.

c. **One** with personal pronoun, as in Scotch *my lane*:

wishinge and wepinge *all min one*, i. 45.
within a gardin *all him one*, i. 148.
tho stood, as who saith, *all him one*, iii. 285, 178.

§ 106. Prefixes **for, to, be**.

for-stormed, i. 160: iii. 322.
for-thenketh, ii. 276: i. 186.
for-blowe, i. 160: iii. 323.
for-slouthen, ii. 190.
for-doth, i. 266.
for-gnawe, i. 326.
for-wept, ii. 15.
for-waked, ii. 15, 309.
for-shape. ii. 100, 338.
for-cast, ii. 167.
for-trode, ii. 330.
for-smite, iii. 308.
for-drive, iii. 330.
for-juged, iii. 192.
for-lain, ii. 234, 337.
for-worth, iii. 10 f.
for-swey, iii. 224, 272, 275, etc.

to-pulled, i. 61.
to-drawe, ii. 330.
to-swolle, ii. 50.
to-throweth, iii. 268.
to-clef, iii, 296.
to-breke, iii. 334.
to-breken, ii. 74, 208: iii. 295, 296.
to-rof(e), iii. 296.
etc.

be-derked, i. 81.
be-bled, i. 183, 326.
be-flain, iii. 183.
be-shineth, iii. 242.
be-reined, iii. 126.
be-snewed, iii. 51.
be-knowe, iii. 10.
etc.

§ 107. Negative Sentences. Double negatives are perhaps the rule, and in some cases more than two are used.

ne hide it *nought*, i. 47.
no vice of which he *nis*, i. 62.
nothing ne lough, i. 101.
ne shall me *nought* asterte, i. 107.
he *ne* wolde him *nought* forbere, i. 147.
no man sigh she *nought*, i. 196.
she *ne* wolde *nought* be shore,
for *no* counseil, i. 101.
I may *nought* well, *ne nought ne* shall, i. 134.
ne for *ne* write *ne* for *ne* taile, ii. 191.
that ther(e) *nis* servaunt in min hous(e),
ne non(e) of tho that ben about(e),
that eche of hem *ne* stant in doute, i. 282.

Single negatives:

it mighte *nought* be to my love,
for so yet was I *never* above, i. 296.
for he therof his part *ne* tath, ii. 129, 130, 131, 146.
man is nought amended, ii. 132.
for ther(e) may be *no* worsë thing, ii. 202.
that reson might him *non* governe (?), i. 361: see *non*, § 108.

But = only takes a negative, as in Saxon and vulgar modern English:

to the *nis but* o wey, iii. 373.

§ 108. Various Particles.

all = although: all nere it worth a stree, ii. 160.

alonge, on (S. gelang), = along of, because of.
howe all is *on* my self *alonge*, ii. 22, 33.
for it is nought *on* me *alonge*, ii. 96, 205.
and all was this *on* him *alonge*, ii. 310, 324: iii. 267.
So, *on* you is ever that I crie, ii. 121.

As, with the fundamental meaning of *considering, with respect to, so far as concerns*, is employed in various shades of distinctness and strength, decreasing to insignificance.

(a tale) that, *as* in conclusion, saith, i. 23.
what saist thou, sone, *as* of thin ere, i. 60.
as for that time, i. 135: *as* for the time that it lasteth, i. 317.
as for the whilë that it lastë, ii. 79: mercy which he dede *as* than, i. 369: ii. 20, 146.
as tho, i. 166: ii. 2, 14, 254: but what *as* after shall befall[ë], i. 234.
that ben nought able *as* of hem selve, i. 162.
now, Johan, quod she, in my powèr
thou must, *as* of thy lovë, stonde, iii. 353.
fame *as* for to speke of armes, ii. 239.
to whom reson in speciall
is yove *as* for the governaunce, iii. 101.
but yet ther(e) ben of londes felë,
in occident *as* for the chelë,
in orient(e) *as* for the hetë,
which of the people be forletë, iii. 104.
wherof *as* to the world no more
ne woll she torne, iii. 316.

as (als, also) intensive, = Latin *quam*:
they cast it over borde *as* blive, = very quickly, immediately; not very different from our *as quick*! ii. 266: 313.
So, als swithe, iii. 306: als tite, ii. 320: als faste, i. 55: ii. 204.
also (al so) faste, ii. 132, 156, 318: iii. 28: also blive, iii. 49, 58.
(als = as: for als moche, i. 51: als fer as, i. 89, 132: als well as, ii. 203, 379: iii. 19.)
al so well as . . . iii. 350 = just as well . . . as.

as he that, etc., = inasmuch as he, seeing that, etc. (*quippe*):

as he that was chival[ë]rous, i. 245.
for every hertë
ye knowe, *as ye that* ben abovë,
the god and the goddesse of lovë, ii. 322.
as he that was tofore unkinde, ii, 325.
but her accord(e)
to lovë mightë no man winnë,
as she whiche hath no lust therinnë,
but swor(e), etc., ii. 336.

at-after = after:

within a time *at-after* mete, iii. 41.
to riden out *at-after* mete, iii. 63.

Still used in the north of England. I do not find the combination in Saxon, but as æt-foran occurs æt-after probably existed. In *Observations on Chaucer*, § 109, after-mete is wrongly explained as a noun, like after-noon.

by, of time, as sometimes Germ. *bei*:

by oldë daies, i. 67, 89, 110, 118, 156.
by oldë tide, ii. 132.
that he all *by* the brodë sunnë
to beddë goth, iii. 255.
the brightë sonnë *by* the morwe, 242,
Also, (they might beholde) *by* thritty mile aboute, ii. 195.
by times seven, i. 138, 140.
by that = because that (see *of that*, further on), i. 226.

ever among = still, continually:

and *ever among* mercy she cride, i. 149.
and *ever among(e)* she wepte, i. 195: ii. 15: iii. 303, 328.
So, ever in on(e), iii. 28, 29.

forth with = with:

herself *forth with* her childe also, i. 194.
the king(e) Allèe *forth with* th'assent, i. 209.
forth with her wivës bothë two, i. 209.
the lond *forth with* the king deceived were, i. 216.
(the fader) *forth with* his sone (weren dede), ii. 67, 154, etc.

in aunter if = if haply, i. 19: iii. 331: = lest, i. 344: ii. 147.

into = until:

into my deth, i. 117.
but *into* now yet dar(e) I sain, ii. 278.
yet *into* now my will hath be, iii. 188.

non, in the sense of no = not:

I not if it be so or *non(e)*, i. 230.
to lovë wher(e) thou wolt or *non(e)*, i. 342.
and malgré wher(e) thou wolde or *non(e)*, iii. 322.
So, i. 86: ii. 66: iii. 134, 362, etc.

nought forthy = nevertheless.

and *nought forthy*, so as they might,
they made hem yongly to the sight, iii. 365.

of, representing the Saxon genitive:

foryete *of* this, i. 157.
the nedeth *of* non(e) other leche, i. 272.
he thonketh God (dat) *of* his miracle, i. 210.
I thonke you *of* that, iii. 273.
they leftë *of* her evil speche, ii. 207.
as he which drad him (dat) *of* vengeaunce, iii. 321.
as pray unto my lady *of* any helpe, iii. 350.
Appollinus *of* whom I mene, iii. 302, 301.
touchend[e] *of* this i. 19.

In these the reason of the *of* is not quite so clear:

that vicë which men call[e] *of* (by the name of?) robbery, ii. 331.
of love make me for to spede, ii. 33: i. 331: (love spede, i. 334, 336).
of that his lond shall spede, iii. 241.
no cause *of* which they hadden to done, ii. 175.
for ther*of* have I nought to done, iii. 353.
and thus though I that lawe obeie
of which that kinges ben put *under*, i. 117.

of = by (Fr. *par*): we ben taught *of* that, i. 1.
of knighthode they overcome, ii. 157.
lost his wit *of* drinke, iii. 4, etc., etc.
of that = be-cause, why (*par ce que*):
deceived were *of that* they wolde
mislokë wher(e) that they ne sholde, i. 56, 157, 161: iii. 361, etc.

the reson of my wit it overpasseth
of that naturë teche[th ?] me the way
to love, and yet . . . iii. 350.

other while — other while = ἄλλοτε — ἄλλοτε, ii. 104.

that with an imperative, like French que:

this Terens (*that* foule him fallë!), ii. 318.
in hastë *that* it werë do! iii. 182.

ther, tho, in relative senses:

whan I am *ther(e)* (= where) my lady is, ii. 372.
and *tho* (= when) this man her tale hath herd, iii. 324, 336, etc.
So thereas, ii. 107: therupon, ii. 136, etc.

till = to, unto:

that none(e) *till* other well accordeth, iii. 98.
as thou *till* other men hast do, iii. 209.
lich *til* other, iii. 370.

to (unto), representing Saxon and Latin dative:

he mot *to* nature obey, i. 291.
I will obeie me ther*to*, i. 288: ii. 48, 135: unto, i. 84.
thilke man obeie, i. 247.
woll nought serve *to* love, ii. 50: i. 80, 322: iii. 190.
thonke *unto* the goddes might, i. 210.
I thonke God, ii. 94.
which hath renounced *to* the heaven, iii. 46.
to the houndes like, i. 261, 282: lich a fox (the rule), i. 261.

unto-ward = to-ward:

stant *untoward* Septemtrion, iii. 127.

up = upon:

up a couche, ii. 132.
up amendëment, ii. 373.

upon = after the manner of:

and she *upon* childehod him tolde, i. 219.

yea — nay, yes — no. The distinction between the two forms of the affirmative and negative particles insisted on by Sir T. More, is not ob-

served by Gower: that is to say, it is not his custom to use *yea* and *nay* exclusively in answer to affirmative questions, and *yes* and *no* in answer to negative questions:

hast thou ben? *ye*, ii. 20: hast thou nought? *ye*, i. 60.
he saidë *nay*, they saiden *yis*, i. 201.
I trowë *yis*: my fader, *nay*, i, 308.
if it be so, tell me: *no*, iii. 24.
tell me, if thou hast: *nay*, ii. 275, 349: iii. 281.
if there be ought, axeth: *yis*, iii, 274.
axeth him, if that this child(e) his sonë were: *ye*, i. 206.

§ 109. Certain Peculiar Words and Phrases.

Adjectives used like nouns:

these other *grete*, i. 155: a sinful, ii. 118.
this envious, i. 171: the jelous (*sing.*), ii. 145.
he by *worthy* and by *wise* was counseiled, ii. 196.
to which no *pouer* may atteigne, iii. 22.
this pouer, iii. 35: the rich[e] (*sing.*), iii. 37.
there is no *certain* for to winne (?), iii. 134, 350.
Bacchus accordant unto his *divine* (?), ii. 132.

at after mete; for a former misapprehension of this phrase, see *at-after*, § 108.

at min (*thin*, *her*) *above*:

as though I were *at min above*, iii. 9.
ne though she were *at her above*, ii. 212.
and how they were *at her above*, ii. 378.
thou might not come *at thin above*
of that thou woldest wel acheve, ii. 32.

This singular phrase seems to signify, in the first two instances, greater than I am (she is) at present; in the third, perhaps, they bore themselves as if superior to what they really were; in the last the meaning is, thou canst not make thyself master of what thou wouldst achieve.

can thonk = *scire gratias*, i. 193, l. 17.

do = *cause, make*:

and tho she hath *do* set up light, ii. 29.
and *doth* to springë gras and floure, iii. 94.
that couthen *do* the monë clipse, iii. 362.
I *do* the to wite, iii. 373.

So, with the addition of *let*,
he *let do* yoken gretë foxes, ii. 63, 208.
let do writen, i. 191.

gan, as auxiliary to form an imperfect tense:

she gan falle, ii. 381, 385, etc.

gesse, in the New England sense of *think*:

there hath be no default, I *gesse*, ii. 11, 59, 368: iii. 180.

go = *walk*, like German *gehen*:

the dede man, which nouther *go* ne spekë can, iii. 3, 5, etc.

hadde lever: I hadde lever to be lewed, i. 295: ii. 211. I hadde lever = *j'aimerais mieux*, hadde being of course in the subjunctive. Germ. *lieb haben*, to like, is much the same. Allë women *levest wolde be* occurs i. 96: lever she wolde have wist, ii. 46: her were levest have, i. 96. *I wolde rather*, ii. 94, = I would sooner. *I had rather* seems to be an imitation of *I had lever*; when the phrase came into use is not known to me.

life = *being, person*:

so fair a *life* as she, was nought in all the town, iii. 264.
no *life* hem shuldë knowë, iii. 253.
lives creature = living creature, ii. 14, as in Cant. Tales, 2397, 8779.

many on(*e*) = *many a one*, i. 56 (deceived were): ii. 313.

moon, masculine as in Saxon:

the *mone* of silver has *his* part, ii. 84: iii. 109.
But, ne yet the *monë* that *she* carie, ii. 112.
go tak(e) the *monë* ther *it* sit, i. 86.

much, in the sense of *great*:

and for to give a *morë* feith, iii, 326.
which was to her a *morë* delit, iii. 335.
the *mostë* joy, iii. 8: my *mostë* care, iii. 254.

past participles in an adverbial sense, like Ger. *er kommt geritten*:

ride *amaied*, for ride a-maying, i. 110.

goth *astraied*, ii. 132 : iii. 175 (goth *astray*, same page).
hem that stonden *misbeleved*, ii. 152.

We find he *cam ride*, i. 53 : ii. 45, 170, where the ride looks more like the infinitive than like the participle : *cam ridend*, present partic., ii. 180, 47.

and *lefte* hem both[ë] *ligge* so, ii. 150, is another extraordinary case of the use of an infinitive.

sight in a peculiar American (?) use :

a wonder *sight* of flowers, i. 121.

slide, let, in the modern vulgar sense :

the highe creator . . . full many chaunce *let slide*, iii. 61.

the, repeatedly used by Gower with abstract nouns etc., one of his gallicisms :

the man (= l'homme), ii. 186.
the men (= les hommes), i. 9 : iii. 61, 188.
the mankinde (= le genre humain), iii. 1.
thexperience, i. 14 : the speche, iii. 136.
the blisse, iii. 276 : the trouthe, iii. 136.
the word, iii. 135, 138 : the derth and the famine, ii. 270.
the gold can make of hatë lovë, ii. 135.
the heven, i. 207 : ii. 159, 185, 362, etc.
the helle, ii. 128, 139, 164 : iii. 47, etc.
Even the God, iii. 177, 187, 201, 204, 223.
etc., etc.

Time : these expressions are somewhat remarkable.

within a monthë day, ii. 27.
within two monthës day, ii. 100.
sometime a (Sax. *on*) yere, iii. 349.

world, in various senses, = *worldly lot, worldly happiness*, etc.

as he that hath his *worlde* acheved, i. 126.
whan that he weneth best acheve
his gode *world*, it is most fro, iii. 170.
her *world* was so miswent, ii. 304.
to set a king in even
bothe in his *worlde* and eke in heven, iii. 152.
all her *worlde* on him she sette, ii. 249.
what *worldes* (= worldly) thing(e) [that] thou wolt crave, i, 323 : ii. 313.
that ye my *worldes* (natural, bodily) deth respite, i. 116.

So in the Wyf of Bathe's Tale :

unto this day it doth myn herte boote,
that I have had my *world* as in my tyme. C. T. 6055.

§ 110. Peculiar Order of Words.

of his visage *and* seeth the make, *for* and seeth the make of, i. 367.
with Frixus *and* this shep(e) forth swam, *for* and this shep swam forth with, ii. 273.
with slepe *and* both his eyen fedde, *for* and fedde both his eyen with, iii. 52.
out of his purs *and* that he nom(e), *for* and that he nom out, etc., ii. 298.
So, i. 37, 56 : ii. 109, 357 l. 17, 368 l. 8 : iii. 37, 75, 215, 216, 239, 258, etc.
as thou might *of* to-forë rede, *for* rede of to-fore, iii. 342.
of gold that I the mantel tok(e), *for* the mantel of gold, ii. 368.
but al this wo is cause *of* man, *for* man is cause of, i. 34.
to reule *with* thy conscience, *for* to reule thy conscience with i. 50.
to tendre *with* the kinges herte, *for* to tendre the kinges herte with, i. 115.
to rockë *with* her child a slepe, *for* to rock her child aslepe with, i. 196.
to stoppë *with* your evil word(e), *for* to stoppe your evil word with, ii. 205.
So, iii. 47, 63, 300, 330, 334.
and him *upon* her herbes cast[e], *for* upon him, etc., ii. 263.
o dampned man *to* helle, *for* o man dampned to helle, i. 189.
on daies *now*, *for* now on daies (now-a-days), ii. 59.
a daies *now*, *for* now-a-daies, i. 307.
of all this world *or* emperesses, *for* or emperesses of all etc., iii. 363. (?)
in perlës whitë *than* forsake, *for* than, in perles white, forsake, ii. 335.
the kingës doughter Lamedon, *for* Lamedon, the kinges, doughter, ii. 375.

§ 111. Ellipsis.

Of the *relative pronoun*.

and speake of thing(e) [that] is nought so strange. i. 41.

within a ship [that] was sterëles, i. 206.
but therto was no man [that] answerde, i. 352.
there is no man [that] withsay it wille, i. 352.
unto the park [that] was fastë by, ii. 45.
So, men beseche [what] his will is, ii. 25.

Of the *personal pronoun*, when subject.

it thought her faire and [she] saidë here, ii. 45.
for than is all my merth away,
and [I] waxe anone of thought so dull, iii. 6.
slain I have
this maidë Thaise and [she] is begrave, iii. 325.
and tho for fere her herte aflight,
and [she] saidë to her self helas, ii. 50.
I have it mad(e) as thilkë same,
which [I] axe for to be excused, iii. 383.
of Demephon right wel her quemeth,
whan he was come, and [she] made him chere, ii. 26.
he was rebuked of hem and [they] saiden, ii. 150.
the king cam with his knightes alle,
and [they] maden him glad welcoming, ii. 255.
thre men he tok(e), . . .
and [they] ete and dranke as well as he, iii. 40.
whan they this straungë vessel sigh
come in, and [it] hath his saile avaled, iii. 329.
the firstë point of slouth I calle
Lachesse, and [it] is the chef(e) of alle, ii. 1.
that if [it] ne were [for] his oignement, ii. 251.

Of *be*, and other verbs, after *shall*.

it is said and ever shal [be], i. 15.
that it is in my hertë lokë
and ever shall [be], i. 222.
all idel was I never yit,
ne never shall [be], ii. 39.
the god was ever and ever shall [be], iii. 88 190, 351.
I wot never whider I shall [go], ii. 21.
that they with him to Tharsë sholdë [go], iii 327.
which wepte as she to water sholde [turn], iii. 260.
and what she sholde [become, come to] she was adrad, iii. 321.

Of *with*: (but note that the instrument, &c., are expressed in Saxon with the Ablative, either with or without the proposition mid = with, and that these are probably cases of the old construction.)

thing which he said [with] his ownë mouth, ii. 310: iii. 155.
fightend, [with] his owne hondes slain, i. 90.
madë cloth [with] her ownë hand, ii. 83, 190, 204: i. 346, 351: iii. 305.
where he [with] his ownë body lay, ii. 198: iii. 208.

Of other prepositions.

I not what thing it may amounte [to ?] ii. 191, 194, 321, 343: iii. 54.
he no childe [of ?] his owne had, ii. 236.
for in the plit(e) [in] which I the finde, iii. 354 (perhaps mere carelessness).

CORRECTIONS.

p. 269. Notation. The Saxon vowel-sounds not distinguished are eá and eâ, eó and eô. I regret that the distinction was not indicated.

p. 270, read hûsbonda for husbonda.

p. 272, read spere for spêre.

pp. 278–80, for §§ 20, 21, 22, 23, 24, 25, 26, 27, 28, read 21, 22, 23, 24, 25, 26, 28, 29, 30.

p. 299, *c*, for but I his gracĕ havĕ so pursued, read but I his grace havĕ so pursued.

p. 299, 3, *a*, read the waie knowe how he shall go.

p. 302, right column, l, 11, read preserve.

p. 303, right column, l, 26, for usē read usĕ.

CORRECTIONS TO OBSERVATIONS ON CHAUCER.

p. 463, good is to be excluded from the list in § 25. See the same section in this paper.

p. 486, § 83, the first example is to be dropped: read, unto a poore ordrë for to gevë.

p. 498, § 109, *at after* was misunderstood. See § 108 of this paper.

p. 483, § 76, *b*, the sixth example, if the reading is correct, is an exception, and should be read, humblessë hath slayn(e) in hir tyrannye.

A few misprints have been noticed in both articles which it is not worth the while to specify.

XII.

Description of a Notation for the Logic of Relatives, resulting from an Amplification of the Conceptions of Boole's Calculus of Logic.

BY C. S. PEIRCE.

Communicated January 26, 1870.

RELATIVE terms usually receive some slight treatment in works upon logic, but the only considerable investigation into the formal laws which govern them is contained in a valuable paper by Mr. De Morgan in the tenth volume of the Cambridge Philosophical Transactions. He there uses a convenient algebraic notation, which is formed by adding to the well-known *spiculæ* of that writer the signs used in the following examples.

X . . LY signifies that X is some one of the objects of thought which stand to Y in the relation L, or is one of the L's of Y.

X . LMY signifies that X is not an L of an M of Y.

X . . (L,M)Y signifies that X is either an L or an M of Y.

LM′ an L of every M. L,M an L of none but M's.

L[−1]Y something to which Y is L. l (small L) non-L.

This system still leaves something to be desired. Moreover, Boole's logical algebra has such singular beauty, so far as it goes, that it is interesting to inquire whether it cannot be extended over the whole realm of formal logic, instead of being restricted to that simplest and least useful part of the subject, the logic of absolute terms, which, when he wrote, was the only formal logic known. The object of this paper is to show that an affirmative answer can be given to this question. I think there can be no doubt that a *calculus*, or art of drawing inferences, based upon the notation I am to describe, would be perfectly possible and even practically useful in some difficult cases, and particularly in the *investigation* of logic. I regret that I am not in a situation to be able to perform this labor, but the account here given of the notation itself will afford the ground of a judgment concerning its probable utility.

In extending the use of old symbols to new subjects, we must of course be guided by certain principles of analogy, which, when formulated, become new and

wider definitions of these symbols. As we are to employ the usual algebraic signs as far as possible, it is proper to begin by laying down definitions of the various algebraic relations and operations. The following will, perhaps, not be objected to.

General Definitions of the Algebraic Signs.

Inclusion in or *being as small as* is a *transitive* relation. The consequence holds that*

$$\begin{array}{ll} \text{If} & x \mathrel{-\!\!<} y\,, \\ \text{and} & y \mathrel{-\!\!<} z\,, \\ \text{then} & x \mathrel{-\!\!<} z\,. \end{array}$$

Equality is the conjunction of being as small as and its converse. To say that $x = y$ is to say that $x \mathrel{-\!\!<} y$ and $y \mathrel{-\!\!<} x$.

Being less than is being as small as with the exclusion of its converse. To say that $x < y$ is to say that $x \mathrel{-\!\!<} y$, and that it is not true that $y \mathrel{-\!\!<} x$.

Being greater than is the converse of being less than. To say that $x > y$ is to say that $y < x$.

Addition is an *associative* operation. That is to say,†

$$(x +_{,} y) +_{,} z = x +_{,} (y +_{,} z)\,.$$

Addition is a *commutative* operation. That is,

$$x +_{,} y = y +_{,} x\,.$$

Invertible addition is addition the corresponding inverse of which is determinative. The last two formulæ hold good for it, and also the consequence that

$$\begin{array}{ll} \text{If} & x + y = z\,, \\ \text{and} & x + y' = z\,, \\ \text{then} & \quad\; y = y'\,. \end{array}$$

* I use the sign $\mathrel{-\!\!<}$ in place of $\leqq$. My reasons for not liking the latter sign are that it cannot be written rapidly enough, and that it seems to represent the relation it expresses as being compounded of two others which in reality are complications of this. It is universally admitted that a higher conception is logically more simple than a lower one under it. Whence it follows from the relations of extension and comprehension, that in any state of information a broader concept is more simple than a narrower one included under it. Now all equality is inclusion in, but the converse is not true; hence inclusion in is a wider concept than equality, and therefore logically a simpler one. On the same principle, inclusion is also simpler than being less than. The sign $\leqq$ seems to involve a definition by enumeration; and such a definition offends against the laws of definition.

† I write a comma below the sign of addition, except when (as is the case in ordinary algebra) the corresponding inverse operation (subtraction) is determinative.

MULTIPLICATION is an operation which is *doubly distributive with reference to addition.* That is,

$$x(y \dotplus z) = xy \dotplus xz\,,$$
$$(x \dotplus y)z = xz \dotplus yz\,.$$

Multiplication is almost invariably an *associative* operation.

$$(xy)z = x(yz)\,.$$

Multiplication is not generally commutative. If we write commutative multiplication with a comma, we have

$$x,y = y,x\,.$$

Invertible multiplication is multiplication whose corresponding inverse operation (division) is determinative. We may indicate this by a dot; and then the consequence holds that

$$\begin{array}{lr} \text{If} & x.y = z\,, \\ \text{and} & x.y' = z\,, \\ \text{then} & y = y'. \end{array}$$

Functional multiplication is the application of an operation to a function. It may be written like ordinary multiplication; but then there will generally be certain points where the associative principle does not hold. Thus, if we write (sin $abc)def$, there is one such point. If we write (log $_{(\text{base } abc)}\, def)ghi$, there are two such points. The number of such points depends on the nature of the symbol of operation, and is necessarily finite. If there were many such points, in any case, it would be necessary to adopt a different mode of writing such functions from that now usually employed. We might, for example, give to "log" such a meaning that what followed it up to a certain point indicated by a † should denote the base of the system, what followed that to the point indicated by a ‡ should be the function operated on, and what followed that should be beyond the influence of the sign "log." Thus log $abc \dagger def \ddagger ghi$ would be (log $abc)ghi$, the base being def. In this paper I shall adopt a notation very similar to this, which will be more conveniently described further on.

The operation of INVOLUTION obeys the formula*

$$(x^y)^z = x^{(yz)}\,.$$

* In the notation of quaternions, Hamilton has assumed

$$(x^y)^z = x^{(zy)}\,, \qquad \text{instead of} \qquad (x^y)^z = x^{(yz)}\,,$$

although it appears to make but little difference which he takes. Perhaps we should assume two involutions, so that

$$(x^y)^z = x^{(yz)}\,, \qquad {}^z({}^yx) = {}^{(zy)}x\,.$$

But in this paper only the former of these is required.

Involution, also, follows the *indexical principle.*

$$x^{y +_{,} z} = x^y , x^z \,.$$

Involution, also, satisfies the *binomial theorem.*

$$(x +_{,} y)^z = x^z +_{,} \Sigma_p \, x^{z-p} , y^p +_{,} y^z \,,$$

where Σ_p denotes that p is to have every value less than z, and is to be taken out of z in all possible ways, and that the sum of all the terms so obtained of the form x^{z-p} , y^p is to be taken.

SUBTRACTION is the operation inverse to addition. We may write indeterminative subtraction with a comma below the usual sign. Then we shall have that

$$\begin{aligned} (x -_{,} y) +_{,} y &= x \,, \\ (x - y) + y &= x \,, \\ (x + y) - y &= x \,. \end{aligned}$$

DIVISION is the operation inverse to multiplication. Since multiplication is not generally commutative it is necessary to have two signs for division. I shall take

$$\begin{aligned} (x{:}y)y &= x \,, \\ x \frac{y}{x} &= y \,. \end{aligned}$$

Division inverse to that multiplication which is indicated by a comma may be indicated by a semicolon. So that

$$(x;y),y = x \,.$$

EVOLUTION and TAKING THE LOGARITHM are the operations inverse to involution.

$$\begin{aligned} (\sqrt[x]{y})^x &= y \,, \\ x^{\log_x y} &= y \,. \end{aligned}$$

These conditions are to be regarded as imperative. But in addition to them there are certain other characters which it is highly desirable that relations and operations should possess, if the ordinary signs of algebra are to be applied to them. These I will here endeavor to enumerate.

1. It is an additional motive for using a mathematical sign to signify a certain operation or relation that the general conception of this operation or relation should

resemble that of the operation or relation usually signified by the same sign. In particular, it will be well that the relation expressed by $-\!\!<$ should involve the conception of one member being in the other; addition, that of taking together; multiplication, that of one factor's being taken relatively to the other (as we write 3×2 for a triplet *of* pairs, and $D\varphi$ for the derivative *of* φ); and involution, that of the base being taken for every unit of the exponent.

2. In the second place, it is desirable that, in certain general circumstances, determinate numbers should be capable of being substituted for the letters operated upon, and that when so substituted the equations should hold good when interpreted in accordance with the ordinary definitions of the signs, so that arithmetical algebra should be included under the notation employed as a special case of it. For this end, there ought to be a number known or unknown, which is appropriately substituted in certain cases, for each one of, at least, some class of letters.

3. In the third place, it is almost essential to the applicability of the signs for addition and multiplication, that a *zero* and a *unity* should be possible. By a *zero* I mean a term such that

$$x +\!\!, \, 0 = x \, ,$$

whatever the signification of x; and by a *unity* a term for which the corresponding general formula

$$x \mathit{1} = x$$

holds good. On the other hand, there ought to be no term a such that $a^x = x$, independently of the value of x.

4. It will also be a strong motive for the adoption of an algebraic notation, if other formulæ which hold good in arithmetic, such as

$$x^z,y^z = (x,y)^z \, ,$$

$$\mathit{1} x = x \, ,$$

$$x^{\mathit{1}} = x \, ,$$

$$x 0 = 0 \, ,$$

continue to hold good; if, for instance, the conception of a differential is possible, and Taylor's Theorem holds, and $\mathfrak{e}$ or $(1 + i)^{\frac{1}{i}}$ plays an important part in the system, if there should be a term having the properties of π (3.14159), or properties similar to those of space should otherwise be brought out by the notation, or if there should be an absurd expression having the properties and uses of J or the square root of the negative.

Application of the Algebraic Signs to Logic.

While holding ourselves free to use the signs of algebra in any sense conformable to the above absolute conditions, we shall find it convenient to restrict ourselves to one particular interpretation except where another is indicated. I proceed to describe the special notation which is adopted in this paper.

Use of the Letters.

The letters of the alphabet will denote logical signs. Now logical terms are of three grand classes. The first embraces those whose logical form involves only the conception of quality, and which therefore represent a thing simply as "a —." These discriminate objects in the most rudimentary way, which does not involve any consciousness of discrimination. They regard an object as it is in itself as *such* (*quale*); for example, as horse, tree, or man. These are *absolute terms.* The second class embraces terms whose logical form involves the conception of relation, and which require the addition of another term to complete the denotation. These discriminate objects with a distinct consciousness of discrimination. They regard an object as over against another, that is as relative; as father of, lover of, or servant of. These are *simple relative terms.* The third class embraces terms whose logical form involves the conception of bringing things into relation, and which require the addition of more than one term to complete the denotation. They discriminate not only with consciousness of discrimination, but with consciousness of its origin. They regard an object as medium or third between two others, that is as conjugative; as giver of — to —, or buyer of — for — from —. These may be termed *conjugative terms.* The conjugative term involves the conception of THIRD, the relative that of second or OTHER, the absolute term simply considers AN object. No fourth class of terms exists involving the conception of *fourth,* because when that of *third* is introduced, since it involves the conception of bringing objects into relation, all higher numbers are given at once, inasmuch as the conception of bringing objects into relation is independent of the number of members of the relationship. Whether this *reason* for the fact that there is no fourth class of terms fundamentally different from the third is satisfactory or not, the fact itself is made perfectly evident by the study of the logic of relatives. I shall denote absolute terms by the Roman alphabet, a, b, c, d, etc.; relative terms by italics, *a*, *b*, *c*, *d*, etc.; and conjugative terms by a kind of type called Madisonian, *a*, *b*, *c*, *d*, etc.

I shall commonly denote individuals by capitals, and generals by small letters. General symbols for numbers will be printed in black-letter, thus, $\mathfrak{a}$, $\mathfrak{b}$, $\mathfrak{c}$, $\mathfrak{d}$, etc. The Greek letters will denote operations.

To avoid repetitions, I give here a catalogue of the letters I shall use in examples in this paper, with the significations I attach to them.

a.	animal.	p.	President of the United States Senate.
b.	black.	r.	rich person.
f.	Frenchman.	u.	violinist.
h.	horse.	v.	Vice-President of the United States.
m.	man.	w.	woman.

a.	enemy.	*h*.	husband.	*o*.	owner.
b.	benefactor.	*l*.	lover.	*s*.	servant.
c.	conqueror.	*m*.	mother.	*w*.	wife.
e.	emperor.	*n*.	not.		

g.	giver to — of —.	*b*.	betrayer to — of —.
w.	winner over of — to — from —.	*t*.	transferrer from — to —.

Numbers corresponding to Letters.

I propose to use the term "universe" to denote that class of individuals *about* which alone the whole discourse is understood to run. The universe, therefore, in this sense, as in Mr. De Morgan's, is different on different occasions. In this sense, moreover, discourse may run upon something which is not a subjective part of the universe; for instance, upon the qualities or collections of the individuals it contains.

I propose to assign to all logical terms, numbers; to an absolute term, the number of individuals it denotes; to a relative term, the average number of things so related to one individual. Thus in a universe of perfect men (men), the number of "tooth of" would be 32. The number of a relative with two correlates would be the average number of things so related to a pair of individuals; and so on for relatives of higher numbers of correlates. I propose to denote the number of a logical term by enclosing the term in square brackets, thus, $[t]$.

The Signs of Inclusion, Equality, etc.

I shall follow Boole in taking the sign of equality to signify identity. Thus, if v denotes the Vice-President of the United States, and p the President of the Senate of the United States,

$$\mathrm{v} = \mathrm{p}$$

means that every Vice-President of the United States is President of the Senate, and every President of the United States Senate is Vice-President. The sign "less than" is to be so taken that

$$\mathrm{f} < \mathrm{m}$$

means every Frenchman is a man, but there are men besides Frenchmen. Drobisch has used this sign in the same sense.* It will follow from these significations of $=$ and $<$ that the sign $-\!\!<$ (or $\leqq$, "as small as") will mean "is." Thus,

$$\mathrm{f} -\!\!< \mathrm{m}$$

means "every Frenchman is a man," without saying whether there are any other men or not. So,

$$m -\!\!< l$$

will mean that every mother of anything is a lover of the same thing; although this interpretation in some degree anticipates a convention to be made further on. These significations of $=$ and $<$ plainly conform to the indispensable conditions. Upon the transitive character of these relations the syllogism depends, for by virtue of it, from

$$\mathrm{f} -\!\!< \mathrm{m}$$

and $$\mathrm{m} -\!\!< \mathrm{a}\,,$$

we can infer that $$\mathrm{f} -\!\!< \mathrm{a}\,;$$

that is, from every Frenchman being a man and every man being an animal, that every Frenchman is an animal. But not only do the significations of $=$ and $<$ here adopted fulfil all absolute requirements, but they have the supererogatory virtue of being very nearly the same as the common significations. Equality is, in fact, nothing but the identity of two numbers; numbers that are equal are those which are predicable of the same collections, just as terms that are identical are those which are predicable of the same classes. So, to write $5 < 7$ is to say that 5 is part of 7, just as to write $\mathrm{f} < \mathrm{m}$ is to say that Frenchmen are part of men. Indeed, if $\mathrm{f} < \mathrm{m}$, then the number of Frenchmen is less than the number of men, and if $\mathrm{v} = \mathrm{p}$, then the number of Vice-Presidents is equal to the number of Presidents of the Senate; so that the numbers may always be substituted for the terms themselves, in case no signs of operation occur in the equations or inequalities.

The Signs for Addition.

The sign of addition is taken by Boole, so that

$$x + y$$

denotes everything denoted by x, and, *besides*, everything denoted by y. Thus

$$\mathrm{m} + \mathrm{w}$$

* According to De Morgan, Formal Logic, p. 334. De Morgan refers to the first edition of Drobisch's Logic. The third edition contains nothing of the sort.

denotes all men, and, besides, all women. This signification for this sign is needed for connecting the notation of logic with that of the theory of probabilities. But if there is anything which is denoted by both the terms of the sum, the latter no longer stands for any logical term on account of its implying that the objects denoted by one term are to be taken *besides* the objects denoted by the other. For example,

$$\mathrm{f} + \mathrm{u}$$

means all Frenchmen besides all violinists, and, therefore, considered as a logical term, implies that all French violinists are *besides themselves*. For this reason alone, in a paper which is published in the Proceedings of the Academy for March 17, 1867, I preferred to take as the regular addition of logic a non-invertible process, such that

$$\mathrm{m} +\!\!,\, \mathrm{b}$$

stands for all men and black things, without any implication that the black things are to be taken besides the men; and the study of the logic of relatives has supplied me with other weighty reasons for the same determination. Since the publication of that paper, I have found that Mr. W. Stanley Jevons, in a tract called "Pure Logic, or the Logic of Quality," had anticipated me in substituting the same operation for Boole's addition, although he rejects Boole's operation entirely and writes the new one with a $+$ sign while withholding from it the name of addition.* It is plain that both the regular non-invertible addition and the invertible addition satisfy the absolute conditions. But the notation has other recommendations. The conception of *taking together* involved in these processes is strongly analogous to that of summation, the sum of 2 and 5, for example, being the number of a collection which consists of a collection of two and a collection of five. Any logical equation or inequality in which no operation but addition is involved may be converted into a numerical equation or inequality by substituting the numbers of the several terms for the terms themselves, — provided all the terms summed are mutually exclusive. Addition being taken in this sense, *nothing* is to be denoted by *zero*, for then

$$x +\!\!,\, 0 = x\,,$$

whatever is denoted by x; and this is the definition of *zero*. This interpretation is given by Boole, and is very neat, on account of the resemblance between the ordinary conception of *zero* and that of nothing, and because we shall thus have

$$[0] = 0\,.$$

* In another book he uses the sign $\cdot|\cdot$ instead of $+$.

The Signs for Multiplication.

I shall adopt for the conception of multiplication *the application of a relation*, in such a way that, for example, lw shall denote whatever is lover of a woman. This notation is the same as that used by Mr. De Morgan, although he appears not to have had multiplication in his mind. $s(\mathrm{m} \,+\!\!,\, \mathrm{w})$ will, then, denote whatever is servant of anything of the class composed of men and women taken together. So that

$$s(\mathrm{m} \,+\!\!,\, \mathrm{w}) = s\mathrm{m} \,+\!\!,\, s\mathrm{w}\,.$$

$(l \,+\!\!,\, s)\mathrm{w}$ will denote whatever is lover or servant to a woman, and

$$(l \,+\!\!,\, s)\mathrm{w} = l\mathrm{w} \,+\!\!,\, s\mathrm{w}\,.$$

$(sl)\mathrm{w}$ will denote whatever stands to a woman in the relation of servant of a lover, and

$$(sl)\mathrm{w} = s(l\mathrm{w})\,.$$

Thus all the absolute conditions of multiplication are satisfied.

The term "identical with —" is a unity for this multiplication. That is to say, if we denote "identical with —" by $\mathit{1}$ we have

$$x\mathit{1} = x\,,$$

whatever relative term x may be. For what is a lover of something identical with anything, is the same as a lover of that thing.

A conjugative term like *giver* naturally requires two correlates, one denoting the thing given, the other the recipient of the gift. We must be able to distinguish, in our notation, the giver of A to B from the giver to A of B, and, therefore, I suppose the signification of the letter equivalent to such a relative to distinguish the correlates as first, second, third, etc., so that "giver of — to —" and "giver to — of —" will be expressed by different letters. Let $\mathfrak{g}$ denote the latter of these conjugative terms. Then, the correlates or multiplicands of this multiplier cannot all stand directly after it, as is usual in multiplication, but may be ranged after it in regular order, so that

$$\mathfrak{g}xy$$

will denote a giver to x of y. But according to the notation, x here multiplies y, so that if we put for x owner (o), and for y horse (h),

$$\mathfrak{g}o\mathrm{h}$$

appears to denote the giver of a horse to an owner of a horse. But let the individual horses be H, H′, H″, etc. Then

$$\mathrm{h} = \mathrm{H} \mathbin{+\!\!,} \mathrm{H}' \mathbin{+\!\!,} \mathrm{H}'' \mathbin{+\!\!,} \text{etc}.$$

$$\mathfrak{g}o\mathrm{h} = \mathfrak{g}o(\mathrm{H} \mathbin{+\!\!,} \mathrm{H}' \mathbin{+\!\!,} \mathrm{H}'' \mathbin{+\!\!,} \text{etc.}) = \mathfrak{g}o\mathrm{H} \mathbin{+\!\!,} \mathfrak{g}o\mathrm{H}' \mathbin{+\!\!,} \mathfrak{g}o\mathrm{H}'' \mathbin{+\!\!,} \text{etc.}$$

Now this last member must be interpreted as a giver of a horse to the owner of *that* horse, and this, therefore, must be the interpretation of $\mathfrak{g}o\mathrm{h}$. This is always very important. *A term multiplied by two relatives shows that* THE SAME INDIVIDUAL *is in the two relations.* If we attempt to express the giver of a horse to a lover of a woman, and for that purpose write

$$\mathfrak{g}l\mathrm{w}\mathrm{h}\,,$$

we have written giver of a woman to a lover of her, and if we add brackets, thus,

$$\mathfrak{g}(l\mathrm{w})\mathrm{h}\,,$$

we abandon the associative principle of multiplication. A little reflection will show that the associative principle must in some form or other be abandoned at this point. But while this principle is sometimes falsified, it oftener holds, and a notation must be adopted which will show of itself when it holds. We already see that we cannot express multiplication by writing the multiplicand directly after the multiplier; let us then affix subjacent numbers after letters to show where their correlates are to be found. The first number shall denote how many factors must be counted from left to right to reach the first correlate, the second how many *more* must be counted to reach the second, and so on. Then, the giver of a horse to a lover of a woman may be written

$$\mathfrak{g}_{12}l_1\mathrm{w}\mathrm{h} = \mathfrak{g}_{11}l_2\mathrm{h}\mathrm{w} = \mathfrak{g}_{2-1}\mathrm{h}l_1\mathrm{w}\,.$$

Of course a negative number indicates that the former correlate follows the latter by the corresponding positive number. A subjacent *zero* makes the term itself the correlate. Thus,

$$l_0$$

denotes the lover of *that* lover or the lover of himself, just as $\mathfrak{g}o\mathrm{h}$ denotes that the horse is given to the owner of itself, for to make a term doubly a correlate is, by the distributive principle, to make each individual doubly a correlate, so that

$$l_0 = L_0 \mathbin{+\!\!,} L_0' \mathbin{+\!\!,} L_0'' \mathbin{+\!\!,} \text{etc.}$$

A subjacent sign of infinity may indicate that the correlate is indeterminate, so that

$$l_{\infty}$$

will denote a lover of something. We shall have some confirmation of this presently.

If the last subjacent number is a *one* it may be omitted. Thus we shall have

$$l_1 \doteq l\,,$$
$$\mathfrak{g}_{11} = \mathfrak{g}_1 = \mathfrak{g}\,.$$

This enables us to retain our former expressions $l\mathrm{w}$, $\mathfrak{g}o\mathrm{h}$, etc.

The associative principle does not hold in this counting of factors. Because it does not hold, these subjacent numbers are frequently inconvenient in practice, and I therefore use also another mode of showing where the correlate of a term is to be found. This is by means of the marks of reference, † ‡ || § ¶, which are placed subjacent to the relative term and before and above the correlate. Thus, giver of a horse to a lover of a woman may be written

$$\mathfrak{g}_{\dagger\ddagger}{}^{\dagger}l_{\|}{}^{\|}\mathrm{w}\,{}^{\ddagger}\mathrm{h}\,.$$

The asterisk I use exclusively to refer to the last correlate of the last relative of the algebraic term.

Now, considering the order of multiplication to be: — a term, a correlate of it, a correlate of that correlate, etc., — there is no violation of the associative principle. The only violations of it in this mode of notation are that in thus passing from relative to correlate, we skip about among the factors in an irregular manner, and that we cannot substitute in such an expression as $\mathfrak{g}o\mathrm{h}$ a single letter for $o\mathrm{h}$. I would suggest that such a notation may be found useful in treating other cases of non-associative multiplication. By comparing this with what was said above concerning functional multiplication, it appears that multiplication by a conjugative term is functional, and that the letter denoting such a term is a symbol of operation. I am therefore using two alphabets, the Greek and Madisonian, where only one was necessary. But it is convenient to use both.

Thus far, we have considered the multiplication of relative terms only. Since our conception of multiplication is the application of a relation, we can only multiply absolute terms by considering them as relatives. Now the absolute term "man" is really exactly equivalent to the relative term "man that is —," and so with any other. I shall write a comma after any absolute term to show that it is so regarded as a relative term. Then man that is black will be written

$$\mathrm{m,b}\,.$$

But not only may any absolute term be thus regarded as a relative term, but any relative term may in the same way be regarded as a relative with one correlate more. It is convenient to take this additional correlate as the first one. Then

$$l{,}s\mathrm{w}$$

will denote a lover of a woman that is a servant of that woman. The comma here after l should not be considered as altering at all the meaning of l, but as only a subjacent sign, serving to alter the arrangement of the correlates. In point of fact, since a comma may be added in this way to any relative term, it may be added to one of these very relatives formed by a comma, and thus by the addition of two commas an absolute term becomes a relative of two correlates. So

$$\mathrm{m}{,}{,}\mathrm{b}{,}\mathrm{r}\ ,$$

interpreted like $\mathfrak{g}o\mathrm{h}$,

means a man that is a rich individual and is a black that is that rich individual. But this has no other meaning than

$$\mathrm{m}{,}\mathrm{b}{,}\mathrm{r}\ ,$$

or a man that is a black that is rich. Thus we see that, after one comma is added, the addition of another does not change the meaning at all, so that whatever has one comma after it must be regarded as having an infinite number. If, therefore, $l{,}{,}s\mathrm{w}$ is not the same as $l{,}s\mathrm{w}$ (as it plainly is not, because the latter means a lover and servant of a woman, and the former a lover of and servant of and same as a woman), this is simply because the writing of the comma alters the arrangement of the correlates. And if we are to suppose that absolute terms are multipliers at all (as mathematical generality demands that we should), we must regard every term as being a relative requiring an infinite number of correlates to its virtual infinite series "that is — and is — and is — etc." Now a relative formed by a comma of course receives its subjacent numbers like any relative, but the question is, What are to be the implied subjacent numbers for these implied correlates? Any term may be regarded as having an infinite number of factors, those at the end being *ones*, thus,

$$l{,}s\mathrm{w} = l{,}s\mathrm{w}{,}\mathit{1}{,}\mathit{1}{,}\mathit{1}{,}\mathit{1}{,}\mathit{1}{,}\mathit{1}{,}\mathit{1}, \text{ etc.}$$

A subjacent number may therefore be as great as we please. But all these *ones* denote the same identical individual denoted by w; what then can be the subjacent numbers to be applied to s, for instance, on account of its infinite "*that is*"'s? What numbers can separate it from being identical with w? There are only two. The first is *zero*, which plainly neutralizes a comma completely, since

$$s{,}_0\mathrm{w} = s\mathrm{w}\ ,$$

and the other is infinity; for as 1^∞ is indeterminate in ordinary algebra, so it will be shown hereafter to be here, so that to remove the correlate by the product of an infinite series of *ones* is to leave it indeterminate. Accordingly,

$$\mathrm{m},_\infty$$

should be regarded as expressing *some* man. Any term, then, is properly to be regarded as having an infinite number of commas, all or some of which are neutralized by zeros.

"Something" may then be expressed by

$$\mathit{1}_\infty .$$

I shall for brevity frequently express this by an antique figure one (1).

"Anything" by

$$\mathit{1}_0 .$$

I shall often also write a straight 1 for *anything.*

It is obvious that multiplication into a multiplicand indicated by a comma is commutative,* that is,

$$s,l = l,s .$$

This multiplication is effectively the same as that of Boole in his logical calculus. Boole's unity is my 1 , that is, it denotes whatever is.

The sum $x + x$ generally denotes no logical term. But $x,_\infty + x,_\infty$ may be considered as denoting some two x's. It is natural to write

$$x + x = \mathit{2}.x ,$$

and

$$x,_\infty + x,_\infty = \mathit{2}.x,_\infty ,$$

where the dot shows that this multiplication is invertible. We may also use the antique figures so that

$$\mathit{2}.x,_\infty = 2x ,$$

just as

$$\mathit{1}_\infty = 1 .$$

Then 2 alone will denote some two things. But this multiplication is not in general commutative, and only becomes so when it affects a relative which imparts a relation such that a thing only bears it to *one* thing, and one thing *alone* bears it to a thing.

* It will often be convenient to speak of the whole operation of affixing a comma and then multiplying as a commutative multiplication, the sign for which is the comma. But though this is allowable, we shall fall into confusion at once if we ever forget that in point of fact it is not a different multiplication, only it is multiplication by a relative whose meaning — or rather whose syntax — has been slightly altered; and that the comma is really the sign of this modification of the foregoing term.

For instance, the lovers of two women are not the same as two lovers of women, that is,

$$l2.\mathrm{w} \text{ and } 2.l\mathrm{w}$$

are unequal; but the husbands of two women are the same as two husbands of women, that is,

$$h2.\mathrm{w} = 2.h\mathrm{w}\,,$$

and in general,

$$x,2.y = 2.x,y\,.$$

The conception of multiplication we have adopted is that of the application of one relation to another. So, a quaternion being the relation of one vector to another, the multiplication of quaternions is the application of one such relation to a second. Even ordinary numerical multiplication involves the same idea, for 2×3 is a pair of triplets, and 3×2 is a triplet of pairs, where "triplet of" and "pair of" are evidently relatives.

If we have an equation of the form

$$xy = z\,,$$

and there are just as many x's per y as there are *per* things things of the universe, then we have also the arithmethical equation,

$$[x]\,[y] = [z]\,.$$

For instance, if our universe is perfect men, and there are as many teeth to a Frenchman (perfect understood) as there are to any one of the universe, then

$$[t]\,[\mathrm{f}] = [t\,\mathrm{f}]$$

holds arithmetically. So if men are just as apt to be black as things in general,

$$[\mathrm{m},]\,[\mathrm{b}] = [\mathrm{m},\mathrm{b}]\,,$$

where the difference between [m] and [m,] must not be overlooked. It is to be observed that

$$[1] = 1\,.$$

Boole was the first to show this connection between logic and probabilities. He was restricted, however, to absolute terms. I do not remember having seen any extension of probability to relatives, except the ordinary theory of *expectation*.

Our logical multiplication, then, satisfies the essential conditions of multiplication, has a unity, has a conception similar to that of admitted multiplications, and contains numerical multiplication as a case under it.

The Sign of Involution.

I shall take involution in such a sense that x^y will denote everything which is an x for every individual of y. Thus l^{w} will be a lover of every woman. Then $(s^l)^{\mathrm{w}}$ will denote whatever stands to every woman in the relation of servant of every lover of hers; and $s^{(l^{\mathrm{w}})}$ will denote whatever is a servant of everything that is lover of a woman. So that

$$(s^l)^{\mathrm{w}} = s^{(l^{\mathrm{w}})} .$$

A servant of every man and woman will be denoted by $s^{\mathrm{m} +, \mathrm{w}}$, and $s^{\mathrm{m}},s^{\mathrm{w}}$ will denote a servant of every man that is a servant of every woman. So that

$$s^{\mathrm{m} +, \mathrm{w}} = s^{\mathrm{m}},s^{\mathrm{w}} .$$

That which is emperor or conqueror of every Frenchman will be denoted by $(e +, c)^{\mathrm{f}}$, and $e^{\mathrm{f}} +, \Sigma_p \, e^{\mathrm{f}-p},c^p +, c^{\mathrm{f}}$ will denote whatever is emperor of every Frenchman or emperor of some Frenchmen and conqueror of all the rest, or conqueror of every Frenchman. Consequently,

$$(e +, c)^{\mathrm{f}} = e^{\mathrm{f}} +, \Sigma_p \, e^{\mathrm{f}-p},c^p +, c^{\mathrm{f}}.$$

Indeed, we may write the binomial theorem so as to preserve all its usual coefficients; for we have

$$(e +, c)^{\mathrm{f}} = e^{\mathrm{f}} +, [\mathrm{f}].e^{\mathrm{f} - \dagger 1},c^{1\dagger} +, \frac{[\mathrm{f}].([\mathrm{f}] - 1)}{2}.e^{\mathrm{f} - \ddagger 2},c^{2\ddagger} +, \text{etc.}$$

That is to say, those things each of which is emperor or conqueror of every Frenchman consist, first, of all those individuals each of which is a conqueror of every Frenchman; second, of a number of classes equal to the number of Frenchmen, each class consisting of everything which is an emperor of every Frenchman but some one and is a conqueror of that one; third, of a number of classes equal to half the product of the number of Frenchmen by one less than that number, each of these classes consisting of every individual which is an emperor of every Frenchman except a certain two, and is conqueror of those two, etc. This theorem holds, also, equally well with invertible addition, and either term of the binomial may be negative provided we assume

$$(-\,x)^y = (-)^{[y]}.x^y .$$

In addition to the above equations which are required to hold good by the definition of involution, the following also holds,

$$(s,l)^{\mathrm{w}} = s^{\mathrm{w}},l^{\mathrm{w}} ,$$

just as it does in arithmetic.

The application of involution to conjugative terms presents little difficulty after the explanations which have been given under the head of multiplication. It is obvious that betrayer to every enemy should be written

$$\mathfrak{b}^{a},$$

just as lover of every woman is written

$$l^{\mathrm{w}}.$$

But $\mathfrak{b} = \mathfrak{b}_{11}$ and therefore, in counting forward as the subjacent numbers direct, we should count the exponents, as well as the factors, of the letter to which the subjacent numbers are attached. Then we shall have, in the case of a relative of two correlates, six different ways of affixing the correlates to it, thus,

$\mathfrak{b}a\mathrm{m}$ betrayer of a man to an enemy of him;
$(\mathfrak{b}a)^{\mathrm{m}}$ betrayer of every man to some enemy of him;
$\mathfrak{b}a^{\mathrm{m}}$ betrayer of each man to an enemy of every man;
$\mathfrak{b}^{a^{\mathrm{m}}}$ betrayer of a man to all enemies of all men;
$\mathfrak{b}^{a}\mathrm{m}$ betrayer of a man to every enemy of him;
$\mathfrak{b}^{a\,\mathrm{m}}$ betrayer of every man to every enemy of him.

If both correlates are absolute terms, the cases are

$\mathfrak{b}\mathrm{m}\mathrm{w}$ betrayer of a woman to a man;
$(\mathfrak{b}\mathrm{m})^{\mathrm{w}}$ betrayer of each woman to some man;
$\mathfrak{b}\mathrm{m}^{\mathrm{w}}$ betrayer of all women to a man;
$\mathfrak{b}^{\mathrm{m}^{\mathrm{w}}}$ betrayer of a woman to every man;
$\mathfrak{b}^{\mathrm{m}}\mathrm{w}$ betrayer of a woman to all men;
$\mathfrak{b}^{\mathrm{m}\,\mathrm{w}}$ betrayer of every woman to every man.

These interpretations are by no means obvious, but I shall show that they are correct further on.

It will be perceived that the rule still holds here that

$$(\mathfrak{b}^{a})^{\mathrm{m}} = \mathfrak{b}^{(a\mathrm{m})},$$

that is to say, that those individuals each of which stand to every man in the relation of betrayer to every enemy of his are identical with those individuals each of which is a betrayer to every enemy of a man of that man.

If the proportion of lovers of each woman among lovers of other women is equal to the average number of lovers which single individuals of the whole universe have, then

$$[l^{\mathrm{w}}] = [l^{\mathrm{W}'},][l^{\mathrm{W}''},][l^{\mathrm{W}'''},] \text{ etc.} = [l]^{[\mathrm{w}]}.$$

Thus arithmetical involution appears as a special case of logical involution.

General Formulæ.

The formulæ which we have thus far obtained, exclusive of mere explanations of signs and of formulæ relating to the numbers of classes, are: —

(1.) If $x \mathrel{-\!\!<} y$ and $y \mathrel{-\!\!<} z$, then $x \mathrel{-\!\!<} z$.

(2.) $(x \mathbin{+\!\!,} y) \mathbin{+\!\!,} z = x \mathbin{+\!\!,} (y \mathbin{+\!\!,} z)$. (Jevons.)

(3.) $x \mathbin{+\!\!,} y = y \mathbin{+\!\!,} x$. (Jevons.)

(4.) $(x \mathbin{+\!\!,} y)z = xz \mathbin{+\!\!,} yz$.

(5.) $x(y \mathbin{+\!\!,} z) = xy \mathbin{+\!\!,} xz$.

(6.) $(xy)z = x(yz)$.

(7.) $x,(y \mathbin{+\!\!,} z) = x,y \mathbin{+\!\!,} x,z$. (Jevons.)

(8.) $(x,y),z = x,(y,z)$. (Boole.)

(9.) $x,y = y,x$. (Boole.)

(10.) $(x^y)^z = x^{(yz)}$.

(11.) $x^{y \mathbin{+\!\!,} z} = x^y,x^z$.

(12.) $(x \mathbin{+\!\!,} y)^z = x^z \mathbin{+\!\!,} \Sigma_p\, (x^{z-p},y^p) \mathbin{+\!\!,} y^p$

$$= x^z \mathbin{+\!\!,} [z].x^{z-\dagger 1},y^{\dagger 1} \mathbin{+\!\!,} \frac{[z].[z-1]}{2}.x^{z-\ddagger 2},y^{\ddagger 2} \mathbin{+\!\!,} \frac{[z].[z-1].[z-2]}{2.3}.x^{z-\| 3},y^{\| 3} \mathbin{+\!\!,} \text{etc.}$$

(13.) $(x,y)^z = x^z,y^z$.

(14.) $x + 0 = x$. (Boole.)

(15.) $x\mathit{1} = x$.

(16.) $(x + y) + z = x + (y + z)$. (Boole.)

(17.) $x + y = y + x$. (Boole.)

(18.) $x + y - y = x$. (Boole.)

(19.) $x,(y + z) = x,y + x,z$. (Boole.)

(20.) $(x + y)^z = x^z + [z].x^{z-\dagger 1},y^{\dagger 1} + \text{etc.}$

We have also the following, which are involved implicitly in the explanations which have been given.

(21.) $x \mathrel{-\!\!<} x \mathbin{+\!\!,} y$.

This, I suppose, is the principle of identity, for it follows from this that $x = x$.

(22.) $x \mathbin{+\!\!,} x = x$. (Jevons.)

(23.) $x,x = x$. (Boole.)

(24.) $x +\!\!, y = x + y - x,y$.

The principle of contradiction is

(25.) $x,n^x = 0$,

where n stands for "not." The principle of excluded middle is

(26.) $x +\!\!, n^x = 1$.

It is an identical proposition, that, if φ be determinative, we have

(27.) If $x = y \quad \varphi x = \varphi y$.

The six following are derivable from the formulæ already given:—

(28.) $(x +\!\!, y),(x +\!\!, z) = x +\!\!, y,z$.

(29.) $(x - y) +\!\!, (z - w) = (x +\!\!, z) - (y +\!\!, w) + y,z,(1 - w) + x,(1 - y),w$.

In the following, φ is a function involving only the commutative operations and the operations inverse to them.*

(30.) $\varphi x = (\varphi 1),x + (\varphi 0),(1 - x)$. (Boole.)

(31.) $\varphi x = \boldsymbol{(}\varphi 1 +\!\!, (1 - x)\boldsymbol{)},(\varphi 0 +\!\!, x)$.

(32.) If $\varphi x = 0 \quad (\varphi 1),(\varphi 0) = 0$. (Boole.)

(33.) If $\varphi x = 1 \quad \varphi 1 +\!\!, \varphi 0 = 1$.

* The reader may wish information concerning the proofs of formulas (30) to (33). When involution is not involved in a function nor any multiplication except that for which $x,x = x$, it is plain that φx is of the first degree, and therefore, since all the rules of ordinary algebra hold, we have as in that

$$\varphi x = \varphi 0 + (\varphi 1 - \varphi 0),x \ .$$

We shall find, hereafter, that when φ has a still more general character, we have,

$$\varphi x = \varphi 0 + (\varphi \mathit{1} - \varphi 0)x \ .$$

The former of these equations by a simple transformation gives (30).

If we regard $(\varphi 1),(\varphi 0)$ as a function of x and develop it by (30), we have

$$(\varphi 1),(\varphi 0) = x,(\varphi 1),(\varphi 0) + (\varphi 1),(\varphi 0),(1 - x) \ .$$

Comparing these terms separately with the terms of the second member of (30), we see that

$$(\varphi 1),(\varphi 0) -\!\!< \varphi x \ .$$

This gives at once (32), and it gives (31) after performing the multiplication indicated in the second member of that equation and equating φx to its value as given in (30). If $(\varphi 1 +\!\!, \varphi 0)$ is developed as a function of x by (31), and the factors of the second member are compared with those of the second member of (31), we get

$$\varphi x -\!\!< \varphi 1 +\!\!, \varphi 0 \ ,$$

from which (33) follows immediately.

Properties of Zero and Unity.

The symbolical definition of zero is

$$x + 0 = x\,,$$

so that by (19) $\qquad x{,}a = x{,}(a + 0) = x{,}a + x{,}0\,.$

Hence, from the invertible character of this addition, and the generality of (14), we have

$$x{,}0 = 0\,.$$

By (24) we have in general,

$$x +_{,} 0 = x + 0 - x{,}0 = x\,,$$

or

$$x +_{,} 0 = x\,.$$

By (4) we have $\qquad ax = (a +_{,} 0)x = ax +_{,} 0x\,.$

But if a is an absurd relation, $\qquad ax = 0\,,$

so that $\qquad 0x = 0\,,$

which must hold invariably.

From (12) we have $\qquad a^x = (a +_{,} 0)^x = a^x +_{,} 0^x +_{,}$ etc.

whence by (21) $\qquad 0^x -\!\!< a^x\,.$

But if a is an absurd relation, and x is not zero,

$$a^x = 0\,.$$

and therefore, unless $x = 0$, $\qquad 0^x = 0\,.$

Any relative x may be conceived as a sum of relatives X, X', X'', etc., such that there is but one individual to which anything is X, but one to which anything is X', etc. Thus, if x denote "cause of," X, X', X'' would denote different kinds of causes, the causes being divided according to the differences of the things they are causes of. Then we have

$$Xy = X(y +_{,} 0) = Xy +_{,} X0\,,$$

whatever y may be. Hence, since y may be taken so that

$$Xy = 0\,,$$

we have $\qquad X0 = 0\,;$

and in a similar way,

$$X'0 = 0\,, \qquad X''0 = 0\,, \qquad X'''0 = 0\,, \quad \text{etc.}$$

We have, then,

$$x0 = (X +_{,} X' +_{,} X'' +_{,} X''' +_{,} \text{etc.})0 = X0 +_{,} X'0 +_{,} X''0 +_{,} X'''0 +_{,} \text{etc.} = 0\,.$$

If the relative x be divided in this way into X, X', X'', X''', etc., so that x is that which is either X or X' or X'' or X''', etc., then non-x is that which is at once non-X and non-X' and non-X'', etc.; that is to say,

$$\text{non-}x = \text{non-}X,\ \text{non-}X',\ \text{non-}X'',\ \text{non-}X''', \text{ etc.};$$

where non-X is such that there is something (Z) such that everything is non-X to Z; and so with non-X', non-X'', etc. Now, non-x may be any relative whatever. Substitute for it, then, y; and for non-X, non-X', etc., Y, Y', etc. Then we have

$$y = Y, Y', Y'', Y''', \text{ etc.};$$

and $$Y'Z' = 1\,, \qquad Y''Z'' = 1\,, \qquad Y'''Z''' = 1\,, \quad \text{etc.},$$

where Z', Z'', Z''' are individual terms which depend for what they denote on Y', Y'', Y'''. Then we have

$$1 = Y'Z' = Y'^{Z'} = Y'^{(Z' +\!, 0)} = Y'^{Z'}, Y'^{0} = Y'Z', Y'^{0},$$

or $$Y'^{0} = 1\,, \quad Y''^{0} = 1\,, \quad Y'''^{0} = 1\,, \quad \text{etc.}$$

Then $$y^{0} = (Y', Y'', Y''', \text{etc.})^{0} = Y'^{0}, Y''^{0}, Y'''^{0}, \text{ etc.} = 1\,.$$

We have by definition, $$x\mathit{1} = x\,.$$

Hence, by (6), $$ax = (a\mathit{1})x = a(\mathit{1}x)\,.$$

Now a may express any relation whatever, but things the same way related to everything are the same. Hence,

$$x = \mathit{1}x\,.$$

We have by definition, $$1 = \mathit{1}_0\,.$$

Then if X is any individual $$X,1 = X,\mathit{1}_0 = X,\mathit{1}X\,.$$

But $$\mathit{1}X = X\,.$$

Hence $$X,1 = X,X\,;$$

and by (23) $$X,1 = X\,;$$

whence if we take $$x = X + X' + X'' + X''' + \text{etc.},$$

where X, X' etc. denote individuals (and by the very meaning of a general term this can always be done, whatever x may be)

$$x,1 = (X + X' + X'' + \text{etc.}),1 = X,1 + X',1 + X'',1 + \text{etc.} = X + X' + X'' + \text{etc.} = x,$$

or $$x,1 = x\,.$$

We have by (24) $$x +\!, 1 = x + 1 - x,1 = x + 1 - x = 1\,,$$

or $$x +\!, 1 = 1\,.$$

We may divide all relatives into limited and unlimited. Limited relatives express such relations as nothing has to everything. For example, nothing is knower of everything. Unlimited relatives express relations such as something has to everything. For example, something is as good as anything. For limited relatives, then, we may write

$$p^1 = 0\,.$$

The converse of an unlimited relative expresses a relation which everything has to something. Thus, everything is as bad as something. Denoting such a relative by q,

$$q1 = 1\,.$$

These formulæ remind one a little of the logical algebra of Boole; because one of them holds good in arithmetic only for *zero*, and the other only for *unity*.

We have by (10) $\quad 1^x = (q^0)^x = q^{(0x)} = q^0 = 1\,,$

or $\quad 1^x = 1\,.$

We have by (4) $\quad 1x = (a +\!, 1)x = ax +\!, 1x\,,$

or by (21) $\quad ax \prec 1x\,.$

But everything is somehow related to x unless x is 0; hence, unless x is 0,

$$1x = 1\,.$$

If a denotes "what possesses," and y "character of what is denoted by x,"

$$x = a^y = a^{(y\mathit{1})} = (a^y)^{\mathit{1}} = x^{\mathit{1}}\,,$$

or $\quad x^{\mathit{1}} = x\,.$

Since $\mathit{1}$ means "identical with," $l,\mathit{1}$w denotes whatever is both a lover of and identical with a woman, or a woman who is a lover of herself. And thus, in general,

$$x,\mathit{1} = x_{0},\,.$$

Nothing is identical with every one of a class; and therefore $\mathit{1}^x$ is zero, unless x denotes only an individual when $\mathit{1}^x$ becomes equal to x. But equations founded on interpretation may not hold in cases in which the symbols have no rational interpretation.

Collecting together all the formulæ relating to *zero* and *unity*, we have

(34.) $x +\!, 0 = x\,.$ (Jevons.)

(35.) $x +\!, 1 = 1\,.$ (Jevons.)

(36.) $x0 = 0\,.$

(37.) $0x = 0\,.$

(38.) $x,0 = 0$. (Boole.)

(39.) $x^0 = 1$.

(40.) $0^x = 0$, provided $x > 0$.

(41.) $\mathit{1}x = x$.

(42.) $x,\mathit{1} = x_0,$.

(43.) $x^{\mathit{1}} = x$.

(44.) $\mathit{1}^x = 0$, unless x is individual, when $\mathit{1}^x = x$.

(45.) $q1 = 1$, where q is the converse of an unlimited relative.

(46.) $1x = 1$, provided $x > 0$.

(47.) $x,1 = x$. (Boole.)

(48.) $p^1 = 0$, where p is a limited relative.

(49.) $1^x = 1$.

These, again, give us the following:—

(50.) $0 \mathbin{+\!\!,} 1 = 1$.

(51.) $0 \mathbin{+\!\!,} \mathit{1} = \mathit{1}$.

(52.) $00 = 0$.

(53.) $0,0 = 0$.

(54.) $0^0 = 1$.

(55.) $\mathit{1}0 = 0$.

(56.) $0\mathit{1} = 0$.

(57.) $0,\mathit{1} = 0$.

(58.) $0^{\mathit{1}} = 0$.

(59.) $\mathit{1}^0 = 1$

(60.) $01 = 0$.

(61.) $10 = 0$.

(62.) $0,1 = 0$.

(63.) $1^0 = 1$.

(64.) $0^1 = 0$.

(65.) $\mathit{1}\mathit{1} = \mathit{1}$.

(66.) $\mathit{1},\mathit{1} = \mathit{1}$.

(67.) $\mathit{1}^{\mathit{1}} = \mathit{1}$.

(68.) $11 = 1$.

(69.) $1,1 = 1$.

(70.) $1^1 = 1$.

(71.) $1\mathit{1} = 1$.

(72.) $\mathit{1}1 = 1$.

(73.) $\mathit{1},1 = \mathit{1}$.

(74.) $1^{\mathit{1}} = 1$.

(75.) $\mathit{1}^1 = 0$.

(76.) $1, = \mathit{1}$.

From (64) we may infer that 0 is a limited relative, and from (60) that it is not the converse of an unlimited relative. From (70) we may infer that 1 is not a limited relative, and from (68) that it is the converse of an unlimited relative.

Formulæ relating to the Numbers of Terms.

We have already seen that

(77.) If $x \prec y$, then $[x] \prec [y]$.

(78.) When $x,y = 0$, then $[x +\!\!, \, y] = [x] +\!\!, \, [y]$.

(79.) When $[xy]{:}[n^x y] = [x]{:}[n^x]$, then $[xy] = [x][y]$.

(80.) When $[x \boldsymbol{\pi} y] = [x][\boldsymbol{\pi} y][1]$, then $[x^y] = [x]^{[y]}$.

It will be observed that the conditions which the terms must conform to, in order that the arithmetical equations shall hold, increase in complexity as we pass from the more simple relations and processes to the more complex.

We have seen that

(81.) $[0] = \mathit{0}$.

(82.) $[\mathit{1}] = \mathit{1}$.

Most commonly the universe is unlimited, and then

(83.) $[1] = \infty$;

and the general properties of 1 correspond with those of infinity. Thus,

$$\begin{array}{llll} x +\!\!, \, 1 = 1 & \text{corresponds to} & & x + \infty = \infty, \\ q1 = 1 & \text{“} & \text{“} & q\infty = \infty, \\ 1x = 1 & \text{“} & \text{“} & \infty x = \infty, \\ p^1 = 0 & \text{“} & \text{“} & p^\infty = 0, \\ 1^x = 1 & \text{“} & \text{“} & \infty^x = \infty. \end{array}$$

The formulæ involving commutative multiplication are derived from the equation $1, = \mathit{1}$. But if 1 be regarded as infinite, it is not an absolute infinite; for $1 0 = 0$. On the other hand, $\mathit{1}^1 = 0$.

It is evident, from the definition of the number of a term, that

(84.) $[x,] = [x]{:}[1]$.

We have, therefore, if the probability of an individual being x to any y is independent of what other y's it is x to, and if x is independent of y,

(85.) $[x^y,] = [x,]^{[y]}$.

General Method of Working with this Notation.

Boole's logical algebra contains no operations except our invertible addition and commutative multiplication, together with the corresponding subtraction and division. He has, therefore, only to expand expressions involving division, by means of (30), so as to free himself from all non-determinative operations, in order to be able to use the ordinary methods of algebra, which are, moreover, greatly simplified by the fact that

$$x{,}x = x\,.$$

Mr. Jevons's modification of Boole's algebra involves only non-invertible addition and commutative multiplication, without the corresponding inverse operations. He is enabled to replace subtraction by multiplication, owing to the principle of contradiction, and to replace division by addition, owing to the principle of excluded middle. For example, if x be unknown, and we have

$$\mathrm{x} +\!\!, \mathrm{m} = \mathrm{a}\,,$$

or what is denoted by x together with men make up animals, we can only conclude, with reference to x, that it denotes (among other things, perhaps) all animals not men; that is, that the x's not men are the same as the animals not men. Let $\overline{\mathrm{m}}$ denote non-men; then by multiplication we have

$$\mathrm{x},\overline{\mathrm{m}} +\!\!, \mathrm{m},\overline{\mathrm{m}} = \mathrm{x},\overline{\mathrm{m}} = \mathrm{a},\overline{\mathrm{m}}\,,$$

because, by the principle of contradiction,

$$\mathrm{m},\overline{\mathrm{m}} = 0\,.$$

Or, suppose, x being again unknown, we have given

$$\mathrm{a},\mathrm{x} = \mathrm{m}\,.$$

Then all that we can conclude is that the x's consist of all the m's and perhaps some or all of the non-a's, or that the x's and non-a's together make up the m's and non-a's together. If, then, $\bar{\mathrm{a}}$ denote non-a, add $\bar{\mathrm{a}}$ to both sides and we have

$$\mathrm{a},\mathrm{x} +\!\!, \bar{\mathrm{a}} = \mathrm{m} +\!\!, \bar{\mathrm{a}}\,.$$

Then by (28) $$(\mathrm{a} +\!\!, \bar{\mathrm{a}}),(\mathrm{x} +\!\!, \bar{\mathrm{a}}) = \mathrm{m} +\!\!, \bar{\mathrm{a}}\,.$$

But by the principle of excluded middle,

$$\mathrm{a} +\!\!, \bar{\mathrm{a}} = 1$$

and therefore $$\mathrm{x} +\!\!, \bar{\mathrm{a}} = \mathrm{m} +\!\!, \bar{\mathrm{a}}\,.$$

I am not aware that Mr. Jevons actually uses this latter process, but it is open to him to do so. In this way, Mr. Jevons's algebra becomes decidedly simpler even than Boole's.

It is obvious that any algebra for the logic of relatives must be far more complicated. In that which I propose, we labor under the disadvantages that the multiplication is not generally commutative, that the inverse operations are usually indeterminative, and that transcendental equations, and even equations like

$$a^{bx} = c^{d^{ex}} + f^x + x \,,$$

where the exponents are three or four deep, are exceedingly common. It is obvious, therefore, that this algebra is much less manageable than ordinary arithmetical algebra.

We may make considerable use of the general formulæ already given, especially of (1), (21), and (27), and also of the following, which are derived from them: —

(86.) If $a \prec b$ then there is such a term x that $a \,+\!\!,\, x = b$.

(87.) If $a \prec b$ then there is such a term x that $b,x = a$.

(88.) If $b,x = a$ then $a \prec b$.

(89.) If $a \prec b$ $c \,+\!\!,\, a \prec c \,+\!\!,\, b$.

(90.) If $a \prec b$ $ca \prec cb$.

(91.) If $a \prec b$ $ac \prec bc$.

(92.) If $a \prec b$ $c^b \prec c^a$.

(93.) If $a \prec b$ $a^c \prec b^c$.

(94.) $a,b \prec a$.

There are, however, very many cases in which the formulæ thus far given are of little avail.

Demonstration of the sort called mathematical is founded on suppositions of particular cases. The geometrician draws a figure; the algebraist assumes a letter to signify a single quantity fulfilling the required conditions. But while the mathematician supposes an individual case, his hypothesis is yet perfectly general, because he considers no characters of the individual case but those which must belong to every such case. The advantage of his procedure lies in the fact that the logical laws of individual terms are simpler than those which relate to general terms, because individuals are either identical or mutually exclusive, and cannot intersect or be subordinated to one another as classes can. Mathematical demonstration is not, therefore, more restricted to matters of intuition than any other kind of reasoning. In-

deed, logical algebra conclusively proves that mathematics extends over the whole realm of formal logic; and any theory of cognition which cannot be adjusted to this fact must be abandoned. We may reap all the advantages which the mathematician is supposed to derive from intuition by simply making general suppositions of individual cases.

In reference to the doctrine of individuals, two distinctions should be borne in mind. The logical atom, or term not capable of logical division, must be one of which every predicate may be universally affirmed or denied. For, let A be such a term. Then, if it is neither true that all A is X nor that no A is X, it must be true that some A is X and some A is not X; and therefore A may be divided into A that is X and A that is not X, which is contrary to its nature as a logical atom. Such a term can be realized neither in thought nor in sense. Not in sense, because our organs of sense are special,— the eye, for example, not immediately informing us of taste, so that an image on the retina is indeterminate in respect to sweetness and non-sweetness. When I see a thing, I do not see that it is not sweet, nor do I see that it is sweet; and therefore what I see is capable of logical division into the sweet and the not sweet. It is customary to assume that visual images are absolutely determinate in respect to color, but even this may be doubted. I know no facts which prove that there is never the least vagueness in the immediate sensation. In thought, an absolutely determinate term cannot be realized, because, not being given by sense, such a concept would have to be formed by synthesis, and there would be no end to the synthesis because there is no limit to the number of possible predicates. A logical atom, then, like a point in space, would involve for its precise determination an endless process. We can only say, in a general way, that a term, however determinate, may be made more determinate still, but not that it can be made absolutely determinate. Such a term as "the second Philip of Macedon" is still capable of logical division,— into Philip drunk and Philip sober, for example; but we call it individual because that which is denoted by it is in only one place at one time. It is a term not *absolutely* indivisible, but indivisible as long as we neglect differences of time and the differences which accompany them. Such differences we habitually disregard in the logical division of substances. In the division of relations, etc., we do not, of course, disregard these differences, but we disregard some others. There is nothing to prevent almost any sort of difference from being conventionally neglected in some discourse, and if I be a term which in consequence of such neglect becomes indivisible in that discourse, we have in that discourse,

$$[I] = 1 .$$

This distinction between the absolutely indivisible and that which is one in number from a particular point of view is shadowed forth in the two words *individual* (τὸ ἄτομον) and *singular* (τὸ καθ' ἕκαστον); but as those who have used the word *individual* have not been aware that absolute individuality is merely ideal, it has come to be used in a more general sense.*

The old logics distinguish between *individuum signatum* and *individuum vagum.* "Julius Cæsar" is an example of the former; "a certain man," of the latter. The *individuum vagum,* in the days when such conceptions were exactly investigated, occasioned great difficulty from its having a certain generality, being capable, apparently, of logical division. If we include under the *individuum vagum* such a term as "any individual man," these difficulties appear in a strong light, for what is true of any individual man is true of all men. Such a term is in one sense not an individual term; for it represents every man. But it represents each man as capable of being denoted by a term which is individual; and so, though it is not itself an individual term, it stands for any one of a class of individual terms. If we call a thought about a thing in so far as it is denoted by a term, a *second intention,* we may say that such a term as "any individual man" is individual by second intention. The letters which the mathematician uses (whether in algebra or in geometry) are such individuals by second intention. Such individuals are one in number, for any individual man is one man; they may also be regarded as incapable of logical division, for any individual man, though he may either be a Frenchman or not, is yet altogether a Frenchman or altogether not, and not some one and some the other. Thus, all the formal logical laws relating to individuals will hold good of such individuals by second intention, and at the same time a universal proposition may at any moment be substituted for a proposition about such an individual, for nothing can be predicated of such an individual which cannot be predicated of the whole class.

There are in the logic of relatives three kinds of terms which involve general suppositions of individual cases. The first are *individual* terms, which denote only individuals; the second are those relatives whose correlatives are individual: I term these *infinitesimal relatives;* the third are *individual infinitesimal* relatives, and these I term *elementary* relatives.

* The absolute individual can not only not be realized in sense or thought, but cannot exist, properly speaking. For whatever lasts for any time, however short, is capable of logical division, because in that time it will undergo some change in its relations. But what does not exist for any time, however short, does not exist at all. All, therefore, that we perceive or think, or that exists, is general. So far there is truth in the doctrine of scholastic realism. But all that exists is infinitely determinate, and the infinitely determinate is the absolutely individual. This seems paradoxical, but the contradiction is easily resolved. That which exists is the object of a true conception. This conception may be made more determinate than any assignable conception; and therefore it is never so determinate that it is capable of no further determination.

Individual Terms.

The fundamental formulæ relating to individuality are two. Individuals are denoted by capitals.

(95.) If $x > 0$ $\quad x = X +\!\!,\, X' +\!\!,\, X'' +\!\!,\, X''' +\!\!,\,$ etc.

(96.) $$y^X = yX\,.$$

We have also the following which are easily deducible from these two:—

(97.) $(y,z)X = (yX),(zX)\,.$ (99.) $[X] = 1\,.$

(98.) $X,y_0 = X,yX\,.$ (100.) $\mathit{1}^X = X\,.$

We have already seen that

$$\mathit{1}^x = 0\,, \quad \text{provided that} \quad [x] > 1\,.$$

As an example of the use of the formulæ we have thus far obtained, let us investigate the logical relations between "benefactor of a lover of every servant of every woman," "that which stands to every servant of some woman in the relation of benefactor of a lover of him," "benefactor of every lover of some servant of a woman," "benefactor of every lover of every servant of every woman," etc.

In the first place, then, we have by (95)

$$s\mathrm{w} = s(\mathrm{W}' +\!\!,\, \mathrm{W}'' +\!\!,\, \mathrm{W}''' +\!\!,\, \text{etc.}) = s\mathrm{W}' +\!\!,\, s\mathrm{W}'' +\!\!,\, s\mathrm{W}''' +\!\!,\, \text{etc.}$$

$$s^{\mathrm{w}} = s^{\mathrm{W}' +\!,\, \mathrm{W}'' +\!,\, \mathrm{W}'' +\!,\, \text{etc.}} = s^{\mathrm{W}'}, s^{\mathrm{W}''}, s^{\mathrm{W}''}, \text{etc.}$$

From the last equation we have by (96)

$$s^{\mathrm{w}} = (s\mathrm{W}'),(s\mathrm{W}''),(s\mathrm{W}'''), \text{etc.}$$

Now by (31) $\quad x' +\!\!,\, x'' +\!\!,\,$ etc. $= x',x'',x'''$, etc. $+\!\!,\,$ etc.,

or

(101.) $$\Pi' \prec \Sigma'\,,$$

where Π' and Σ' signify that the addition and multiplication with commas are to be used. From this it follows that

(102.) $$s^{\mathrm{w}} \prec s\mathrm{w}\,.$$

If w vanishes, this equation fails, because in that case (95) does not hold.

From (102) we have

(103.) $$(ls)^{\mathrm{w}} \prec ls\mathrm{w}\,.$$

Since
$$\mathrm{a} = \mathrm{a},\mathrm{b} \mathbin{+_{\!,}} \text{etc.}\,,$$
$$\mathrm{b} = \mathrm{a},\mathrm{b} \mathbin{+_{\!,}} \text{etc.}\,,$$
we have
$$l\mathrm{a} = l(\mathrm{a},\mathrm{b} \mathbin{+_{\!,}} \text{etc.}) = l(\mathrm{a},\mathrm{b}) \mathbin{+_{\!,}} l(\text{etc.})\,,$$
$$l\mathrm{b} = l(\mathrm{a},\mathrm{b} \mathbin{+_{\!,}} \text{etc.}) = l(\mathrm{a},\mathrm{b}) \mathbin{+_{\!,}} l(\text{etc.})\,.$$

Multiplying these two equations commutatively we have
$$(l\mathrm{a}),(l\mathrm{b}) = l(\mathrm{a},\mathrm{b}) \mathbin{+_{\!,}} \text{etc.}$$
or

(104.)
$$l\Pi' \prec \Pi' l\,.$$

Now
$$(ls)^{\mathrm{w}} = (ls)^{\mathrm{W}' \mathbin{+_{\!,}} \mathrm{W}'' \mathbin{+_{\!,}} \mathrm{W}''' \mathbin{+_{\!,}} \text{etc.}} = \Pi'(ls)^{\mathrm{W}} = \Pi' ls\mathrm{W}\,,$$
$$ls^{\mathrm{w}} = ls^{\mathrm{W}' \mathbin{+_{\!,}} \mathrm{W}'' \mathbin{+_{\!,}} \mathrm{W}''' \mathbin{+_{\!,}} \text{etc.}} = l\Pi' s^{\mathrm{W}'} = l\Pi' s\mathrm{W}\,.$$

Hence,

(105.)
$$ls^{\mathrm{w}} \prec (ls)^{\mathrm{w}}\,,$$

or every lover of a servant of all women stands to every woman in the relation of lover of a servant of hers.

From (102) we have

(106.)
$$l^{s^{\mathrm{w}}} \prec ls^{\mathrm{w}}\,.$$

By (95) and (96) we have
$$l^s\mathrm{w} = l^s(\mathrm{W}' \mathbin{+_{\!,}} \mathrm{W}'' \mathbin{+_{\!,}} \mathrm{W}''' \text{ etc.}) = l^s\mathrm{W}' \mathbin{+_{\!,}} l^s\mathrm{W}'' \mathbin{+_{\!,}} l^s\mathrm{W}''' + \text{etc.}$$
$$= l^{s\mathrm{W}'} \mathbin{+_{\!,}} l^{s\mathrm{W}''} \mathbin{+_{\!,}} l^{s\mathrm{W}'''} + \text{etc.}$$

Now
$$s^{\mathrm{w}} = s^{\mathrm{W}' \mathbin{+_{\!,}} \mathrm{W}'' \mathbin{+_{\!,}} \mathrm{W}''' \mathbin{+_{\!,}} \text{etc.}} = s^{\mathrm{W}'},s^{\mathrm{W}''},s^{\mathrm{W}'''},\text{etc.}$$

So that by (94)
$$s^{\mathrm{w}} \prec s^{\mathrm{W}'} \prec s\mathrm{W}'\,.$$

Hence by (92)
$$l^{s\mathrm{W}'} \prec l^{s^{\mathrm{w}}}\,, \qquad l^{s\mathrm{W}''} \prec l^{s^{\mathrm{w}}}\,, \qquad l^{s\mathrm{W}'''} \prec l^{s^{\mathrm{w}}}\,.$$

Adding,
$$l^{s\mathrm{W}'} \mathbin{+_{\!,}} l^{s\mathrm{W}''} \mathbin{+_{\!,}} l^{s\mathrm{W}'''} \prec l^{s^{\mathrm{w}}}\;;$$
or

(107.)
$$l^s\mathrm{w} \prec l^{s^{\mathrm{w}}}\,.$$

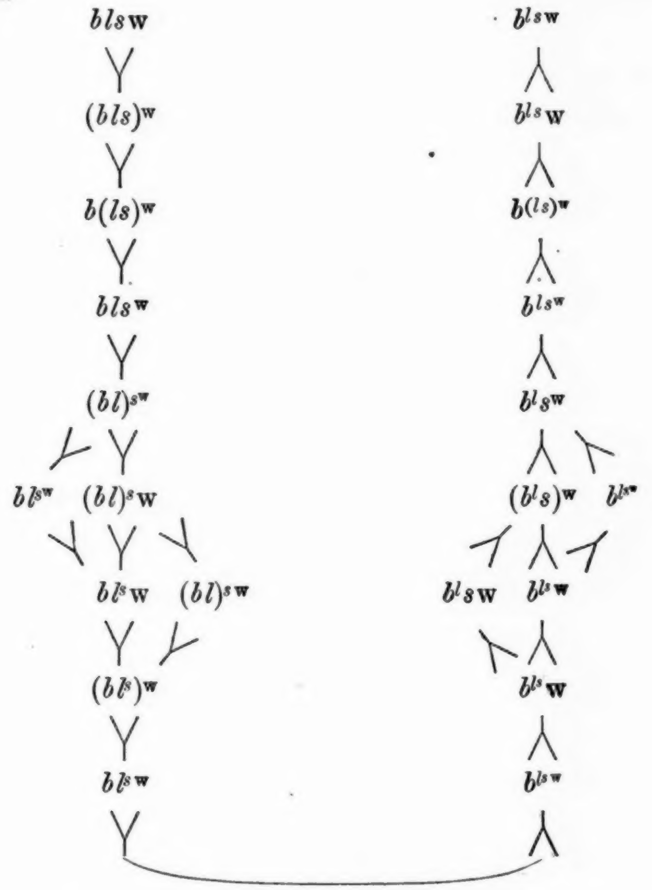
That is, every lover of every servant of any particular woman is a lover of every servant of all women.

By (102) we have

(108.)
$$l^{s\mathrm{w}} \prec l^s\mathrm{w}\,.$$

Thus we have
$$l^{s\mathrm{w}} \prec l^s\mathrm{w} \prec l^{s^{\mathrm{w}}} \prec ls^{\mathrm{w}} \prec (ls)^{\mathrm{w}} \prec ls\mathrm{w}\,.$$

By similar reasoning we can easily make out the relations shown in the following table. It must be remembered that the formulæ do not generally hold when exponents vanish.



It appears to me that the advantage of the algebraic notation already begins to be perceptible, although its powers are thus far very imperfectly made out. At any rate, it seems to me that such a *prima facie* case is made out that the reader who still denies the utility of the algebra ought not to be too indolent to attempt to write down the above twenty-two terms in ordinary language with logical precision. Having done that, he has only to disarrange them and then restore the arrangement by ordinary logic, in order to test the algebra so far as it is yet developed.

Infinitesimal Relatives.

We have by the binomial theorem by (49) and by (47),

$$(1+x)^n = 1 + \Sigma_p x^{n-p} + x^n .$$

Now, if we suppose the number of individuals to which any one thing is x to be reduced to a smaller and smaller number, we reach as our limit

$$x^2 = 0 ,$$

$$\Sigma_p x^{n-p} = [n] . \mathrm{I}^{n-\dagger\mathrm{I}}, x^{\dagger\mathrm{I}} = xn ,$$

$$(1+x)^n = 1 + xn .$$

If, on account of the vanishing of its powers, we call x an infinitesimal here and denote it by i, and if we put

$$xn = in = y ,$$

our equation becomes

(109.) $$(1+i)^{\frac{y}{i}} = 1 + y .$$

Putting $y = \mathit{1}$, and denoting $(1+i)^{\frac{\mathit{1}}{i}}$ by $\mathfrak{e}$, we have

(110.) $$\mathfrak{e} = (1+i)^{\frac{\mathit{1}}{i}} = 1 + \mathit{1} .$$

In fact, this agrees with ordinary algebra better than it seems to do; for $\mathit{1}$ is itself an infinitesimal, and $\mathfrak{e}$ is $\mathfrak{e}^{\mathit{1}}$. If the higher powers of $\mathit{1}$ did not vanish, we should get the ordinary development of $\mathfrak{e}$.

Positive powers of $\mathfrak{e}$ are absurdities in our notation. For negative powers we have

(111.) $$\mathfrak{e}^{-x} = 1 - x .$$

There are two ways of raising $\mathfrak{e}^{-x}$ to the y^{th} power. In the first place, by the binomial theorem,

$$(1-x)^y = 1 - [y] . 1^{y-\dagger\mathrm{I}}, x^{\dagger\mathrm{I}} + \frac{[y] . [y-\mathrm{I}]}{2} . 1^{y-\ddagger 2}, x^{\ddagger 2} - \text{etc.} ;$$

and, in the second place, by (111) and (10).

$$\mathfrak{e}^{-xy} = 1 - xy .$$

It thus appears that the sum of all the terms of the binomial development of $(1-x)^y$, after the first, is $-xy$. The truth of this may be shown by an example. Suppose the number of y's are four, viz. Y', Y'', Y''', and Y''''. Let us use x', x'', x''', and x'''' in such senses that

$$xY' = x', \quad xY'' = x'', \quad xY''' = x''', \quad xY'''' = x'''' .$$

Then the negatives of the different terms of the binomial development are,

$$[y].1^{y-\dagger 1}, x^{\dagger 1} = x' + x'' + x''' + x''''.$$

$$-\frac{[y].[y-1]}{2}.1^{y-\ddagger 2}, x^{\ddagger 2} = -x', x'' - x', x''' - x', x'''' - x'', x''' - x'', x'''' - x''', x''''.$$

$$+\frac{[y].[y-1].[y-2]}{2.3}.1^{-\|3}, x^{\|3} = x', x'', x''' + x', x'', x'''' + x', x''', x'''' + x'', x''', x''''.$$

$$x^y = -x', x'', x''', x''''.$$

Now, since this addition is invertible, in the first term, x' that is x'', is counted over twice, and so with every other pair. The second term subtracts each of these pairs, so that it is only counted once. But in the first term the x' that is x'' that is x''' is counted in three times only, while in the second term it is subtracted three times; namely, in (x', x'') in (x', x''') and in (x'', x'''). On the whole, therefore, a triplet would not be represented in the sum at all, were it not added by the third term. The whole quartette is included four times in the first term, is subtracted six times by the second term, and is added four times in the third term. The fourth term subtracts it once, and thus in the sum of these negative terms each combination occurs once, and once only; that is to say the sum is

$$x' +\!, x'' +\!, x''' +\!, x'''' = x(Y' +\!, Y'' +\!, Y''' +\!, Y'''') = xy.$$

If we write $(ax)^3$ for $[x].[x-1].[x-2].1^{x-\dagger 3}, a^{\dagger 3}$, that is for whatever is a to any three x's, regard being had for the order of the x's; and employ the modern numbers as exponents with this signification generally, then

$$1 - ax + \frac{1}{2!}(ax)^2 - \frac{1}{3!}(ax)^3 + \text{etc.}$$

is the development of $(1-a)^x$ and consequently it reduces itself to $1 - ax$. That is,

(112.) $$x = x - \frac{1}{2!}x^2 + \frac{1}{3!}x^3 - \frac{1}{4!}x^4 + \text{etc.}$$

$1 - x$ denotes everything except x, that is, whatever is other than every x; so that $\mathfrak{G}-$ means "not." We shall take $\log x$ in such a sense that

$$\mathfrak{G}^{\log x} = x^*.$$

* It makes another resemblance between 1 and infinity that $\log 0 = -1$.

I define the first difference of a function by the usual formula,

$$(113.)\qquad \Delta\varphi x = \varphi(x + \Delta x) - \varphi x\,,$$

where Δx is an indefinite relative which never has a correlate in common with x. So that

$$(114.)\qquad x,(\Delta x) = 0 \qquad x + \Delta x = x \,+\!\!,\, \Delta x\,.$$

Higher differences may be defined by the formulæ

$$(115.)\qquad \Delta^{\mathfrak{n}\cdot} x = 0 \quad \text{if} \quad \mathfrak{n} > 1\,.$$

$$\Delta^{2\cdot}\varphi x = \Delta\Delta x = \varphi(x + 2.\Delta x) - 2.\varphi(x + \Delta x) + \varphi x\,,$$

$$\Delta^{3\cdot}\varphi x = \Delta\Delta^{2\cdot}x = \varphi(x + 3.\Delta x) - 3.\varphi(x + 2.\Delta x) + 3.\varphi(x + \Delta x) - \varphi x\,.$$

$$(116.)\qquad \Delta^{\mathfrak{n}\cdot}\varphi x = \varphi(x + \mathfrak{n}.\Delta x) - \mathfrak{n}.\varphi(x + (\mathfrak{n} - 1).\Delta x)$$

$$+ \frac{\mathfrak{n}.(\mathfrak{n} - 1)}{2}.\varphi(x + (\mathfrak{n} - 2).\Delta x) - \text{etc.}$$

The exponents here affixed to Δ denote the number of times this operation is to be repeated, and thus have quite a different signification from that of the numerical coefficients in the binomial theorem. I have indicated the difference by putting a period after exponents significative of operational repetition. Thus, m^2 may denote a mother of a certain pair, $m^{2\cdot}$ a maternal grandmother.

Another circumstance to be observed is, that in taking the second difference of x, if we distinguish the two increments which x successively receives as $\Delta' x$ and $\Delta'' x$, then by (114)

$$(\Delta' x),(\Delta'' x) = 0$$

If Δx is relative to so small a number of individuals that if the number were diminished by one $\Delta^{\mathfrak{n}\cdot}\varphi x$ would vanish, then I term these two corresponding differences *differentials*, and write them with d instead of Δ.

The difference of the invertible sum of two functions is the sum of their differences; for by (113) and (18),

$$(117.)\qquad \Delta(\varphi x + \psi x) = \varphi(x + \Delta x) + \psi(x + \Delta x) - \varphi x - \psi x$$

$$= \varphi(x + \Delta x) - \varphi x + \psi(x + \Delta x) - \psi x = \Delta\varphi x + \Delta\psi x\,.$$

If a is a constant, we have

(118.) $$\Delta a\varphi x = a(\varphi x \mathbin{+\!\!,} \Delta\varphi x) - a\varphi x = a\Delta\varphi x - (a\Delta\varphi x),a\varphi x\ ,$$

$$\Delta^{2\cdot} a\varphi x = - \Delta a\varphi x, a\Delta x, \text{ etc.}$$

$$\Delta(\varphi x)a = (\Delta\varphi x)a - ((\Delta\varphi x)a),\varphi x a\ ,$$

$$\Delta^{2\cdot}(\varphi x)a = - \Delta(\varphi x)a, \text{ etc.}$$

(119.) $$\Delta(a,\varphi x) = a,\Delta\varphi x\ .$$

Let us differentiate the successive powers of x. We have in the first place,

$$\Delta(x^2) = (x + \Delta x)^2 - x^2 = 2.x^{2-\dagger 1},(\Delta x)^{\dagger 1} + (\Delta x)^2\ .$$

Here, if we suppose Δx to be relative to only one individual, $(\Delta x)^2$ vanishes, and we have, with the aid of (115),

$$d(x^2) = 2.x^1,dx\ .$$

Considering next the third power, we have, for the first differential,

$$\Delta(x^3) = (x + \Delta x)^3 - x^3 = 3.x^{3-\dagger 1},(\Delta x)^{\dagger 1} + 3.x^{3-\ddagger 2},(\Delta x)^{\ddagger 2} + (\Delta x)^3\ ,$$

$$d(x^3) = 3.x^2,(dx)\ .$$

To obtain the second differential, we proceed as follows:—

$$\begin{aligned}
\Delta^{2\cdot}(x^3) &= (x + 2.\Delta x)^3 - 2.(x + \Delta x)^3 + x^3 \\
&= x^3 + 6.x^{3-\dagger 1},(\Delta x)^{\dagger 1} + 12.x^{3-\ddagger 2},(\Delta x)^{\ddagger 2} + 8.(\Delta x)^3 \\
&\quad - 2.x^3 - 6.x^{3-\| 1},(\Delta x)^{\| 1} - 6.x^{3-\S 2},(\Delta x)^{\S 2} - 2.(\Delta x)^3 \\
&\quad + x^3 \\
&= 6.x^{3-\ddagger 2},(\Delta x)^{\ddagger 2} + 6.(\Delta x)^3\ .
\end{aligned}$$

Here, if Δx is relative to less than two individuals, $\Delta\varphi x$ vanishes. Making it relative to two only, then, we have

$$d^{2\cdot}(x^3) = 6.x^1,(dx)^2\ .$$

These examples suffice to show what the differentials of $x^{\mathbf{n}}$ will be. If for the number $\mathbf{n}$ we substitute the logical term n, we have

$$\Delta(x^n) = (x + \Delta x)^n - x^n = [n].x^{n-\dagger 1},(\Delta x)^{\dagger 1} + \text{etc.}$$

$$d(x^n) = [n].x^{n-1},(dx)\ .$$

We should thus readily find

$$(120.)\quad d^{\mathfrak{m}}(x^n) = [n].[n-1].[n-2]....[n-\mathfrak{m}+1].x^{n-{}^{\dagger}\mathfrak{m}},(dx)^{\dagger\mathfrak{m}}.$$

Let us next differentiate l^x. We have, in the first place,

$$\Delta l^x = l^{x \,+\!\!,\, \Delta x} - l^x = l^x, l^{\Delta x} - l^x = l^x, (l^{\Delta x} - 1)\,.$$

The value of $l^{\Delta' x} - 1$ is next to be found.

We have by (111) $\qquad \mathfrak{S}^{l^{\Delta x}-1} = l^{\Delta x}\,.$

Hence, $\qquad l^{\Delta x} - 1 = \log l^{\Delta x}\,.$

But by (10) $\qquad \log l^{\Delta x} = (\log l)\Delta x\,.$

Substituting this value of $l^{dx} - 1$ in the equation lately found for dl^x we have

$$(121.)\qquad dl^x = l^x, (\log, l)\, dx = l^x, (l-1)\, dx = -\, l^x, (1-l)\, dx\,.$$

In printing this paper, I here make an addition which supplies an omission in the account given above of involution in this algebra. We have seen that every term which does not vanish is conceivable as logically divisible into individual terms. Thus we may write

$$s = S' \,+\!\!,\, S'' \,+\!\!,\, S''' \,+\!\!,\, \text{etc.}$$

where not more than one individual is in any one of these relations to the same individual, although there is nothing to prevent the same person from being so related to many individuals. Thus, "bishop of the see of" may be divided into first bishop, second bishop, etc., and only one person can be $\mathfrak{n}^{\text{th}}$ bishop of any one see, although the same person may (where translation is permitted) be $\mathfrak{n}^{\text{th}}$ bishop of several sees. Now let us denote the converse of x by $\mathcal{K}x$; thus, if s is "servant of," $\mathcal{K}s$ is "master or mistress of." Then we have

$$\mathcal{K}s = \mathcal{K}S' \,+\!\!,\, \mathcal{K}S'' \,+\!\!,\, \mathcal{K}S''' \,+\!\!,\, \text{etc.};$$

and here each of the terms of the second member evidently expresses such a relation that the same person cannot be so related to more than one, although more than one may be so related to the same. Thus, the converse of "bishop of the see of —" is "see one of whose bishops is —," the converse of "first bishop of —" is "see whose first bishop is —," etc. Now, the same see cannot be a see whose $\mathfrak{n}^{\text{th}}$ bishop is more than one individual, although several sees may be so related to the same indi-

vidual. Such relatives I term infinitesimal on account of the vanishing of their higher powers. Every relative has a converse, and since this converse is conceivable as divisible into individual terms, the relative itself is conceivable as divisible into infinitesimal terms. To indicate this we may write

(122.) $$\text{If } x > 0 \quad . \quad x = X_{\prime} +\!\!, X_{\prime\prime} +\!\!, X_{\prime\prime\prime} +\!\!, \text{etc.}$$

As a term which vanishes is not an individual, nor is it composed of individuals, so it is neither an infinitesimal nor composed of infinitesimals.

As we write $$lS', lS'', lS''', \text{etc} = l^s,$$
so we may write
(123.) $$L_{\prime}s, L_{\prime\prime}s, L_{\prime\prime\prime}s, \text{etc} = {}^{l}s\,.$$

But as the first formula is affected by the circumstance that *zero* is not an individual, so that $l^s w$ does not vanish on account of no woman having the particular kind of servant denoted by S'', $l^s w$ denoting merely every lover of whatever servant there is of any woman; so the second formula is affected in a similar way, so that the vanishing of $L_{\prime}s$ does not make ${}^{l}s$ to vanish, but this is to be interpreted as denoting everything which is a lover, *in whatever way it is a lover at all*, of a servant. Then just as we have by (112), that

(124.) $$l^s = 1 - (1 - l)s\,;$$
so we have
(125.) $$^{l}s = 1 - l(1 - s)\,.$$

Mr. De Morgan denotes l^s and ${}^{l}s$ by LS' and $L_{\prime}S$ respectively, and he has traced out the manner of forming the converse and negative of such functions in detail. The following table contains most of his results in my notation. For the converse of m, I write ɯ; and for that of n, u.

x	$\mathcal{K}x$	$\mathcal{G}^{-x}$	$\mathcal{K}\mathcal{G}^{-x}$
mn	$u\text{ɯ}$	$(1-m)^n = {}^{m}(1-n)$	${}^{\text{ɯ}}(1-u) = (1-\text{ɯ})^u$
$m^n = {}^{(1-m)}(1-n)$	${}^{u}\text{ɯ} = (1-u)^{(1-\text{ɯ})}$	$(1-m)n$	$\text{ɯ}(1-u)$
${}^{m}n = (1-m)^{(1-n)}$	$u^{\text{ɯ}} = {}^{(1-u)}(1-\text{ɯ})$	$m(1-n)$	$(1-\text{ɯ})u$

I shall term the operation by which w is changed to ^{l}w, *backward involution.* All the laws of this but one are the same as for ordinary involution, and the one exception is of that kind which is said to prove the rule. It is that whereas with ordinary

involution we have,

$$(l^s)^w = l^{(sw)} \, ;$$

in backward involution we have

$$(126.) \qquad {}^{l}({}^{s}w) = {}^{(ls)}w \, ;$$

that is, the things which are lovers to nothing but things that are servants to nothing but women are the things which are lovers of servants to nothing but women.

The other fundamental formulæ of backward involution are as follows:—

$$(127.) \qquad {}^{l \,+\!\!,\, s}w = {}^{l}w, {}^{s}w \, ,$$

or, the things which are lovers or servants to nothing but women are the things which are lovers to nothing but women and servants to nothing but women.

$$(128.) \qquad {}^{l}(f, u) = {}^{l}f, {}^{l}u \, ,$$

or, the things which are lovers to nothing but French violinists are the things that are lovers to nothing but Frenchmen and lovers to nothing but violinists. This is perhaps not quite axiomatic. It is proved as follows. By (125) and (30)

$${}^{l}(f, u) = \mathfrak{S}^{-l(1-f, u)} = \mathfrak{S}^{-(l(1-f) \,+\!\!,\, l(1-u))}$$

By (125), (13), and (7),

$${}^{l}f, {}^{l}u = \mathfrak{S}^{-l(1-f)}, \mathfrak{S}^{-l(1-u)} = \mathfrak{S}^{-(l(1-f) \,+\!\!,\, l(1-u))} \, .$$

Finally, the binomial theorem holds with backward involution. For those persons who are lovers of nothing but Frenchmen and violinists consist first of those who are lovers of nothing but Frenchmen; second, of those who in some ways are lovers of nothing but Frenchmen and in all other ways of nothing but violinists, and finally of those who are lovers only of violinists. That is,

$$(129.) \qquad {}^{l}(u \,+\!\!,\, f) = {}^{l}u \,+\!\!,\, \Sigma_p {}^{l-p}u, {}^{p}f \,+\!\!,\, {}^{l}f \, .$$

In order to retain the numerical coefficients, we must let $\{l\}$ be the number of persons that one person is lover of. We can then write

$${}^{l}(u + f) = {}^{l}u + \{l\} {}^{l-\dagger 1}u, {}^{\dagger 1}f + \frac{\{l\}.\{l-1\}}{2} {}^{l-\ddagger 2}u, {}^{\ddagger 2}f + \text{etc.}$$

We have also the following formula which combines the two involutions:—

$$(130.) \qquad {}^{l}(s^w) = ({}^{l}s)^w \, ;$$

that is, the things which are lovers of nothing but what are servants of all women

are the same as the things which are related to all women as lovers of nothing but their servants.

It is worth while to mention, in passing, a singular proposition derivable from (128). Since, by (124) and (125)

$$^{x}y = (1 - x)^{(1-y)} ,$$

and since

$$1 - (\mathrm{u} +\!\!, \mathrm{f}) = \infty^{-(\mathrm{u} +, \mathrm{f})} = \infty^{-\mathrm{u}}, \infty^{-\mathrm{f}} = (1 - \mathrm{u}),(1 - \mathrm{f}) ,$$

(128) gives us,

$$(1 - l)^{(1-\mathrm{u}),(1-\mathrm{f})} = (1 - l)^{(1-\mathrm{u})} +\!\!, \Sigma_p(1 - (l - p))^{(1-\mathrm{u})},(1 - p)^{(1-\mathrm{f})}$$
$$+\!\!, (1 - l)^{(1-\mathrm{f})} .$$

This is, of course, as true for u and f as for $(1 - u)$ and $(1 - f)$. Making those substitutions, and taking the negative of both sides, we have, by (124)

(131.) $$l(\mathrm{u},\mathrm{f}) = (l\mathrm{u}),\Pi'_p((l - p)u +\!\!, p\mathrm{f}),(l\mathrm{f}) ,$$

or, the lovers of French violinists are those persons who, in reference to every mode of loving whatever, either in that way love some violinists or in some other way love some Frenchmen. This logical proposition is certainly not self-evident, and its practical importance is considerable. In a similar way, from (12) we obtain

(132.) $$(e,c)\mathrm{f} = \Pi'_p(e(\mathrm{f} - p) +\!\!, cp) ,$$

that is, to say that a person is both emperor and conqueror of the same Frenchman is the same as to say that, taking any class of Frenchmen whatever, this person is either an emperor of some one of this class, or conqueror of some one among the remaining Frenchmen.

The properties of zero and unity, with reference to backward involution, are easily derived from (125). I give them here in comparison with the corresponding formulæ for forward involution.

(133.) $^{0}x = 1 \qquad x^{0} = 1 .$

(134.) $^{q}0 = 0 \qquad 0^{r} = 0 ,$

where q is the converse of an unlimited relative, and r is greater than zero.

(135.) $^{1}x = x \qquad x^{1} = x .$

(136.) $^{y}1 = y \qquad 1^{z} = z ,$

where y is infinitesimal, and z is individual. Otherwise, both vanish.

(137.) $\qquad {}^{1}s = 0 \qquad\qquad p^{1} = 0\,,$

where s is less than unity and p is a limited relative.

(138.) $\qquad {}^{x}1 = 1 \qquad\qquad 1^{x} = 1\,.$

In other respects the formulæ for the two involutions are not so analogous as might be supposed; and this is owing to the dissimilarity between individuals and infinitesimals. We have, it is true, if $X_{,}$ is an infinitesimal and X' an individual,

(139.) $\qquad X_{,}(y,z) = X_{,}y, X_{,}z \quad$ like $\quad (y,z)X' = yX', zX'\,;$

(140.) $\qquad X_{,},y_0 = X_{,}, X_{,}y \quad$ " $\quad X',y_0 = X',yX'\,;$

(141.) $\qquad \{X_{,}\} = \mathrm{I} \quad$ " $\quad [X'] = \mathrm{I}\,.$

We also have

(142.) $\qquad X_{,}y \mathrel{-\!\!<} {}^{X_{,}}y\,.$

But we have *not* ${}^{X_{,}}y = X_{,}y$, and consequently we have *not* ${}^{s}\mathrm{w} \mathrel{-\!\!<} s\mathrm{w}$, for this fails if there is anything which is not a servant at all, while the corresponding formula $s^{\mathrm{w}} \mathrel{-\!\!<} s\mathrm{w}$ only fails if there is not anything which is a woman. Now, it is much more often the case that there is something which is not x, than that there is not anything which is x. We have with the backward involution, as with the forward, the formulæ

(143.) $\qquad$ If $x \mathrel{-\!\!<} y \qquad {}^{y}z \mathrel{-\!\!<} {}^{x}z\,;$

(144.) $\qquad$ If $x \mathrel{-\!\!<} y \qquad {}^{z}x \mathrel{-\!\!<} {}^{z}y\,.$

The former of these gives us

(145.) $\qquad {}^{ls}\mathrm{w} \mathrel{-\!\!<} {}^{(l^{s})}\mathrm{w}\,,$

or, whatever is lover to nothing but what is servant to nothing but women stands to nothing but a woman in the relation of lover of every servant of hers. The following formulæ can be proved without difficulty.

(146.) $\qquad l^{s}\mathrm{w} \mathrel{-\!\!<} {}^{l_s}\mathrm{w}\,,$

or, every lover of somebody who is servant to nothing but a woman stands to nothing but women in the relation of lover of nothing but a servant of them.

(147.) $\qquad {}^{l}s\mathrm{w} \mathrel{-\!\!<} {}^{l}(s\mathrm{w})\,,$

or, whatever stands to a woman in the relation of lover of nothing but a servant of hers is a lover of nothing but servants of women.

The differentials of functions involving backward involution are

(148.) $$d^n x = \{n\}^{n-1} x , dx \, .$$

(149.) $$d^x l = {}^x l , dx \log. x \, .$$

In regard to powers of $\mathfrak{S}$ we have

(150.) $${}^x\mathfrak{S} = \mathfrak{S}^x .$$

Exponents with a dot may also be put upon either side of the letters which they affect.

The greater number of functions of x in this algebra may be put in the form

$$\varphi x = \Sigma_p \, \Sigma_q \, {}_p\mathrm{A}_q \, {}^p x^q \, {}_p\mathrm{B}_q \, .$$

For all such functions Taylor's and Maclaurin's theorems hold good in the form,

(151.) $$\boxed{\frac{y}{dx}} \; \boxed{\frac{0}{y}} \; \sum_0^\infty {}_\mathfrak{p} \frac{1}{\mathfrak{p}!} . d^{\mathfrak{p}.} = \mathit{1} \, .$$

The symbol $\boxed{\frac{\mathrm{a}}{\mathrm{b}}}$ is used to denote that a is to be substituted for b in what follows. For the sake of perspicuity, I will write Maclaurin's theorem at length.

$$\varphi x = \boxed{\frac{x}{dx}} \; \boxed{\frac{0}{x}} \left(\frac{1}{0!} . d^{0.} + \frac{1}{1!} . d^{1.} + \frac{1}{2!} . d^{2.} + \frac{1}{3!} . d^{3.} + \text{etc.} \right) \varphi x \, .$$

The proof of these theorems is very simple. The $(p+q)^{\text{th}}$ differential of ${}^p x^q$ is the only one which does not vanish when x vanishes. This differential then becomes $[p+q]! . {}^p(dx)^q$. It is plain, therefore, that the theorems hold when the coefficients ${}^p\mathrm{A}^q$ and ${}^p\mathrm{B}^q$ are $\mathit{1}$. But the general development, by Maclaurin's theorem, of $a\varphi x$ or $(\varphi x)a$ is in a form which (112) reduces to identity. It is very likely that the application of these theorems is not confined within the limits to which I have restricted it. We may write these theorems in the form

(152.) $$\boxed{\frac{y}{dx}} \; \boxed{\frac{0}{y}} \; \mathfrak{S}^d = \mathit{1},$$

provided we assume that when the first differential is positive

$$\mathfrak{S}^d = \frac{1}{0!} d^0 + \frac{1}{1!} d^1 + \frac{1}{2!} d^2 + \text{etc.},$$

but that when the first differential is negative this becomes by (111),

$$\mathfrak{S}^d = 1 + d.$$

As another illustration of the use which may be made of differentiation in logic, let us consider the following problem. In a certain institution all the officers (x) and also all their common friends (f) are privileged persons (y). How shall the class of privileged persons be reduced to a minimum? Here we have

$$y = x + f^x ,$$
$$dy = dx + df^x = dx - f^x,(1 - f)dx .$$

When y is at a minimum it is not diminished either by an increase or diminution of x. That is,

$$[dy] \succ 0 ,$$

and when [x] is diminished by one,

$$[dy] \prec 0 .$$

When x is a minimum, then

$$[dx - f^x,(1 - f)dx] \succ 0 \qquad [dx - f^{x-1},(1 - f)dx] \prec 0$$

$$\text{(A.)} \qquad [dx] - [f^x,(1 - f)dx] \succ 0 \qquad [dx] - [f^{x-1},(1 - f)dx] \prec 0 .$$

Now we have by (30)

$$f^x,(1 - f)dx = f^x - (0;0),(1 - f)dx .$$

Hence,

$$[f^x] \prec [dx] + [0;0,].[(1 - f)dx] \qquad [f^{x-1}] \succ [dx] + [0;0,].[(1 - f)dx].$$

But [0;0,] lies between the limits o and 1, and

$$(153.) \qquad [dx] = 1 .$$

We have, therefore,

$$[f^x] \prec 1 + [(1 - f)1] \qquad [f^{x-1}] \succ 1 .$$

This is the general solution of the problem. If the event of a person who may be an official in the institution being a friend of a second such person is independent of and equally probable with his being a friend of any third such person, and if we take p, or the whole class of such persons, for our universe, we have,

$$p = 1 ;$$
$$[f^x,] = \frac{[f^x]}{[p]} = \left(\frac{[f]}{[p]}\right)^{[x]},$$
$$[(1 - f)dx] = [1 - f].[dx] = ([p] - [f]).[dx] ,$$
$$[f^x,(1 - f)dx] = \left(\frac{[f]}{[p]}\right)^{[x]}.([p] - [f]).[dx] .$$

Substituting these values in our equations marked (A) we get, by a little reduction,

$$[\mathrm{x}] > \frac{\log([\mathrm{p}] - [f])}{\log[\mathrm{p}] - \log[f]},$$

$$[\mathrm{x}] < \frac{\log([\mathrm{p}] - [f])}{\log[\mathrm{p}] - \log[f]} + 1 .$$

The same solution would be reached through quite a different road by applying the calculus of finite differences in the usual way.

Elementary Relatives.

By an elementary relative I mean one which signifies a relation which exists only between mutually exclusive pairs (or in the case of a conjugative term, triplets, or quartettes, etc.) of individuals, or else between pairs of classes in such a way that every individual of one class of the pair is in that relation to every individual of the other. If we suppose that in every school, every teacher teaches every pupil (a supposition which I shall tacitly make whenever in this paper I speak of a school), then *pupil* is an elementary relative. That every relative may be conceived of as a logical sum of elementary relatives is plain, from the fact that if a relation is sufficiently determined it can exist only between two individuals. Thus, a *father* is either father in the first ten years of the Christian era, or father in the second ten years, in the third ten years, in the first ten years, B. C., in the second ten years, or the third ten years, etc. Any one of these species of father is father for the first time or father for the second time, etc. Now such a relative as "father for the third time in the second decade of our era, of —" signifies a relation which can exist only between mutually exclusive pairs of individuals, and is therefore an elementary relative; and so the relative *father* may be resolved into a logical sum of elementary relatives.

The conception of a relative as resolvable into elementary relatives has the same sort of utility as the conception of a relative as resolvable into infinitesimals or of any term as resolvable into individuals.

Elementary simple relatives are connected together in systems of four. For if A:B be taken to denote the elementary relative which multiplied into B gives A, then this relation existing as elementary, we have the four elementary relatives

$$\mathrm{A{:}A} \qquad \mathrm{A{:}B} \qquad \mathrm{B{:}A} \qquad \mathrm{B{:}B} .$$

An example of such a system is — colleague : teacher : pupil : schoolmate. In the same way, obviously, elementary conjugatives are in systems the number of members in

which is $(\mathfrak{n} + 1)^{\mathfrak{n}+1}$ where $\mathfrak{n}$ is the number of correlates which the conjugative has. At present, I shall consider only the simple relatives.

The existence of an elementary relation supposes the existence of mutually exclusive pairs of classes. The first members of those pairs have something in common which discriminates them from the second members, and may therefore be united in one class, while the second members are united into a second class. Thus *pupil* is not an elementary relative unless there is an absolute distinction between those who teach and those who are taught. We have, therefore, two general absolute terms which are mutually exclusive, "body of teachers in a school," and "body of pupils in a school." These terms are general because it remains undetermined what school is referred to. I shall call the two mutually exclusive absolute terms which any system of elementary relatives supposes, the *universal extremes* of that system. There are certain characters in respect to the possession of which both members of any one of the pairs between which there is a certain elementary relation agree. Thus, the body of teachers and the body of pupils in any school agree in respect to the country and age in which they live, etc., etc. Such characters I term *scalar characters* for the system of elementary relatives to which they are so related; and the relatives written with a comma which signify the possession of such characters, I term *scalars* for the system. Thus, supposing French teachers have only French pupils and *vice versa*, the relative

$$\mathrm{f},$$

will be a scalar for the system "colleague : teacher : pupil : schoolmate." If r is an elementary relative for which s, is a scalar,

$$\mathrm{s},r = r\mathrm{s},\,. \qquad (154.)$$

Let $c, t, p, s,$ denote the four elementary relatives of any system; such as colleague, teacher, pupil, schoolmate; and let a,, b,, c,, d,, be scalars for this system. Then any relative which is capable of expression in the form

$$\mathrm{a},c + \mathrm{b},t + \mathrm{c},p + \mathrm{d},s\,.$$

I shall call a *logical quaternion.* Let such relatives be denoted by q, q', q'', etc. It is plain, then, from what has been said, that any relative may be regarded as resolvable into a logical sum of logical quaternions.

The multiplication of elementary relatives of the same system follows a very simple law. For if u and v be the two universal extremes of the system $c, t, p, s,$ we may write

$$c = \mathrm{u:u} \quad t = \mathrm{u:v} \quad p = \mathrm{v:u} \quad s = \mathrm{v:v}\,,$$

and then if w and w′ are each either u or v, we have

$$(155.)\qquad (\mathrm{w}':\mathrm{w})\,\text{б}-^{\mathrm{w}} = 0\,.$$

This gives us the following multiplication-table, where the multiplier is to be entered at the side of the table and the multiplicand at the top, and the product is found in the middle:—

(156.)

	c	*t*	*p*	*s*
c	*c*	*t*	0	0
t	0	0	*c*	*t*
p	*p*	*s*	0	0
s	0	0	*p*	*s*

The sixteen propositions expressed by this table are in ordinary language as follows:—

The colleagues of the colleagues of any person are that person's colleagues;
The colleagues of the teachers of any person are that person's teachers;
There are no colleagues of any person's pupils;
There are no colleagues of any person's schoolmates;
There are no teachers of any person's colleagues;
There are no teachers of any person's teachers;
The teachers of the pupils of any person are that person's colleagues;
The teachers of the schoolmates of any person are that person's teachers;
The pupils of the colleagues of any person are that person's pupils;
The pupils of the teachers of any person are that person's schoolmates;
There are no pupils of any person's pupils;
There are no pupils of any person's schoolmates;
There are no schoolmates of any person's colleagues;
There are no schoolmates of any person's teachers;
The schoolmates of the pupils of any person are that person's pupils;
The schoolmates of the schoolmates of any person are that person's schoolmates.

This simplicity and regularity in the multiplication of elementary relatives must clearly enhance the utility of the conception of a relative as resolvable into a sum of logical quaternions.

It may sometimes be convenient to consider relatives each one of which is of the form

$$\mathrm{a},i + \mathrm{b},j + \mathrm{c},k + \mathrm{d},l + \text{etc.}$$

where a,, b,, c,, d,, etc. are scalars, and i, j, k, l, etc. are each of the form

$$\mathrm{m},u + \mathrm{n},v + \mathrm{o},w + \text{etc.}$$

where m,, n,, o,, etc. are scalars, and u, v, w, etc. are elementary relatives. In all such cases (155) will give a multiplication-table for i, j, k, l, etc. For example, if we have three classes of individuals, u_1, u_2, u_3, which are related to one another in pairs, we may put

$$\begin{array}{lll} \mathrm{u}_1:\mathrm{u}_1 = i & \mathrm{u}_1:\mathrm{u}_2 = j & \mathrm{u}_1:\mathrm{u}_3 = k \\ \mathrm{u}_2:\mathrm{u}_1 = l & \mathrm{u}_2:\mathrm{u}_2 = m & \mathrm{u}_2:\mathrm{u}_3 = n \\ \mathrm{u}_3:\mathrm{u}_1 = o & \mathrm{u}_3:\mathrm{u}_2 = p & \mathrm{u}_3:\mathrm{u}_3 = q \end{array}$$

and by (155) we get the multiplication-table

	i	j	k	l	m	n	o	p	q
i	i	j	k	0	0	0	0	0	0
j	0	0	0	i	j	k	0	0	0
k	0	0	0	0	0	0	i	j	k
l	l	m	n	0	0	0	0	0	0
m	0	0	0	l	m	n	0	0	0
n	0	0	0	0	0	0	l	m	n
o	o	p	q	0	0	0	0	0	0
p	0	0	0	o	p	q	0	0	0
q	0	0	0	0	0	0	o	p	q

If we take

$$i = \mathrm{u}_1\!:\!\mathrm{u}_2 + \mathrm{u}_2\!:\!\mathrm{u}_3 + \mathrm{u}_3\!:\!\mathrm{u}_4\,,$$
$$j = \mathrm{u}_1\!:\!\mathrm{u}_3 + \mathrm{u}_2\!:\!\mathrm{u}_4\,,$$
$$k = 2.\mathrm{u}_1\!:\!\mathrm{u}_4\,,$$

we have

	i	j	k
i	j	k	0
j	k	0	0
k	0	0	0

If we take

$$i = \mathrm{u}_1\!:\!\mathrm{u}_2 + \mathrm{u}_2\!:\!\mathrm{u}_3 + \mathrm{u}_3\!:\!\mathrm{u}_4 + \mathrm{u}_5\!:\!\mathrm{u}_6 + \mathrm{u}_7\!:\!\mathrm{u}_8\,,$$
$$j = \mathrm{u}_1\!:\!\mathrm{u}_3 + \mathrm{u}_2\!:\!\mathrm{u}_4\,,$$
$$k = 2.\mathrm{u}_1\!:\!\mathrm{u}_4\,,$$
$$l = \mathrm{u}_6\!:\!\mathrm{u}_8 + \mathfrak{a}.\mathrm{u}_5\!:\!\mathrm{u}_7 + 2\mathfrak{b}.\mathrm{u}_1\!:\!\mathrm{u}_9 + \mathrm{u}_9\!:\!\mathrm{u}_4 + \mathfrak{c}.\mathrm{u}_5\!:\!\mathrm{u}_6\,.$$
$$m = \mathrm{u}_5\!:\!\mathrm{u}_8\,,$$

we have

	i	j	k	l	m
i	j	k	0	m	0
j	k	0	0	0	0
k	0	0	0	0	0
l	$\mathfrak{a}.m$	0	0	$\mathfrak{b}.k+\mathfrak{c}.m$	0
m	0	0	0	0	0

These multiplication-tables have been copied from Professor Peirce's monograph on Linear Associative Algebras.* I can assert, upon reasonable inductive evidence, that all such algebras can be interpreted on the principles of the present notation in the

* *Linear Associative Algebra.* By BENJAMIN PEIRCE. 4to, lithographed. Washington. 1870.

same way as those given above. In other words, all such algebras are complications and modifications of the algebra of (156). It is very likely that this is true of all algebras whatever. The algebra of (156), which is of such a fundamental character in reference to pure algebra and our logical notation, has been shown by Professor Peirce to be the algebra of Hamilton's quaternions. In fact, if we put

$$1 = i + l.$$

$$i' = \sqrt{1-b^2}\,\mathrm{J}\,i - (\sqrt{1-a^2}\,b + ab\,\mathrm{J})j + (\sqrt{1-a^2}\,b - ab\,\mathrm{J})k - \sqrt{1-b^2}\,\mathrm{J}\,l.$$

$$j' = -b\sqrt{1-c^2}\,\mathrm{J}i + \left(ac - \sqrt{1-a^2}\sqrt{1-b^2}\sqrt{1-c^2} - (\sqrt{1-a^2}\,c + a\sqrt{1-b^2}\sqrt{1-c^2})\mathrm{J}\right)j$$
$$-\left(ac - \sqrt{1-a^2}\sqrt{1-b^2}\sqrt{1-c^2} + (\sqrt{1-a^2}\,c + a\sqrt{1-b^2}\sqrt{1-c^2})\mathrm{J}\right)k + b\sqrt{1-c^2}\,\mathrm{J}\,l.$$

$$k' = bc\,\mathrm{J}i + \left(\sqrt{1-a^2}\sqrt{1-b^2}\,c + a\sqrt{1-c^2} + (a\sqrt{1-b^2}\,c - \sqrt{1-a^2}\sqrt{1-c^2})\mathrm{J}\right)j$$
$$-\left(\sqrt{1-a^2}\sqrt{1-b^2}\,c + a\sqrt{1-c^2} - (a\sqrt{1-b^2}\,c - \sqrt{1-a^2}\sqrt{1-c^2})\mathrm{J}\right)k - bc\,\mathrm{J}\,l.$$

where a,b,c, are scalars, then $1,i',j',k'$ are the four fundamental factors of quaternions, the multiplication-table of which is as follows: —

	1	i'	j'	k'
1	1	i'	j'	k'
i'	i'	-1	k'	$-j'$
j'	j'	$-k'$	-1	i'
k'	k'	j'	$-i'$	-1

It is no part of my present purpose to consider the bearing upon the philosophy of space of this occurrence, in pure logic, of the algebra which expresses all the properties of space; but it is proper to point out that one method of working with this notation would be to transform the given logical expressions into the form of Hamilton's quaternions (after representing them as separated into elementary relatives), and then to make use of geometrical reasoning. The following formulæ will assist this process. Take the quaternion relative

$$q = xi + yj + zk + wl,$$

where x,y,z, and w are scalars. The conditions of q being a *scalar*, *vector*, etc. (that

is, being denoted by an algebraic expression which denotes a scalar, a vector, etc., in geometry), are

(157.) Form of a scalar: $x(i+l)$.

(158.) Form of a vector: $xi + yi + zk - xl$.

(159.) Form of a versor:

$$\frac{x}{y}\left(\frac{x}{z}-1\right)^{-\frac{1}{2}}i+\frac{y}{x}\left(\frac{x}{z}-1\right)^{-\frac{1}{2}}j+\frac{z}{y}\left(\frac{z}{x}-1\right)^{-\frac{1}{2}}k+\frac{y}{z}\left(\frac{z}{x}-1\right)^{-\frac{1}{2}}l.$$

(160.) Form of zero: $xi + xyj + \frac{z}{y}k + zl$.

(161.) Scalar of q: $Sq = \frac{1}{2}(x+w)(i+l)$.

(162.) Vector of q: $Vq = \frac{1}{2}(x-w)i + yj + zk + \frac{1}{2}(w-x)l$.

(163.) Tensor of q: $Tq = \sqrt{xw - yz}(i+l)$.

(164.) Conjugate of q: $Kq = wi - yj - zk + xl$.

In order to exhibit the logical interpretations of these functions, let us consider a universe of married monogamists, in which husband and wife always have country, race, wealth, and virtue, in common. Let i denote "man that is —," j "husband of —," k "wife of —," and l "woman that is —"; x negro that is —, y rich person that is —, z American that is —, and w thief that is —. Then, q being defined as above, the q's of any class will consist of so many individuals of that class as are negro-men or women-thieves together with all persons who are rich husbands or American wives of persons of this class. Then, $2Sq$ denotes, by (160), all the negroes and besides all the thieves, while Sq is the indefinite term which denotes half the negroes and thieves. Now, those persons who are self-q's of any class (that is, the q's of themselves among that class) are $xi + wl$; add to these their spouses and we have $2Sq$. In general, let us term $(j+k)$ the "correspondent of —." Then, the double scalar of any quaternion relative, q, is that relative which denotes all self-q's, and, besides, all correspondents of self-q's of —." $(Tq)^2$ denotes all persons belonging to pairs of corresponding self-q's *minus* all persons belonging to pairs of corresponding q's of each other.

As a very simple example of the application of geometry to the logic of relatives, we may take the following. Euclid's axiom concerning parallels corresponds to the quaternion principle that the square of a vector is a scalar. From this it follows, since by (157) $yz + zk$ is a vector, that the rich husbands and American wives of the

rich husbands and American wives of any class of persons are wholly contained under that class, and can be described without any discrimination of sex. In point of fact, by (156), the rich husbands and American wives of the rich husbands and American wives of any class of persons, are the rich Americans of that class.

Lobatchewsky has shown that Euclid's axiom concerning parallels may be supposed to be false without invalidating the propositions of spherical trigonometry. In order, then, that corresponding propositions should hold good in logic, we need not resort to elementary relatives, but need only take S and V in such senses that every relative of the class considered should be capable of being regarded as a sum of a scalar and a vector, and that a scalar multiplied by a scalar should be a scalar, while the product of a scalar and a vector is a vector. Now, to fulfil these conditions we have only to take Sq as "self-q of," and Vq as "alio-q of" (q of another, that other being —), and q may be any relative whatever. For, "lover," for example, is divisible into self-lover and alio-lover; a self-lover of a self-benefactor of persons of any class is contained under that class, and neither the self-lover of an alio-benefactor of any persons nor the alio-lover of the self-benefactor of any persons are among those persons. Suppose, then, we take the formula of spherical trigonometry,

$$\cos \mathrm{a} = \cos \mathrm{b} \cos \mathrm{c} + \cos \mathrm{A} \sin \mathrm{b} \sin \mathrm{c} \ .$$

In quaternion form, this is,

$$\text{(165.)} \qquad S(pq) = (Sp)(Sq) + S((Vp)(Vq)) \, .$$

Let p be "lover," and q be "benefactor." Then this reads, lovers of their own benefactors consist of self-lovers of self-benefactors together with alio-lovers of alio-benefactors of themselves. So the formula

$$\sin \mathrm{b} \cos \mathrm{pb}' = - \sin \mathrm{a} \cos \mathrm{c} \cos \mathrm{pa}' - \sin \mathrm{c} \cos \mathrm{a} \cos \mathrm{pc}' + \sin \mathrm{a} \sin \mathrm{c} \sin \mathrm{b} \cos \mathrm{pb} \ ,$$

where A′, B′, C′, are the positive poles of the sides a, b, c, is in quaternions

$$\text{(166.)} \qquad V(pq) = (Vp)(Sq) + (Sp)(Vq) + V((Vp)(Vq)) \, ,$$

and the logical interpretation of this is: lovers of benefactors of others consist of alio-lovers of self-benefactors, together with self-lovers of alio-benefactors, together with alio-lovers of alio-benefactors of others. It is a little striking that just as in the non-Euclidean or imaginary geometry of Lobatchewsky the axiom concerning parallels holds good only with the ultimate elements of space, so its logical equivalent holds good only for elementary relatives.

It follows from what has been said that for every proposition in geometry there is a proposition in the pure logic of relatives. But the method of working with logical algebra which is founded on this principle seems to be of little use. On the other hand, the fact promises to throw some light upon the philosophy of space.*

Properties of Particular Relative Terms.

Classification of Simple Relatives.

Any particular property which any class of relative terms may have may be stated in the form of an equation, and affords us another premise for the solution of problems in which such terms occur. A good classification of relatives is, therefore, a great aid in the use of this notation, as the notation is also an aid in forming such a classification.

The first division of relatives is, of course, into simple relatives and conjugatives. The most fundamental divisions of simple relatives are based on the distinction

* The researches of Lobatchewsky furnish no solution of the question concerning the apriority of space. For though he has shown that it is conceivable that space should have such properties that two lines might be in a plane and inclined to one another without ever meeting, however far produced, yet he has not shown that the facts implied in that supposition are inconsistent with supposing space to retain its present nature and the properties only of the things in it to change. For example, in Lobatchewsky's geometry a star at an infinite distance has a finite parallax. But suppose space to have its present properties, and suppose that there were one point in the universe towards which anything being moved should expand, and away from which being moved should contract. Then this expansion and contraction might obey such a law that a star, the parallax of which was finite, should be at an infinite distance measured by the number of times a yard-stick must be laid down to measure off that distance. I have not seen Beltrami's investigations, but I understand that they do show that something of this sort is possible. Thus, it may be that, make what suppositions you will concerning phenomena, they can always be reconciled to our present geometry or be shown to involve implicit contradictions. If this is so, — and whether it is or not is a completely open question, — then the principles of geometry are necessary, and do not result from the specialities of any object cognized, but from the conditions of cognition in general. In speaking of the conditions of cognition, in general, I have in view no psychological conception, but only a distinction between principles which, if the facts should present a sufficient difficulty, I may always logically doubt, and principles which it can be shown cannot become open to doubt from any difficulty in my facts, as long as they continue to be supposed in all logical procedure.

But, waiving this point, Lobatchewsky's conclusions do not positively overthrow the hypothesis that space is *a priori*. For he has only shown that a certain proposition, *not usually believed to be axiomatical*, is conceivably false. That people may be doubtful or even mistaken about *a priori* truth does not destroy all important practical distinction between the two kinds of necessity. It may be said that if Lobatchewsky's geometry is the true one, then space involves an arbitrary constant, which value cannot be given *a priori*. This may be; but it may be that the general properties of space, with the general fact that there is such a constant, are *a priori*, while the value of the constant is only empirically determined.

It appears to me plain that no geometrical speculations will settle the philosophy of space, which is a logical question. If space is *a priori*, I believe that it is in some recondite way involved in the logic of relatives.

between elementary relatives of the form (A:A), and those of the form (A:B). These are divisions in regard to the amount of opposition between relative and correlative.

a. Simple relatives are in this way primarily divisible into relatives all of whose elements are of the form (A:A) and those which contain elements of the form (A:B). The former express a mere agreement among things, the latter set one thing over against another, and in that sense express an opposition (ἀντικεῖσθαι); I shall therefore term the former *concurrents*, and the latter *opponents*. The distinction appears in this notation as between relatives with a comma, such as (w,), and relatives without a comma, such as (*w*); and is evidently of the highest importance. The character which is signified by a concurrent relative is an absolute character, that signified by an opponent is a relative character, that is, one which cannot be prescinded from reference to a correlate.

b. The second division of simple relatives with reference to the amount of opposition between relative and correlative is into those whose elements may be arranged in collections of squares, each square like this,

A:A	A:B	A:C
B:A	B:B	B:C
C:A	C:B	C:C

and those whose elements cannot be so arranged. The former (examples of which are, "equal to —," "similar to —") may be called *copulatives*, the latter *non-copulatives*. A copulative multiplied into itself gives itself. Professor Peirce calls letters having this property, *idempotents*. The present distinction is of course very important in pure algebra. All concurrents are copulatives.

c. Third, relatives are divisible into those which for every element of the form (A:B) have another of the form (B:A), and those which want this symmetry. This is the old division into *equiparants* and *disquiparants*,* or in Professor De Morgan's language, convertible and inconvertible relatives. Equiparants are their own correlatives. All copulatives are equiparant.

* "Quædam sunt relationes equiparantiæ, quædam disquiparantiæ. Primæ sunt relationes similium nominum, secundæ relationes dissimilium nominum. Exemplum primi est quando idem nomen ponitur in recto et in obliquo, sicut simile simili est simile. . . . Exemplum secundi est quando unum nomen ponitur in recto sed aliud in obliquo, sicut pater est filii pater et non oportet quod sit patris pater." Ockham Quodlibetum 6, qu 20. See also his Summa Logices, pars 1, cap. 52. "Relativa equiparantiæ: quæ sunt synonyma cum suis correlativis. . . . Relativa diquiparantiæ: quæ non sunt synonyma cum suis correlativis." Pschlacher in Petr. Hisp. The same definitions substantially may be found in many late mediæval logics.

d. Fourth, simple relatives are divisible into those which contain elements of the form (A:A) and those which do not. The former express relations such as a thing may have to itself, the latter (as cousin of —, hater of —) relations which nothing can have to itself. The former may be termed *self-relatives*, the latter *alio-relatives.* All copulatives are self-relatives.

e. The fifth division is into relatives some power (i. e. repeated product) of which contains elements of the form (A:A), and those of which this is not true. The former I term *cyclic*, the latter *non-cyclic* relatives. As an example of the former, take

$$(A:B) +\!\!, (B:A) +\!\!, (C:D) +\!\!, (D:E) +\!\!, (E:C)\,.$$

The product of this into itself is

$$(A:A) +\!\!, (B:B) +\!\!, (C:E) +\!\!, (D:C) +\!\!, (E:D)\,.$$

The third power is

$$(A:B) +\!\!, (B:A) +\!\!, (C:C) +\!\!, (D:D) +\!\!, (E:E)\,.$$

The fourth power is

$$(A:A) +\!\!, (B:B) +\!\!, (C:D) +\!\!, (D:E) +\!\!, (E:C)\,.$$

The fifth power is

$$(A:B) +\!\!, (B:A) +\!\!, (C:E) +\!\!, (D:C) +\!\!, (E:D)\,.$$

The sixth power is

$$(A:A) +\!\!, (B:B) +\!\!, (C:C) +\!\!, (D:D) +\!\!, (E:E)\,.$$

where all the terms are of the form (A:A). Such relatives, as *cousin of* —, are cyclic. All equiparants are cyclic.

f. The sixth division is into relatives no power of which is zero, and relatives some power of which is zero. The former may be termed *inexhaustible*, the latter *exhaustible.* An example of the former is "spouse of —," of the latter, "husband of —." All cyclics are inexhaustible.

g. Seventh, simple relatives may be divided into those whose products into themselves are not zero, and those whose products into themselves are zero. The former may be termed *repeating*, the latter, *non-repeating* relatives. All inexhaustible relatives are repeating.

h. Repeating relatives may be divided (after De Morgan) into those whose products into themselves are contained under themselves, and those of which this is not true. The former are well named by De Morgan *transitive*, the latter *intransitive.* All transitives are inexhaustible; all copulatives are transitive; and all transitive equiparants are copulative. The class of transitive equiparants has a character, that of being self-relatives, not involved in the definitions of the terms.

i. Transitives are further divisible into those whose products by themselves are equal to themselves, and those whose products by themselves are less than themselves; the former may be termed *continuous*, the latter *discontinuous*. An example of the second is found in the pure mathematics of a continuum, where if a is greater than b it is greater than something greater than b; and as long as a and b are not of the same magnitude, an intervening magnitude always exists. All concurrents are continuous.

j. Intransitives may be divided into those the number of the powers (repeated products) of which not contained in the first is infinite, and those some power of which is contained in the first. The former may be called *infinites*, the latter, *finites*. Infinite inexhaustibles are cyclic.

In addition to these, the old divisions of relations into relations of reason and real relations, of the latter into aptitudinal and actual, and of the last into extrinsic and intrinsic, are often useful.*

"*Not.*"

We have already seen that "not," or "other than," is denoted by $\mathfrak{G}^{-1}$. It is often more convenient to write it, n. The fundamental property of this relative has been given above (111). It is that,

$$\mathfrak{G}^{-x} = 1 - x\,.$$

Two other properties are expressed by the principles of contradiction and excluded middle. They are,

$$x, \mathfrak{G}^{-x} = 0\,;$$

$$x \dotplus \mathfrak{G}^{-x} = 1\,.$$

The following deduced properties are of frequent application: —

(167.) $$\mathfrak{G}^{-(x,y)} = \mathfrak{G}^{-x} \dotplus \mathfrak{G}^{-y}\,;$$

(168.) $$\mathfrak{G}^{-xy} = \mathfrak{G}^{-x}y\,.$$

The former of these is the counterpart of the general formula, $z^{x \dotplus y} = z^x, z^y$. The

* "Duplex est relatio: scilicet rationis et realis. Unde relatio rationis est quæ fit per actum comparativum intellectus, ut sunt secundæ intentiones; sed relatio realis est duplex, scilicet aptitudinalis et actualis. Aptitudinalis est quæ non requirit terminum actu existere sed solum in aptitudine; cujusmodi sunt omnes propriæ passiones, omnes aptitudines, et omnes inclinationes; et tales sunt in illo prædicamento reductive in quo sunt illa quorum sunt propriæ passiones. Sed relatio actualis est duplex, scilicet, intrinsecus adveniens, et extrinsecus adveniens. Intrinsecus adveniens est quæ necessario ponitur positis extremis in quacunque etiam distantia ponantur, ut similitudo, paternitas, equalitas. Extrinsecus adveniens est quæ necessario non ponitur, positis extremis, sed requiritur debita approximatio extremorum; cujusmodi sunt sex ultima prædicamenta, scilicet, actio, passio, quando, ubi, situs, et habitus." Tartaretus.

latter enables us always to bring the exponent of the exponent of $\mathfrak{S}-$ down to the line, and make it a factor. By the former principle, objects not French violinists consist of objects not Frenchmen, together with objects not violinists; by the latter, individuals not servants of all women are the same as non-servants of some women.

Another singular property of $\mathfrak{S}-$ is that,

$$\text{(169.)} \qquad \text{If } [x] > 1 \qquad \mathfrak{S}^{-1}x = 1 .$$

"Case of the existence of —," and "case of the non-existence of —."

That which first led me to seek for the present extension of Boole's logical notation was the consideration that as he left his algebra, neither hypothetical propositions nor particular propositions could be properly expressed. It is true that Boole was able to express hypothetical propositions in a way which answered some purposes perfectly. He could, for example, express the proposition, "Either the sun will shine, or the enterprise will be postponed," by letting x denote "the truth of the proposition that the sun will shine," and y "the truth of the proposition that the enterprise will be postponed"; and writing,

$$\mathrm{x} \mathbin{+\!\!,} \mathrm{y} = 1 ,$$

or, with the invertible addition,

$$\mathrm{x} + (1 - \mathrm{x}),\mathrm{y} = 1.$$

But if he had given four letters denoting the four terms, "sun," "what is about to shine," "the enterprise," and "what is about to be postponed," he could make no use of these to express his disjunctive proposition, but would be obliged to assume others. The imperfection of the algebra here was obvious. As for particular propositions, Boole could not accurately express them at all. He did undertake to express them, and wrote

Some Y's are X's: $\qquad \mathrm{v},\mathrm{y} = \mathrm{v},\mathrm{x}$;

Some Y's are not X's: $\qquad \mathrm{v},\mathrm{y} = \mathrm{v},(1 - \mathrm{x})$.

The letter v is here used, says Boole, for an "indefinite class symbol." This betrays a radical misapprehension of the nature of a particular proposition. To say that some Y's are X's, is not the same as saying that a logical species of Y's are X's. For the logical species need not be the name of anything existing. It is only a certain description of things fully expressed by a mere definition, and it is a question of fact whether such a thing really exist or not. St. Anselm wished to infer existence from a definition, but that argument has long been exploded. If, then, v is a mere logical species in general, there is not necessarily any such thing, and the equation means

nothing. If it is to be a logical species, then, it is necessary to suppose in addition that it exists, and further that *some v is y*. In short, it is necessary to assume concerning it the truth of a proposition, which, being itself particular, presents the original difficulty in regard to its symbolical expression. Moreover, from

$$\mathrm{v},\mathrm{y} = \mathrm{v},(1-\mathrm{x})$$

we can, according to algebraic principles, deduce successively

$$\mathrm{v},\mathrm{y} = \mathrm{v} - \mathrm{v},x$$

$$\mathrm{v},x = \mathrm{v} - \mathrm{v},\mathrm{y} = \mathrm{v},(1-\mathrm{y}).$$

Now if the first equation means that some Y's are not X's, the last ought to mean that some X's are not Y's; for the algebraic forms are the same, and the question is, whether the algebraic forms are adequate to the expression of particulars. It would appear, therefore, that the inference from Some Y's are not X's to Some X's are not Y's, is good; but it is not so, in fact.

What is wanted, in order to express hypotheticals and particulars analytically, is a relative term which shall denote "case of the existence of —," or "what exists only if there is any —"; or else "case of the non-existence of —," or "what exists only if there is not —." When Boole's algebra is extended to relative terms, it is easy to see what these particular relatives must be. For suppose that having expressed the propositions "it thunders," and "it lightens," we wish to express the fact that "if it lightens, it thunders." Let

$$\mathrm{A} = 0 \quad \text{and} \quad \mathrm{B} = 0\ ,$$

be equations meaning respectively, it lightens and it thunders. Then, if φx vanishes when x does not and *vice versa*, whatever x may be, the formula

$$\varphi\mathrm{A} \prec \varphi\mathrm{B}$$

expresses that if it lightens it thunders; for if it lightens, A vanishes; hence φA does not vanish, hence φB does not vanish, hence B vanishes, hence it thunders. It makes no difference what the function φ is, provided only it fulfils the condition mentioned. Now, 0^x is such a function, vanishing when x does not, and not vanishing when x does. *Zero*, therefore, may be interpreted as denoting "that which exists if, and only if, there is not —." Then the equation

$$0^0 = 1$$

means, everything which exists, exists only if there is not anything which does not exist. So,

$$0x = 0$$

means that there is nothing which exists if, and only if, *some* x does not exist. The reason of this is that *some* x means some existing x.

It "lightens" and "it thunders" might have been expressed by equations in the forms

$$\mathrm{A} = 1\,, \qquad \mathrm{B} = 1\,.$$

In that case, in order to express that if it lightens it thunders, in the form

$$\varphi\mathrm{A} \prec \varphi\mathrm{B}\,,$$

it would only be necessary to find a function, φx, which should vanish unless x were 1, and should not vanish if x were 1. Such a function is ${}^1x$. We must therefore interpret 1 as "that which exists if, and only if, there is —," ${}^1x$ as "that which exists if and only if, there is nothing but x," and $1x$ as "that which exists if, and only if, there is some x." Then the equation

$$1^x = 1\,,$$

means everything exists if, and only if, whatever x there is exists.

Every hypothetical proposition may be put into four equivalent forms, as follows:—

If X, then Y.

If not Y, then not X.

Either not X or Y.

Not both X and not Y.

If the propositions X and Y are $\mathrm{A} = 1$ and $\mathrm{B} = 1$, these four forms are naturally expressed by

$$
\begin{aligned}
&{}^1\mathrm{A} \prec {}^1\mathrm{B}\,,\\
&1(1 - \mathrm{A}) \prec 1(1 - \mathrm{B})\,,\\
&1(1 - \mathrm{A}) \dotplus \mathrm{B} = 1\,,\\
&{}^1\mathrm{A}{,}1(1 - \mathrm{B}) = 0\,.
\end{aligned}
$$

For ${}^1x$ we may always substitute $0^{(1-x)}$.

Particular propositions are expressed by the consideration that they are contradictory of universal propositions. Thus, as $\mathrm{h},(1 - \mathrm{b}) = 0$ means every horse is black, so $0^{\mathrm{h},(1-\mathrm{b})} = 0$ means that some horse is not black; and as $\mathrm{h},\mathrm{b} = 0$ means that no

horse is black, so $0^{h,b} = 0$ means that some horse is black. We may also write the particular affirmative $1(h,b) = 1$, and the particular negative $1(h,n^b) = 1$.

Given the premises, every horse is black, and every horse is an animal; required the conclusion. We have given

$$h \prec b;$$

$$h \prec a.$$

Commutatively multiplying, we get

$$h \prec a,b.$$

Then, by (92) or by (90),

$$0^{a,b} \prec 0^h, \quad \text{or} \quad 1h \prec 1(a,b).$$

Hence, by (40) or by (46),

$$\text{If } h > 0 \quad 0^{a,b} = 0, \quad \text{or} \quad 1(a,b) = 1;$$

or if there are any horses, some animals are black. I think it would be difficult to reach this conclusion, by Boole's method unmodified.

Particular propositions may also be expressed by means of the signs of inequality. Thus, some animals are horses, may be written

$$a,h > 0;$$

and the conclusion required in the above problem might have been obtained in this form, very easily, from the product of the premises, by (1) and (21).

We shall presently see that conditional and disjunctive propositions may also be expressed in a different way.

Conjugative Terms.

The treatment of conjugative terms presents considerable difficulty, and would no doubt be greatly facilitated by algebraic devices. I have, however, studied this part of my notation but little.

A relative term cannot possibly be reduced to any combination of absolute terms, nor can a conjugative term be reduced to any combination of simple relatives; but a conjugative having more than two correlates can always be reduced to a combination of conjugatives of two correlates. Thus for "winner over of —, from —, to —," we may always substitute u, or "gainer of the advantage — to —," where the first correlate is itself to be another conjugative v, or "the advantage of winning over of — from —." Then we may write,

$$w = uv.$$

It is evident that in this way all conjugatives may be expressed as production of conjugatives of two correlates.

The interpretation of such combinations as $\ell^{a^{\mathrm{m}}}$, etc., is not very easy. When the conjugative and its first correlative can be taken together apart from the second correlative, as in $(\ell a)\mathrm{m}$ and $(\ell a)^{\mathrm{m}}$ and $(\ell^{a})\mathrm{m}$ and $(\ell^{a})^{\mathrm{m}}$, there is no perplexity, because in such cases (ℓa) or (ℓ^{a}) is a simple relative. We have, therefore, only to call the betrayer to an enemy an inimical betrayer, when we have

$(\ell a)\mathrm{m}$ = inimical betrayer of a man = betrayer of a man to an enemy of him,
$(\ell a)^{\mathrm{m}}$ = inimical betrayer of every man = betrayer of every man to an enemy of him.

And we have only to call the betrayer to every enemy an unbounded betrayer, in order to get

$(\ell^{a})\mathrm{m}$ = unbounded betrayer of a man = betrayer of a man to every enemy of him,
$(\ell^{a})^{\mathrm{m}}$ = unbounded betrayer of every man = betrayer of every man to every enemy of him.

The two terms ℓa^{m} and $\ell^{a^{\mathrm{m}}}$ are not quite so easily interpreted. Imagine a separated into infinitesimal relatives, $A_{\prime}, A_{\prime\prime}, A_{\prime\prime\prime}$, etc., each of which is relative to but one individual which is m. Then, because all powers of $A_{\prime}, A_{\prime\prime}, A_{\prime\prime\prime}$, etc., higher than the first, vanish, and because the number of such terms must be [m], we have,

$$a^{\mathrm{m}} = (\mathrm{A}_{\prime} \dotplus \mathrm{A}_{\prime\prime} \dotplus \mathrm{A}_{\prime\prime\prime} \dotplus \text{etc.})^{\mathrm{m}} = (A_{\prime}\mathrm{m}),(A_{\prime\prime}\mathrm{m}),(A_{\prime\prime\prime}\mathrm{m}), \text{etc.}$$

or if M′, M″, M‴, etc., are the individual m's,

$$a^{\mathrm{m}} = (A_{\prime}\mathrm{M}'),(A_{\prime\prime}\mathrm{M}''),(A_{\prime\prime\prime}\mathrm{M}'''), \text{etc.}$$

It is evident from this that ℓa^{m} is a betrayer to an $A_{\prime}$ of M′, to an $A_{\prime\prime}$ of M″, to an $A_{\prime\prime\prime}$ of M‴, etc., in short of all men to some enemy of them all. In order to interpret $\ell^{a^{\mathrm{m}}}$ we have only to take the negative of it. This, by (124), is $(1 - \ell)a^{\mathrm{m}}$, or a non-betrayer of all men to some enemy of them. Hence, $\ell^{a^{\mathrm{m}}}$, or that which is *not* this, is a betrayer of some man to each enemy of all men. To interpret $\ell^{(a\mathrm{m})}$, we may put it in the form $(1 - \ell)^{(1-a)^{\mathrm{m}}}$. This is "non-betrayer of a man to all non-enemies of all men." Now, a non-betrayer of some X to every Y, is the same as a betrayer of all X's to nothing but what is not Y; and the negative of "non-enemy of all men," is "enemy of a man." Thus, $\ell^{(a\mathrm{m})}$ is, "betrayer of all men to nothing but an enemy of a man." To interpret $\ell^{a}\mathrm{m}$ we may put it in the form $(1 - \ell)^{(1-a)}\mathrm{m}$, which is, "non-betrayer of a man to every non-enemy of him." This is a logical sum of terms, each of which is "non-betrayer of an individual man M to every non-enemy of M." Each of these terms is the same as "betrayer of M to nothing but an enemy of M."

The sum of them, therefore, which is $\mathfrak{t}a\,\mathrm{m}$ is "betrayer of some man to nothing but an enemy of him." In the same way it is obvious that $^{\mathfrak{t}a}\mathrm{m}$ is "betrayer of nothing but a man to nothing but an enemy of him." We have $\mathfrak{t}^{a}\mathrm{m} = \mathfrak{t}\,(1 - a)^{(1-\mathrm{m})}$, or "betrayer of all non-men to a non-enemy of all non-men." This is the same as "that which stands to something which is an enemy of nothing but a man in the relation of betrayer of nothing but men to what is not it." The interpretation of $\mathfrak{t}^{a}\mathrm{m}$ is obviously "betrayer of nothing but a man to an enemy of him." It is equally plain that $\mathfrak{t}\mathrm{a}^{\mathrm{m}}$ is "betrayer of no man to anything but an enemy of him," and that $\mathfrak{t}^{a}\mathrm{m}$ is "betrayer of nothing but a man to every enemy of him. By putting $\mathfrak{t}^{a\mathrm{m}}$ in the form $\mathfrak{t}^{(1-a)(1-\mathrm{m})}$ we find that it denotes "betrayer of something besides a man to all things which are enemies of nothing but men." When an absolute term is put in place of a, the interpretations are obtained in the same way, with greater facility.

The sign of an operation is plainly a conjugative term. Thus, our commutative multiplication might be denoted by the conjugative

$$\mathit{1},.$$

For we have,

$$l,s\,\mathrm{w} = \mathit{1},l,s\,\mathrm{w}.$$

As conjugatives can all be reduced to conjugatives of two correlates, they might be expressed by an operative sign (for which a Hebrew letter might be used) put between the symbols for the two correlates. There would often be an advantage in doing this, owing to the intricacy of the usual notation for conjugatives. If these operational signs happened to agree in their properties with any of the signs of algebra, modifications of the algebraic signs might be used in place of Hebrew letters. For instance, if $\mathfrak{r}$ were such that

$$\mathfrak{r}x\mathfrak{r}yz = \mathfrak{r}_{13}\mathfrak{r}yz,$$

then, if we were to substitute for $\mathfrak{r}$ the operational sign $\daleth$ we have

$$x\daleth(y\daleth z) = (x\daleth y)\daleth z,$$

which is the expression of the associative principle. So, if

$$\mathfrak{r}xy = \mathfrak{r}yx$$

we may write,

$$x\daleth y = y\daleth x$$

which is the commutative principle. If both these equations held for any conjugative, we might conveniently express it by a modified sign $+$. For example, let us consider the conjugative "what is denoted by a term which either denotes — or else —."

For this, the above principles obviously hold, and we may naturally denote it by $\dotplus$. Then, if p denotes Protestantism, r Romanism, and f what is false,

$$\mathrm{p} \dotplus \mathrm{r} \prec \mathrm{f}$$

means either all Protestantism or all Romanism is false. In this way it is plain that all hypothetical propositions may be expressed. Moreover, if we suppose any term as "man" (m) to be separated into its individuals, M′, M″, M‴, etc., then,

$$\mathrm{M}' \dotplus \mathrm{M}'' \dotplus \mathrm{M}''' \dotplus \text{etc.},$$

means "some man." This may very naturally be written

$$\text{‘m’}$$

and this gives us an improved way of writing a particular proposition; for

$$\text{‘x’} \prec \mathrm{y}$$

seems a simpler way of writing "Some X is Y" than

$$0^{\mathrm{x},y} = 0\,.$$

Converse.

If we separate *lover* into its elementary relatives, take the reciprocal of each of these, that is, change it from

$$\mathrm{A}:\mathrm{B} \qquad \text{to} \qquad \mathrm{B}:\mathrm{A}\,,$$

and sum these reciprocals, we obtain the relative *loved by*. There is no such operation as this in ordinary arithmetic, but if we suppose a science of discrete quantity in quaternion form (a science of equal intervals in space), the sum of the reciprocals of the units of such a quaternion will be the conjugate-quaternion. For this reason, I express the conjugative term "what is related in the way that to — is —, to the latter" by $\mathcal{K}$. The fundamental equations upon which the properties of this term depend are

(169.) $$\mathcal{K}\mathcal{K} = 1\,.$$

(170.) $$\text{If } x < y^z \quad \text{then} \quad z \prec (\mathcal{K}y)^x\,,$$

or $$1(x,yz) = 1(z,\mathcal{K}yx\,.$$

We have, also,

(171.) $$\mathcal{K}\Sigma = \Sigma\,\mathcal{K},$$

(172.) $$\mathcal{K}\Pi = \amalg\,\mathcal{K}\,,$$

where $\amalg$ denote the product in the reverse order. Other equations will be found in Mr. De Morgan's table, given above.

Conclusion.

If the question is asked, What are the axiomatic principles of this branch of logic, not deducible from others? I reply that whatever rank is assigned to the laws of contradiction and excluded middle belongs equally to the interpretations of all the general equations given under the head of "Application of the Algebraic Signs to Logic," together with those relating to backward involution, and the principles expressed by equations (95), (96), (122), (142), (156), (25), (26), (14), (15), (169), (170).

But these axioms are mere substitutes for definitions of the universal logical relations, and so far as these can be defined, all axioms may be dispensed with. The fundamental principles of formal logic are not properly axioms, but definitions and divisions; and the only *facts* which it contains relate to the identity of the conceptions resulting from those processes with certain familiar ones.

XIII.

The Uses and Origin of the Arrangements of Leaves in Plants.

BY CHAUNCEY WRIGHT.

Communicated October 10, 1871.

In proposing to treat in this paper of the *origin* of some of the more common arrangements of leaves and leaf-like organs in the higher orders of plants, I do not intend to make this question the principal object of discussion, but propose only to consider it so far as it affords useful hypotheses for the interpretation of some of the obscurer features in the main object of this inquiry; namely, questions of the *uses* of these arrangements, or of their adaptations to the outward economy of the plant's life, and to the conditions of its existence. If by such a discussion hypothesis can be made to throw light on physiological questions, while seeking more directly to connect in a continuous series the simpler and more general with the more specific and complicated forms in vegetable life, it will gain for itself a much greater interest and value than it would otherwise possess. It is, indeed, in this value of the principle of Natural Selection, its value and use as a working hypothesis, that its principal claim to respect consists. If any subsidiary hypothesis under the theory serve only as a principle of connection, a thread on which we may arrange and more clearly regard relationships that are the objects of a more promising scientific inquiry, it will at least serve a useful purpose, and even, perhaps, give greater plausibility to the theory in general of the origin of organic forms through the agency of their utilities, or through the advantages these have given to surviving forms of life.

There is hardly any animal or plant, especially of the higher orders, that has not in many of the characteristics of its structure very conspicuous adaptations to the outward conditions of its life, — to "the part it has to play in the world," or at least to the many values or advantages it has to secure. This fact has led many naturalists, whose opinion, until lately, and for a long time, has prevailed, to regard a living structure as principally, if not entirely, made up of subordinate parts or organs which exist for specific purposes, or are essentially

concerned with special services to the general life of the organism, or even to life external to it, the general life of the world, or ultimately even to the highest and best life of the world. This doctrine deprived of its grander features, as the doctrine of Final Causes in natural history, and limited simply to the conception of the parts and characters of organic structures as all, or nearly all, related essentially to the preservation and continuance of the life itself which they embody, or to the principle of self-conservation, is the ground of the importance claimed for the principle of Natural Selection in the generation of organic species. But another school of naturalists, whose influence has been steadily gaining ground, has always strenuously opposed this view, and questioned the validity of the induction on which it rests. Though it is true that the higher animals and plants exhibit a great many special adaptations to the conditions of their existence, yet, it is objected, in a far greater number of characteristics they, in common with the rest of the organic world, exhibit no such adaptations. In those most important features of organic structures, which are now called genetic characters, and were formerly called affinities, few or no specific uses can in general be discovered; and it is considered unphilosophical to base an induction on the comparatively few cases of this class of characters which have obvious utilities. It is thought unphilosophical to presume on such meagre grounds that all these characters are either now, or have been, of service to the life of the organism; thus confounding these genetic characters with those that are properly called adaptive. By positing this distinction of genetic and adaptive characters as a fundamental and absolute one, the theory of organic types opposes itself to the conception of utility as a property of organic structures in general, and conceives, on the other hand, that an organism consists essentially of certain constituent parts and characters which are of no service to its general life, and are ends, so far as we can know, in themselves; though other and subordinate ones may stand incidentally in this menial relation.

This contrast being a merely speculative difference of opinion, a reference to it, in a scientific inquiry, would be out of place were it not that scientific inquiries are almost never free from such biases. These almost always exert an unperceived influence, unless specially guarded against; and in calling attention here to this question in biological philosophy, it is only for the purpose of characterizing it as a strictly open question. As is so often the case in such debates, both sides are right and both wrong; right, so far as each refuses credence to the other's main and exclusive position, and wrong, so far as each

claims it for its own. In other words, they are not properly inductive theories, awaiting and subject to verification, but arrogant dogmas, demanding unconditional assent. The bearing of this debate on the proper questions of science relates only to *method*, or to what are the directions in which scientific pursuit and hypothesis are legitimate. It is oftener by diverting or misdirecting scientific pursuit than in any other way that such speculative opinions are of serious importance; and in this way they are purely mischievous. The theory of types is undoubtedly right in refusing assent to the doctrine, as an established induction, that every part, arrangement, or function of an organism is of some special, though it may be unrecognized, service to its life; but it is wrong in assuming, on the other hand, that all attempts at discovering uses which are not present or obvious must be futile; or, in assuming that there are characteristic features in all organisms, which are not only at present of no use, but never could have been grounds of advantage. Again, the theory of the essential reference of every feature of an organism to the conditions of its existence is undoubted right in refusing assent to this assumption of essentially useless forms, and in affirming the legitimacy of inquiries concerning the utility of any feature whatever to the life of an organism, however far removed in appearance from any relations to its present conditions of existence. It is wrong, on the other hand, in confounding the legitimacy of this pursuit with the dogma in which, as a theory, it essentially consists, or in assuming as an established induction what is only a legitimate question or line of inquiry. It is obvious, however, that a proper scientific judgment of these theories cannot be absolutely impartial, since one of them is opposed to scientific pursuit, and the other invites it. The theory of types, assuming that utility is only a superficial or incidental character, and not a property of organic forms and functions generally, occupies a negative and forbidding attitude towards what are really legitimate questions of science; and, from this point of view, judgment must be made in favor of the rival dogma. We ought to be on our guard, moreover, against this theory, since there is a strong natural, but erroneous and mischievous, tendency in the mind to fall back upon it from the difficulties of a baffled pursuit; and to regard as really ultimate those facts of which the causes and dependences elude our researches. This resort can never be justified so long as there remain any suggestions of explanation not altogether frivolous, or incapable of some degree of verification. We may safely maintain that this tendency to rest from the difficulties of scientific pursuit is the chief

cause of the prevalence at the present time of the doctrine, which, when first propounded, was regarded as heterodox and dangerous, especially as it then seemed opposed to the doctrine of Final Causes. This apparent opposition has since, however, been made to disappear by a modification of the latter doctrine, which has incorporated in it this theory of types, by representing a type of structure as an ultimate feature in the general plan of creation, or as an end for which the successive manifestations and the adaptations of life exist, or to which they tend. According to this doctrine, it is not for the sake of the maintenance and continuance of the mere life, such as it is, or such as it can be, under the conditions of its existence, that adaptations exist in organisms; but it is for the sake of realizing in it certain predetermined special types of structure, which are ends in themselves, and to which the adaptive characters of the structure are subservient. Thus an elaborate and formidable philosophical theory has grown up, which stands in direct and forbidding opposition to such inquiries as the one proposed in this discussion. If the theory were true, it would, indeed, be idle to ask what are the uses, and how could these have determined the origin of those special leaf arrangements in the higher plants, which have been observed by botanists, and discussed by mathematicians in the theory of Phyllotaxy. There is a sufficiently obvious utility in the general character of these arrangements with reference to the general external economy of vegetable life and the functions of leaf-like bodies; but this does not at first sight appear to regard the particular details, or the special laws of arrangement, with which the theory of Phyllotaxy is concerned. In these we have apparently reached ultimate features of structure, the origin or value of which in the plant's life it would, on the theory of types, be idle to seek. These are such excellent examples of what the theory of types supposes to be finalities in biological science, that botanists and mathematicians, with hardly an exception, have consented to regard them in this light. There is a difference of opinion, it is true, as to whether the several angular intervals between successive leaves around the stem, or the several angles of divergence between successive leaves in the spiral arrangements, ought to be regarded as modifications of a single typical angle to which they approximate in value, or as several distinct types. There is no difference of opinion, however, in regard to another distinction of types in leaf arrangements, which, to all appearance, are separated by entirely distinct characters; namely, the so-called spiral arrangements and those of the verticil or whorl. It is with the former chiefly that the mathematical theory

of Phyllotaxy is concerned. The latter, or the verticil arrangements, though presenting a great variety of forms, are so obviously all of the same general and simple type, that they present no difficulties or problems for the exercise of mathematical skill. Their varieties consist simply in the number of leaves in the whorl. From two leaves placed oppositely, these whorls vary through all numbers to very large ones, and in all these varieties the simple law holds that the leaves of successive whorls, being of the same number and placed in each whorl at equal distances around the stem, like the spokes of a wheel, are so disposed that the leaves of the upper whorl stand directly over the angular spaces between those of the lower one. These features of arrangement are so obviously the same adaptations as those we shall find in the more complicated spiral arrangements, that I will consider them both together. They appear to be two solutions of the same problem in the economy of the higher vegetable life; though it is probable that the whorl arrangement is the inferior one. It approaches in simplicity most nearly to the alternate system among the spiral forms, though it is perfectly distinct from this. An opposition of leaves in the whorl is an accident or trivial circumstance dependent on the fact that the number of leaves in the whorl is in many cases an even one; while in the alternate arrangement this opposition is an essential character. This would not be strictly the case, indeed, if the theory were true that the alternate as well as the other spiral arrangements are only modifications of a single typical one. But an examination of the evidence will show very slight grounds for this opinion. No doubt, in the doctrine of development, all these arrangements must be considered as modifications of some single ancient form, though this, it is quite likely, was very different from the typical arrangement, or the perfect form, in the theory of Phyllotaxy. The important point, however, to be considered here, is, that on the theory of development there is properly no genetic connection between the opposition of leaves in whorls and those of the alternate arrangement. And, indeed, in the three-leaved systems of the two types the contrast is very marked; for the three leaves of such a whorl stand over the angular spaces between the three of the whorl below it, as in other arrangements of this type; while the three leaves of the spiral system or cycle stand severally directly over the three below them. The genetic relationships of the two great types will be specially considered when we come to the problem of the origin of both from simpler vegetable forms.

The names "system" and "cycle" are not so properly applicable to groups

of leaves in the spiral arrangements as to those of whorls, and refer rather to abstract numbers, counted from any point we please, than to actually definite groups. The actual system, cycle, or group in these arrangements is of indefinite extent, or comprises the whole stem, so far as it is developed, and even extends into the undeveloped leaves of the terminal bud. In speaking of a cycle of leaves in these arrangements no definitely situated group is meant, but only a definite number counted from any one we may choose for an origin. In almost all arrangements of this type we find that, after thus counting some definite number of leaves from some one assumed as the first, we arrive next at a leaf which stands directly over the first. Such a group, so determined, makes what is called a cycle; or, as we may sometimes prefer to call it, a system. Within it leaves succeed each other at successively greater and greater heights, and are so placed around the stem that the same angular interval or angle of divergence is contained between any two successive ones. This angle of divergence is commensurate with the circumference, but is not always an aliquot part of it, as in the angular interval of the leaves of whorls. It is in many plants some multiple of an aliquot part, and in counting the leaves successively through the cycle, we have to turn several times around the stem. This number of revolutions, divided by the number of leaves in the cycle, is the ratio of the angle of divergence to the whole circumference; and the fraction expressing this ratio is used to denote the particular arrangement of such a system. Thus the fraction $\frac{1}{2}$ denotes the alternate arrangement, in which there are two leaves in one turn, the third leaf falling over the first. $\frac{1}{3}$ is the name of the three-leaved system, in which there are three leaves in one turn, the fourth falling over the first. $\frac{2}{5}$ is the name of the system in which five leaves occur in two turns, and the sixth falls over the first. In order that such definite numerical systems, or cycles, should exist in the leaves of any plant, it is only necessary that the ratio of the angle of divergence to the circumference should be some proper fraction, and this fraction would be in the same way the name of the system. But any proper fraction whatever would have the property I have pointed out; namely, that after the number of leaves denoted by its denominator, and the number of turns denoted by its numerator, the next succeeding leaf would fall over the first. Whatever may be the purpose or advantage of the spiral arrangement, and of this feature in it, it is obvious that some other purpose is sought, or some other advantage gained, by the actual arrangements of this sort in nature; or else it would appear on the

theory of types, that the typical properties of them are not fully determined by what we have yet observed respecting them. For, although there is a great variety of such arrangements, these do not include all the possible ones, nor even all the simplest. There must still be another principle of choice besides what determines the rational fraction and the spiral arrangement. What this is, is the problem of the mathematical theory of Phyllotaxy. The result of this investigation was a classification of all the fractions that occur in natural arrangements under the general form of the continued fraction

$$\cfrac{1}{a+\cfrac{1}{1+\cfrac{1}{1+\&c.,}}}$$

in which a may have the values 1, 2, 3, or 4. The successive approximations of these four continued fractions give four series of proper fractions, which include all the arrangements that occur in nature. These series are for

$a=1$ . . . . $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{8}{13}$, &c.

$a=2$ . . . . $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{5}{13}$, &c.

$a=3$ . . . . $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{7}$, $\frac{3}{11}$, $\frac{5}{18}$, &c.

$a=4$ . . . . $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{9}$, $\frac{3}{14}$, $\frac{5}{23}$, &c.

The first series is not usually given, since they are the complements of the fractions of the second series, and express the same arrangements, but in an opposite direction around the circumference; or by supposing that the spiral line connecting the leaves is drawn from leaf to leaf the longer way round. Omitting then the first series, we shall still have in the others, as they stand, developed to five terms, many more fractions than have actually been observed, or could be observed in actual plants.

I propose in what follows to subject the mathematical induction expressed by these series to careful critical examination, to distinguish what is matter of actual observation from what is deduced from theory, and to ascertain with precision the amount of inductive evidence on which the theory of the typical angle rests. Pursuing the subject afterwards by a strictly inductive investigation, I shall estimate what there is of truth in the theory. This will lead, I think, to the rejection of the theory as it stands, or under the form of the typical angle, but will not render the observation on which it depends wholly nugatory. On the contrary, it will show that this observation really leads to the true explanation of the occurrence of only certain fractions in the spiral

arrangements, and the more frequent occurrence of some of them than of others. It is a well-known property of the fractions of these series, that after the first two in each, the others can be deduced from the preceding ones, and continued indefinitely, by a very simple process. The numerator of each after the first two is equal to the sum of the numerators of the two preceding, and its denominator to the sum of their denominators. This law, as a matter of observation, was actually discovered only in the first four fractions of the first or second series, which are by far the commonest of actually observed arrangements in nature. Other less frequently occurring fractions were arranged on the same principle, and extended so as to give the last two series. The four series, or the three lower ones, contain, therefore, more than all the fractions that are known to belong to natural arrangements. This will be sufficiently evident when we observe that the fractions $\frac{5}{8}$ and $\frac{8}{13}$ in the first series, or their complements, $\frac{3}{8}$ and $\frac{5}{13}$, in the second series, would be indistinguishable in actual measurement; since they differ from each other by $\frac{1}{104}$, or by less than a hundredth, which is much less than can be observed, or than stems are often twisted by irregular growth. For the same reason we must reject all but the first three terms of the third and fourth series as being distinguishable only in theory. We are thus left with a very slight basis of facts on which to erect the superstructure of theory. We shall see further on a still more cogent reason for calling in question the validity of this induction; namely, that limiting the evidence as we are thus obliged to do, we have still left so large a number of actually observed arrangements, that they include almost all that are possible among equally simple and distinguishable fractions within the observed limits of natural arrangements; all, in fact, but two; namely, the fractions $\frac{1}{6}$ and $\frac{2}{7}$. The range is not a narrow one, but extends from $\frac{1}{5}$ to $\frac{4}{5}$, or from $\frac{1}{5}$ to $\frac{1}{2}$, since the fractions above $\frac{1}{2}$ are complements of those below, and express the same arrangements, but in an opposite direction around the circumference. The problem of Phyllotaxy, therefore, seems at first sight to be reduced to this; not why the other fractions do occur in nature, but why these two do not? But to answer the latter question is really also to answer the former, though it will go but very little way towards justifying the theory of the typical or unique angle. It will go much further if we exclude from this list of fractions those which are of very infrequent occurrence, namely, those peculiar to the third and fourth series; or, in other words, take account of the relative frequency in nature of the several arrangements. This, indeed, entirely changes the aspects of the question, for we

find that, instead of two, there are six fractions of the simpler denominations (or within the limits of distinguishable values), which either do not occur in nature at all, or occur very rarely; while those that are common are four in number, or less than half of all. But we shall find that those of the six which occur rarely differ from the two really unique ones among them, and agree with the common ones in respect to the law on which the answer to our question really depends. This answer will be found to depend on the law which was observed in the first four fractions of the first or second series, and was extended in the continuation of these and the formation of the others. This law, or the dependence of these fractions on each other, was seen to be a simple case of the relations of dependence in the successive approximations of continued fractions, and thus lead to the induction of these fractions; namely, the continued fraction

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\&c.}}}$$

for the first series, or

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\&c.}}}$$

for the second. The ultimate values of these continued fractions extended infinitely are complements of each other, as their successive approximations are, and are in effect the same fraction; namely, the irrational or incommensurate interval which is supposed to be the perfect form of the spiral arrangement. This does, in fact, possess in a higher degree than any rational fraction the property common to those which have been observed in nature; though practically, or so far as observation can go, this higher degree is a mere refinement of theory. For, as we shall find, the typical irrational interval differs from that of the fraction $\frac{3}{8}$ (and its complement differs from $\frac{5}{8}$) by almost exactly $\frac{7}{1000}$, a quantity much less than can be observed in the actual angles of leaf-arrangements. The conception of such a typical angle as an actual value in nature, and as a point of departure for more specialized ones, existing either among the normal patterns, or formative principles of vegetable life, as the theory of types supposes, or in some unknown law of development or physiological necessity,—such a conception is a very attractive one. And as exhibiting in the abstract and in its most perfect form a property peculiar, as we shall see, to natural arrangements, but belonging to them in inferior and in various degrees,—as exhibiting this separated from the property which such arrangements

also have, by which they are divisible into limited systems or cycles, — from this point of view the conception acquires a valid scientific utility. But we should be on our guard against a misconstruction of it. There is no evidence whatever, and there *could* be none from observation, that any such separation of properties actually occurs in nature, or that one is superposed on the other in successive stages of development in the bud, or that this typical arrangement is first produced and subsequently modified into the more special ones, — into the limited systems or cycles represented by simple rational fractions. To suppose this is to confound abstractions with concrete existences, or would be an instance of the so-called "realism" in science, against which it is always so necessary to be on our guard. There is no reason to suppose that one rather than the other of these properties appears first in the incipient parts of the bud, or that either exists in any degree of perfection before the development of these parts has made considerable advance.

I now propose to show what this property is, which the typical or unique angle has in the abstract and in perfection, and to show what its utility is in the economy of vegetable life. And to avoid all theoretical biases I propose, as I have said, to make the inquiry a strictly inductive investigation. Taking the first of the series of fractions given above and the complements of the third and fourth, we have,

$$\begin{array}{cccccc}
\frac{1}{2}, & \frac{2}{3}, & \frac{3}{5}, & \frac{5}{8}, & \frac{8}{13}, & \&c. \\
\frac{2}{3}, & \frac{3}{4}, & \frac{5}{7}, & \frac{8}{11}, & \frac{13}{18}, & \&c. \\
\frac{3}{4}, & \frac{4}{5}, & \frac{7}{9}, & \frac{11}{14}, & \frac{16}{23}, & \&c.
\end{array}$$

These contain all, and more than all, the distinguishable arrangements of the spiral type; but they are the intervals reckoned the longer way round. I have adopted this mode of expressing these arrangements, partly for the purpose of varying the investigation, and partly because it is better adapted to the graphical representation of these, as well as other possible arrangements, which are given in the accompanying diagram. It will be seen that the same law holds in the series here given as in those given above; yet these cannot be represented by the same general formula; but the formula becomes,

$$\cfrac{1}{1+\cfrac{1}{a+\cfrac{1}{1+\cfrac{1}{1+\&c.,}}}}$$

in which a is 1, 2, or 3. For the first of these series, or for the intervals most

frequent in nature, $a = 1$, and if we denote by k the ultimate value to which the fractions of this series more and more approximate, or what is supposed to be the type form of them, then, since $k = \cfrac{1}{1 + \cfrac{1}{1 + id.\ ad\ inf.}}$, we have $k = \frac{1}{1+k}$. Hence $k + k^2 = 1$, or $k^2 = 1 - k$. In the form of a proportion this is, $1 : k = k : 1 - k$; or k is the ratio of the extreme and mean proportion. Its value found by solving this equation is $k = \frac{1}{2}(\sqrt{5} - 1) = 0.6180$, approximately. From the above equation we obtain by multiplying by k successively the following: $k^2 = 1 - k$; $k^3 = k - k^2$; $k^4 = k^2 - k^3$, and in general $k^n = k^{n-2} - k^{n-1}$; that is, any power of this quantity is equal to the difference between the two next lower powers. Its square is equal to its complement; the cube to the difference between it and its square or complement, and so on. Or $k = 0.618$; $k^2 = 0.382$; $k^3 = 0.236$; $k^4 = 0.146$; $k^5 = 0.090$, etc. On this peculiar arithmetical property of k depends the geometrical one of the spiral arrangement, which it represents; namely, that such an arrangement would effect the most thorough and rapid distribution of the leaves around the stem, each new or higher leaf falling over the angular space between the two older ones which are nearest in direction so as to subdivide it in the same ratio, k, in which the first two, or any two successive ones, divide the circumference. But according to such an arrangement there could be no limited systems or cycles, or no leaf would ever fall exactly over any other; and, as I have said, we have no evidence, and could have none, that this arrangement actually exists in nature. To realize simply and purely the property of the most thorough distribution, the most complete exposure of the leaves to light and air around the stem, and the most ample elbow-room or space for expansion in the bud, is to realize a property that exists separately only in abstraction, like a line without breadth. Nevertheless practically, and so far as observation can go, we find that the last two fractions, $\frac{5}{8}$ and $\frac{8}{13}$, and all further ones of the first series, like $\frac{13}{21}$, etc., which are all indistinguishable as measured values in the plant, do actually realize this property with all needful accuracy. Thus $\frac{5}{8} = 0.625$; $\frac{8}{13} = 0.615$; and $\frac{13}{21} = 0.619$; and differ from k by 0.007, 0.003, and 0.001, respectively; or they all differ by inappreciable values from the quantity which might therefore be made to stand for all of them. But in putting k for all the values of the first series after the first three, it should be with the understanding that it is not so employed in its capacity as the grand type, or the source of the distributive character which they have;

in its capacity as an irrational fraction, — but simply as being indistinguishable practically from these rational ones, and as being entirely consistent practically with the property that rational proper fractions also have of forming limited systems or cycles. Much mystification has come from the irrational character of this fraction; scepticism on the part of non-mathematical botanists, and mysticism on the part of mathematicians. The simpler or the first three fractions of this series have also in a less degree the same distributive quality, and so in a still less degree have the fractions of the two lower series. But all the fractions left among possible ones, within the limits considered, that are sufficiently simple to be readily identified, are the fractions $\frac{4}{7}$ and $\frac{5}{9}$, or their complements $\frac{3}{7}$ and $\frac{4}{9}$; and these exceptions, as I have said, are all the grounds of fact which at first sight give any plausibility to the theory of Phyllotaxy, or make its laws anything other apparently than the necessary consequences of purely numerical properties in the simpler fractions. Yet beside the fact that these two have not the distributive character of the others, the fact should be taken account of, that by confining ourselves to the limits $\frac{1}{2}$ to $\frac{4}{5}$ we have neglected several other simple fractions, that are even worse adapted for the purpose which the great majority appear to serve. These fractions are $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, and $\frac{8}{9}$, or their complements. Moreover, we should consider that as the fractions peculiar to the two lower series are much less fitted for this purpose than those of the first series, so they are much less frequently found in nature. Taking account of all these facts, we find the hypothesis that nature has chosen certain intervals in the spiral arrangements of leaves, and for the purpose I have indicated, to be sufficiently probable to justify a more careful consideration of it. Wide divergences from the most perfect realization of this purpose, such as we have among the more frequent forms in the fractions $\frac{1}{2}$ and $\frac{2}{3}$, or in the alternate and three-leaved systems, and also among the less frequent forms, indicate the existence of other conditions or purposes in these arrangements, which I propose to consider further on. I may remark here, however, that these two classes of exceptions from the most perfect realization of the distributive property, namely, those of the first series which belong to the most advanced forms of life, and those peculiar to the two other series, are probably due to widely different causes; the one having, in fact, a high degree of specialization, and the other falling short in respect to this distributive property on account of a low degree of specialization. This view, which is one of the consequences of theoretical considerations on the *origin* of these arrangements, that will be presented when we come to consider the

origin of spiral arrangements in general, and of the whorl, is significantly in accordance with the observation that the forms peculiar to the two lower series are more frequent among fossil plants than among surviving ones.

But waiving these theoretical considerations for the present, I will now examine, quite independently of theory, the properties in the spiral arrangements of all the fractions between $\frac{1}{5}$ and $\frac{4}{5}$, or rather between $\frac{1}{2}$ and $\frac{4}{5}$, and of a less denomination than 14ths. I adopt these limits because the character of all fractions greater than $\frac{4}{5}$, or less than $\frac{1}{5}$, will be sufficiently shown by this limit, and because fractions less simple than 13ths cannot be distinguished in nature from simpler ones. The fractions between $\frac{1}{2}$ and $\frac{1}{5}$, being complements of those greater than $\frac{1}{2}$, need not, of course, be separately studied; since they express the same arrangements, only counted in the opposite direction around the stem. I have chosen to represent these possible arrangements by the larger fractions rather than by the smaller (their complements), for reasons I have given, although theoretical considerations on the origin of spiral arrangements in general suggest the latter and more usual mode as the proper one. This may be given as an additional reason for the choice, since we shall not thus be led to confound a conventional mode of representation with a law of nature, or have any undue bias in consequence, but shall be able to judge the hypothesis on its own merits. The best reason, however, for the choice is, that by representing the cycle by the larger number of turns, or by counting the longer way round, we are able to spread out into greater detail in the accompanying diagram the steps of the distribution, and see more clearly its character.

In the following table the first column contains all the fractions I have defined, arranged in the order of their denominations, with their decimal equivalents to thousandths, and at the end the irrational quantity k, with its approximate decimal value. The second column contains the same fractions, arranged in the order of their magnitudes, with their decimal values and the differences between successive ones. The third contains the complements of these decimal values. They represent the smaller of the two parts or angles of divergence into which two successive leaves would divide the circumference, or represent these angles reckoned the shorter way round. The fourth contains the differences between the decimals of the second and third, together with the ratios of these to those of the third. (These differences are occasionally corrected by a unit, to allow for the inexactness or the approximate character of some of these decimals.) They are the subintervals or angles of divergence introduced by the third leaf of a cycle between it and the first:

$\frac{1}{2}$	.500	$\frac{1}{2}$	.500	38	.500	.000	∞
$\frac{2}{3}$	.667	$\frac{7}{13}$	.538	7	.462	.076	6
$\frac{3}{4}$	.750	$\frac{6}{11}$	.545	11	.455	.091	5
$\frac{3}{5}$	.600	$\frac{5}{9}$	.556	15	.444	.111	4
$\frac{4}{5}$	.800	$\frac{4}{7}$	.571	12	.429	.143	3
$\frac{4}{7}$	.571	$\frac{7}{12}$	.583	17	.417	.167	$2\frac{1}{2}$
$\frac{5}{7}$	.714	$\frac{3}{5}$	.600	15	.400	.200	2
$\frac{5}{8}$	.625	$\frac{8}{13}$	.615	3	.385	.231	$1\frac{2}{3}$
$\frac{5}{9}$	.556	k	.618	7	.382	.236	$\frac{1}{k}$
$\frac{7}{9}$	.778	$\frac{5}{8}$	.625	11	.375	.250	$1\frac{1}{2}$
$\frac{7}{10}$	.700	$\frac{7}{11}$	.636	31	.364	.272	$1\frac{1}{3}$
$\frac{6}{11}$	.545	$\frac{2}{3}$	.667	25	.333	.333	1
$\frac{7}{11}$	.636	$\frac{9}{13}$	.692	8	.308	.384	$\frac{4}{5}$
$\frac{8}{11}$	.727	$\frac{7}{10}$	.700	14	.300	.400	$\frac{3}{4}$
$\frac{7}{12}$	.583	$\frac{5}{7}$	.714	13	.286	.428	$\frac{2}{3}$
$\frac{7}{13}$	.538	$\frac{8}{11}$	.727	23	.273	.454	$\frac{3}{5}$
$\frac{8}{13}$	.615	$\frac{3}{4}$	.750	19	.250	.500	$\frac{1}{2}$
$\frac{9}{13}$	.692	$\frac{10}{13}$	.769	9	.231	.538	$\frac{3}{7}$
$\frac{10}{13}$	.769	$\frac{7}{9}$	.778	22	.222	.556	$\frac{2}{5}$
k	.618	$\frac{4}{5}$	.800		.200	.600	$\frac{1}{3}$

The first point to be noticed in this table is the character of the ratios in the last column. For all the fractions here given of less magnitude than $\frac{3}{5}$, (or whose complements are greater than $\frac{2}{5}$,) the first subinterval is contained in the preceding, or in the smaller of the primary intervals, several times, or more than twice. To the interval $\frac{1}{2}$ or in the alternate arrangement there is no secondary interval. The cycle is completed at once, and no distribution is effected, except the simple opposition of successive leaves. The next following fractions $\frac{7}{13}$ and $\frac{6}{11}$ have a similar character in respect to the property of distribution; that is, the subinterval introduced by the third leaf would, in these, be very small, being respectively 6 and 5 times smaller than the smaller primary interval. But they have not the cyclic simplicity of the alternate system, and thus lack whatever advantage belongs to it. The same is true in diminishing degrees of the following fractions, until we arrive at $\frac{3}{5}$; and none of these occur in nature. $\frac{3}{5}$ is the first in order of magnitude after $\frac{1}{2}$ which is found in nature, and immediately following it, we find all the other phyllotactic fractions of the first series, or all the fractions that are of common occurrence in nature, except $\frac{1}{2}$. The bracket includes these, and also one intruder, namely, $\frac{7}{11}$. The ratios in the last column range for these fractions from 2 to 1; indicating that for all these the subinterval introduced by the third leaf, though less than the smaller primary interval, is not contained in it more than twice. For the last fraction of this series, $\frac{2}{3}$,

this subinterval is equal to the smaller primary one; the third leaf falls exactly in the middle of the larger interval and completes the cycle. This fraction is thus next in cyclic simplicity to $\frac{1}{2}$, and has but little more of the distributive character. Following $\frac{2}{3}$ we find two fractions, $\frac{9}{13}$ and $\frac{7}{10}$, which resemble it in this respect as $\frac{7}{13}$ and $\frac{6}{11}$ resemble $\frac{1}{2}$, and lacking like them the cyclic simplicity of their type they have still no superiority to it in the distributive property. The same is true in diminishing degrees of the following fractions. But these onward to the end are, with one intrusive exception, phyllotactic fractions of the two lower series, or those of infrequent occurrence in nature. They include all the fractions peculiar to these series, that is, all but $\frac{2}{3}$, just as the group above includes all of the first series except $\frac{1}{2}$; and these two fractions have the least of the distributive character and most of cyclic simplicity. They are the fractions of the smallest denominations, and might properly be separated from the others as a special type; with more propriety, indeed, than the purely distributive fraction k could be separated from others of the first group. In all the fractions of the lower group the disparity of the primary intervals, or the great difference between these fractions and their complements (the differences exceeding these complements, and the ratios being proper fractions), unfit them for a distributive arrangement, so far at least as the earlier steps of the cycle or the first three leaves are concerned. In other words, the primary intervals being in greater ratios than two to one, the distribution is imperfect at the outset. But it is better in subsequent leaves, for all those fractions that are found in nature, or for all but the one intrusive exception I have referred to. This exception is the fraction $\frac{10}{13}$, which, as well as $\frac{7}{11}$ in the first group, seems at first sight a remarkable anomaly. They are not, as we shall see, anomalies at all; but, though differing from the other fractions of these groups in the character of the distribution higher up in the cycles than the steps we have yet considered, namely, the first three leaves, yet they occur thus isolated in these groups only on account of the arbitrary limit I have assumed for the denominations of the fractions in the table, or the limit of 13ths. If fractions of higher denominations had been included in the table, other exceptional fractions would have appeared within the limits of these groups. But before proceeding to show this, I will call attention to one other fact shown by this table, namely, how large within the limits assumed for the table the number of phyllotactic fractions is, compared to the whole number, namely, more than half. Out of the nineteen possible proper fractions given in the table, ten are phyllotactic; that is, either actually

occur in nature, or are deduced from theory. The theoretical source of many of them, of more than half, is more than probable; for the limit of the denominations assumed for the table is undoubtedly beyond the limits of distinguishable forms in actual measurements. If, however, this assumed limit had been less, the number of the other fractions would have been reduced in greater proportion than the phyllotactic ones. If it had been greater, they would have been increased in a greater proportion. In other words, the ratio of the number of fractions given by the theory of Phyllotaxy to all other possible ones within the same range of denominations decreases from a very large value (nearly the whole) to smaller and smaller values. This ratio, therefore, is a fact of no importance as a fact of observation, since it is almost wholly a formal fact, not a material one, as the logicians say; or is involved in the form of expression, or of representation, or in the nature of the method of investigation. The important fact is that there are fractions, however few, which would be distinguishable if they existed in nature, but are not found, though their magnitudes are within the range of those that do exist. Such are the fractions $\frac{4}{7}$ and $\frac{5}{9}$. If the fractions of our table were arranged, not only in the order of their magnitudes, but at corresponding distances, and if we disregarded altogether the character of simplicity or complexity in these fractions, and the numbers of them within any limit of denomination, and considered only the ranges of geometrical values between them, we should find between $\frac{1}{2}$ and $\frac{3}{5}$ a difference of $\frac{1}{10} = 0.100$ of the circumference, which is greater than the whole range of the other fractions of Phyllotaxy in the first group, namely, $\frac{1}{15} = \frac{2}{3} - \frac{3}{5} = 0.067$ of the circumference; and between the last of these and the first fraction of the second group we have the difference $\frac{5}{7} - \frac{2}{3} = \frac{1}{21} = 0.047$ of the circumference, which is not much less than the range $\frac{1}{15}$. In these two spaces, therefore, of $\frac{1}{10}$ and $\frac{1}{21}$, there would be room for fractional intervals as distinguishable from each other as those of the first group are; though, in the space $\frac{1}{21}$, or 0.047, between the first and second groups, no simple fractions, or of less denomination than $\frac{7}{10}$, could occur, and no interval has been observed which belongs to either of these spaces. It is therefore sufficiently obvious that the fractions $\frac{5}{9}$ and $\frac{4}{7}$ (which would, perhaps, be with difficulty distinguished from each other, since they differ by only $\frac{1}{63}$) are real omissions from natural arrangements. $\frac{9}{13}$ and $\frac{7}{10}$ would be really indistinguishable from each other if they existed; but the former could be as readily distinguished from any real arrangements as these are from one another. It ought, therefore, to be regarded as also a real exception,

though it may have been omitted not only on account of its defective character as a distributive fraction, but also for its lack of simplicity. Taking account of the relative frequency of the fractions that are found in nature, we have sufficient grounds to suppose that the distributive character of them is an utility of actual importance to the welfare of plants.

But I have not yet shown this distributive character throughout the cycle, or what distinguishes them from the abnormal forms I have referred to. I must first show, however, that these are only apparent anomalies, and that others of the same character would occur if the table were extended. There is a ready means of extending the table; the same law in fact which holds in the series given above. Each value of the proper fractions, as arranged in the second column, except the extreme ones, can, it will be seen, be derived from the preceding and following ones by adding their numerators for a new numerator, and their denominators for a new denominator, and, in some cases, reducing the fraction thus obtained to lower terms. Moreover, in the table as it stands, the difference of any two successive fractions is the reciprocal of the product of their denominators, or when reduced to this as a common denominator, their numerators differ by a unit. From this property it follows in the theory of numbers that intermediate values obtained in this way cannot be reduced to lower terms, and are the fractions of the smallest denomination intermediate in value between any two. It is obvious that by this process the table could be extended indefinitely, without omitting any fraction of less than any assignable denomination. Indeed, it could have been constructed from its limits by successive interpolations of this sort. Thus taking the extreme limits $\frac{1}{2}$ and 1, or, as we may express the latter, $\frac{1}{1}$, which differ by $\frac{1}{2}$, or the reciprocal of the product of their denominators, we obtain between them by this process $\frac{2}{3}$. Between $\frac{1}{2}$ and $\frac{2}{3}$ we find $\frac{3}{5}$. Between $\frac{2}{3}$ and $\frac{1}{1}$ we find $\frac{3}{4}$. Continuing this process, as in the subjoined example, we arrive at the results in the lower line, which are, many of them, of higher denomination than those of the table.

$$\frac{1}{2} \qquad\qquad\qquad\qquad \frac{2}{3} \qquad\qquad\qquad\qquad \frac{1}{1}$$

$$\frac{3}{5} \qquad\qquad\qquad\qquad \frac{3}{4}$$

$$\frac{4}{7} \qquad \frac{5}{8} \qquad \frac{5}{7} \qquad \frac{4}{5}$$

$$\frac{5}{9} \quad \frac{7}{12} \quad \frac{8}{13} \quad \frac{7}{11} \quad \frac{7}{10} \quad \frac{8}{11} \quad \frac{7}{9} \quad \frac{5}{6}$$

$$\frac{1}{2}, \frac{6}{11}, \frac{5}{9}, \frac{9}{16}, \frac{4}{7}, \frac{11}{19}, \frac{7}{12}, \frac{10}{17}, \underbrace{\frac{3}{5}, \frac{11}{18}, \frac{8}{13}, \frac{13}{21}, \frac{5}{8}, \frac{12}{19}, \frac{7}{11}, \frac{9}{14}, \frac{2}{3}}, \frac{9}{13}, \frac{7}{10}, \frac{12}{17}, \underbrace{\frac{5}{7}, \frac{13}{18}, \frac{8}{11}, \frac{11}{15}, \frac{3}{4}, \frac{10}{13}, \frac{7}{9}, \frac{11}{14}, \frac{4}{5}}, \frac{9}{11}, \frac{5}{6}, \frac{6}{7}, 1.$$

By interpolating one more value between the first two of these fractions, or between $\frac{1}{2}$ and $\frac{6}{11}$, namely, $\frac{7}{13}$, and by omitting those of higher denominations, we obtain all the proper fractions of our table. Among these higher denomi-

nations we find several new intruders in the phyllotactic groups, namely, $\frac{11}{18}$, $\frac{13}{19}$, and $\frac{9}{14}$ in the first, and $\frac{11}{16}$ in the second, as well as new fractions of the series, namely, $\frac{13}{21}$ of the first, $\frac{13}{18}$ of the second, and $\frac{11}{14}$ of the third series, which our limits had excluded from the table; and several others beyond these groups. If, on the other hand, we make the denominations of our table smaller, and exclude all above 9ths, as being either actually indistinguishable, or with difficulty distinguished from the remaining ones, we have remaining $\frac{1}{2}$, $\frac{5}{9}$, $\frac{4}{7}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{5}{7}$, $\frac{3}{4}$, $\frac{7}{9}$, $\frac{4}{5}$, all of which are included in observed arrangements, except the two, $\frac{5}{9}$ and $\frac{4}{7}$. Our problem would therefore, as I have said, appear to be why these have been excluded from natural arrangements, rather than why the others have been adopted or preserved. But the problem is more correctly as follows: Account ought to be taken of the relative frequency of them, or weights ought to be attached to them according to this consideration. The weight of these two fractions would then be nothing. The weights of several others would be very small. Now, what has determined these weights? This is our problem. The superior distributive character of the more frequently occurring ones, is the only conceivable answer. We see from this, however, that we ought not to attribute to Natural Selection the existence of the spiral arrangements in general, at least not on account of the distributive property we have considered, for, in fact, they include almost all possible intervals, as the arrangements of the whorl do, and little selection is shown in them independently of their relative frequency. This relative frequency, or infrequency, in nature, amounting to total exclusion in the case of these two fractions, is, then, the only way in which Natural Selection could have been concerned in producing or modifying the spiral arrangements, so far as that property of distribution is concerned which is exhibited most perfectly by the typical or unique angle of the theory of Phyllotaxy. But, supposing these arrangements to have come into existence through some other agency, or by Natural Selection acting on some other ground of utility, or under some other phase of this one, we then see sufficient reasons why, on this principle, they should be what they are.

We might exclude altogether from our consideration the intrusive angle in our table, $\frac{7}{11}$, as being a purely theoretical product, and indistinguishable in nature from the simpler fraction above it, $\frac{5}{8}$; but I will include it in the further discussion of this group, for the sake of showing that it would, probably, have been excluded from nature, if plants had been more accurately constructed; and would not, therefore, be found, even if we had the power to distinguish it.

This discussion will also show the resemblance of the distributive character of the other arrangements of this group, as we ascend the cycle beyond the third leaf, to the theoretical unique angle. In these, as in this angle k, the successive subintervals are simply the differences of the two preceding ones, continually diminishing, but growing nearer and nearer in value until they become all the same aliquot parts of the circumference; namely, that expressed by the denominator of the fraction. This will be best seen in the accompanying diagram. For the numerical illustration of it, let us take each fraction of this group, and its complement; then the difference of these; then the difference of this from the complement, and so on. We have in this way:—

$$\tfrac{3}{5} \cdot \tfrac{2}{5} \cdot \tfrac{1}{5} \cdot \tfrac{1}{5} \cdot 0.$$
$$\tfrac{8}{13} \cdot \tfrac{5}{13} \cdot \tfrac{3}{13} \cdot \tfrac{2}{13} \cdot \tfrac{1}{13} \cdot \tfrac{1}{13} \cdot 0.$$
$$k.\ k^2.\ k^3.\ k^4.\ k^5.\ k^6.\ \&c.$$
$$\tfrac{5}{8} \cdot \tfrac{3}{8} \cdot \tfrac{2}{8} \cdot \tfrac{1}{8} \cdot \tfrac{1}{8} \cdot 0.$$
$$\tfrac{7}{11} \cdot \tfrac{4}{11} \cdot \tfrac{3}{11} \cdot \tfrac{1}{11} \cdot \tfrac{2}{11} \cdot (?)$$
$$\tfrac{2}{3} \cdot \tfrac{1}{3} \cdot \tfrac{1}{3} \cdot 0.$$

It will be seen by the (?) in the series for $\tfrac{7}{11}$, that it violates the law which holds in all the other fractions. The fourth interval, or the second subinterval, is contained three times in the preceding one, instead of once, with a smaller remainder, or exactly twice without remainder as in the others. It completes the cycle finally like the others, but introduces into it at the end of the second and in subsequent turns great inequalities side by side. If these were of sufficient absolute amount to be of importance in nature, we might be sure that such an arrangement would never exist. But a twist of the stem by only one eighth of the circumference in the length covered by eleven leaves, or a twist of one eleventh in the range of eight leaves, would convert this arrangement into the $\tfrac{5}{8}$ system, or the $\tfrac{5}{8}$ into this. We may see from this illustration how much the mathematical theory of Phyllotaxy has refined upon the facts of observation.

The property thus exhibited by the first group belongs also to the second group, or the less frequently occurring fractions, but only after the first revolution. The complement of each of these fractions, or the smaller of the primary intervals, is contained more than twice in the larger, or in the fraction itself. We must, therefore, subtract from these fractions the largest multiple of their complements contained in them for a first difference or subinterval, and then proceed as in the above cases. This gives:—

$$\tfrac{5}{7} \cdot \tfrac{2}{7} \times 2 \cdot \tfrac{1}{7} \cdot \tfrac{1}{7} \cdot 0.$$
$$\tfrac{8}{11} \cdot \tfrac{3}{11} \times 2 \cdot \tfrac{2}{11} \cdot \tfrac{1}{11} \cdot \tfrac{1}{11} \cdot 0.$$
$$\tfrac{3}{4} \cdot \tfrac{1}{4} \times 2 \cdot \tfrac{1}{4} \cdot 0.$$
$$\tfrac{10}{13} \cdot \tfrac{3}{13} \times 3 \cdot \tfrac{1}{13} \cdot \tfrac{2}{13} \cdot (?)$$
$$\tfrac{7}{9} \cdot \tfrac{2}{9} \times 3 \cdot \tfrac{1}{9} \cdot \tfrac{1}{9} \cdot 0$$
$$\tfrac{4}{5} \cdot \tfrac{1}{5} \times 3 \cdot \tfrac{1}{5} \cdot 0.$$

Here, as before, the intrusive angle ($\tfrac{10}{13}$) is found to violate the law which holds for the other fractions beyond the first turn. The first difference or subinterval is contained three times in the second or the smaller primary interval. But here, also, as

before, this fraction differs insensibly from a simpler one near it, namely, $\frac{7}{9}$. All the other fractions of our table will be found by an inspection of the diagram to violate in the same way the law in which the observed values of natural fractions, as well as the deduced ones of theory, agree. They all introduce side by side in the more advanced phases of the cycle intervals in greater ratios than two to one.

The diagram is constructed as follows: The fractions, for convenience, are in an order the inverse of that in the table. The horizontal lines represent the developed helices or spiral paths connecting ideally the successive leaves on the stem the longer way round. These are divided by the vertical lines into lengths representing single revolutions or turns around the stem. Above each line, except for k, the smaller dots (when their places are not occupied by the larger triangular dots) represent the horizontal places or directions in which the leaves fall in the cycle, and are distant successively from each other by that part of the circumference denoted by the denominator of the fraction. Above the lines are also placed the larger dots to represent the leaves as they are introduced at the constant angle represented by the fraction. After the turn in which each is introduced, dots are placed below the line in corresponding positions for all subsequent turns; and when the cycle is completed (as happens with all but k and two rational fractions within the length of the eight turns here represented), the completed cycle is repeated on parallel lines below. We are thus enabled by mere inspection to see how each new leaf would be introduced in these several arrangements in relation to the two older ones that are nearest it in horizontal direction. Thus the fractions $\frac{7}{13}$ and $\frac{6}{11}$ resemble $\frac{1}{2}$, or the alternate system, in crowding the leaves together on opposite sides of the stem, and permitting large intermediate spaces; but they do not bring them into the perfect vertical allignment of this system. The same is true in diminishing degrees of the fractions above them, as $\frac{5}{9}$, $\frac{4}{7}$, $\frac{7}{12}$, until we come to $\frac{3}{8}$. In all these cases spaces or subintervals exist side by side in greater ratio than two to one. It can be seen among the fractions next following of the first group how little the theoretical value k differs from $\frac{8}{13}$, or even from $\frac{5}{8}$, the fourteenth leaf falling only a little (by about $\frac{1}{30}$ of a turn) beyond the position of the first, instead of falling exactly over it, as it does for $\frac{8}{13}$. All the fractions of the actual arrangements of nature, as well as the less simple theoretical ones of Phyllotaxy, have the property, that after the first turn of the cycle, and also in this first turn for all the fractions of

the first series, or for those most commonly occurring in nature, *each leaf of the cycle is so placed over the space between older leaves nearest in direction to it as always to fall near the middle, and never beyond the middle third of the space, or by more than one sixth of the space from the middle, until the cycle is completed, when the new leaf is placed exactly over an older one.* This property depends mathematically on the character of the continued fractions, of which these fractions are the approximations, according to the theory of Phyllotaxy. The denominators in the characteristic part of the continued fractions, or for the whole in the case of the fractions of the first group, are each a unit *plus* a fraction, which, at the end, is also a unit, or the last denominator is 2, or $1 + \frac{1}{1}$. The first denominator is the ratio of the larger primary interval to the whole circumference. These denominators are, in fact, the ratios of the successive intervals and subintervals of our diagram. The other fractions, expressed in the form of continued ones, would have denominators expressing, in the same way, the ratios of the successive subintervals, which the diagram represents; and fractions in general may be classified according to their special forms as continued fractions. Thus we have: —

$$\tfrac{2}{3} = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}} \qquad \tfrac{3}{5} = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2}}} \qquad \tfrac{4}{7} = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3}}} \qquad \tfrac{5}{9} = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4}}} \qquad \tfrac{6}{11} = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{5}}} \qquad \tfrac{7}{13} = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{6}}}$$

Again we have: —

$$\tfrac{3}{4} = \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1}}} \qquad \tfrac{5}{7} = \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2}}} \qquad \tfrac{7}{10} = \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{3}}} \qquad \tfrac{9}{13} = \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{4}}} \qquad \tfrac{11}{16} = \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{5}}}$$

The numerators and the denominators of the proper fractions of these series have constant successive differences.

The last denominators in these continued fractions represent the ratios of the contiguous intervals of the diagram introduced in the second or third turns by the third or fourth leaves. Only the first two fractions in each of these series conform to the above law. The others, like $\frac{4}{7}$ and $\frac{5}{9}$, violate the law early in the cycle; and this explains the absence of them from natural arrangements of the spiral type. The property common to the latter resembles what we have observed in the arrangements of whorls, namely, that the leaves of successive whorls are so placed that those of the upper one fall over the middle positions of the spaces between those of the lower one; but those of the next one above, or in the third whorl, are thus made to fall directly over the leaves of the first. Two whorls thus constitute a cycle, in the sense in

which this name is applied to the spiral arrangements; and in respect to their distributive and cyclic characters, whorls are thus most closely related to the $\frac{1}{2}$, or alternate system. But there is, as I have said, no fundamental or genetic relationship between them and this particular form of the spiral arrangement. The relationship is rather an adaptive or analogical one. They are, so to speak, two distinct solutions of the same problem, two modes of realizing the same utilities, or securing the same advantages; like the wings of birds and bats.

One of these utilities we have now sufficiently considered, namely, that which the theoretical angle k would realize most perfectly; by which the leaves would be distributed most thoroughly and rapidly around the stem, exposed most completely to light and air, and provided with the greatest freedom for symmetrical expansion, together with a compact arrangement in the bud. Neither this property, nor an exact cyclical arrangement, ought, as I have said, to be found, or expected, in the incipient parts at the centre of the bud, any more than the perfected proportions and adaptations of the mature animal could be expected, or are found, in the embryo. Both are fully determined, no doubt, in the vital forces of the individual's growth. Our question is, what has determined such an action in these vital forces? "Their very nature, or an ultimate creative power," is the answer which the theory of types gives to this question. "The necessities of their lives, both outward and inward, or the conditions past and present of their existence," is the answer of the theory of adaptation. Science ought to be entirely neutral between these theories, and ready to receive any confirmation of either of them which can be adduced; though, from this point of view, the theory of adaptation has a decided advantage; since the theory of types can have no confirmation from observation except of a negative sort, the failure of its rival to show conclusive proofs. But we have seen that whatever can be said in favor of the view, that there is a unity of type in the intervals of spiral arrangements, is directly convertible to the advantage of the theory of adaptation; since this unity consists in the distributive property common to these arrangements.* Natural Selection, however,

* There is a remarkable analogy between this relation and that of the two theories of the structure of the honey-cell. The work of the bees suggests to the geometrician a perfectly definite and regular form, which he finds to be the most economical form of compartments into which space can be divided; or he finds that the honeycomb would be the lightest, or be composed of the least material for the same capacity and number of compartments, if partitioned into such figures as the typical cell. From the definition of this figure he is able to compute its angles and proportions with a degree of precision to which the bees' work only roughly approximates at its best, and from which it often deviates widely. The theory of types regards this ideal figure as a determining

or the indirect agency of utility in producing adaptations, cannot, so far as we have yet seen, be appealed to for the explanation of the spiral arrangements in general; nor for the explanation of the verticil arrangements; though the character in the latter, in which they resemble the alternate system, may come within the range of this explanation through the utility I have pointed out. The only ground for the action of Natural Selection which I have yet shown is in the choice there is among possible spiral arrangements with reference to this utility; and it appears that the principle is fully competent to account for the relative frequency of these, and the entire absence of some of them from the actual forms of nature.

We now come to the special study of two other features which have appeared in these arrangements, namely, the spiral character itself and the simplicity of their cycles. The cyclic character is entirely wanting in the ideal arrangement of the interval k; but, as I have said, this interval cannot be proved to exist in nature; for even if it did, it would be indistinguishable even from the simple fraction $\frac{5}{8}$. This very fact, however, makes the interval $\frac{5}{8}$, a sufficiently exact realization of the distributive property, according to the degree of exactness with which actual plants are constructed. But $\frac{5}{8}$ is also a comparatively simple cycle, though there would not be sufficient evidence that its cyclic character is an essential one, or other than incidental to the scale of exactness in the structure of plants, if there did not exist several distinguishable and simpler cycles, namely, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{5}$. The cyclic character of leaf arrangements is, indeed, a more noticeable feature in plants generally than the distributive one. It is obviously essential, and involves on the theory of adaptation some important utility. Whatever this may be, it is clear that it has to be gained by means directly opposed to those which secure distribution; that is, its utility depends on leaves coming together in direction, or being brought nearer to each other than they would otherwise be; instead of their being dispersed as widely and as thoroughly as possible. This utility is obviously to be sought in the internal relations of leaves to each other, or their connections through the

cause of the structure, or as the pattern which guides the bees' instinct towards an ideally perfect economy. But a plainer order of economy, a simple housewifely one, saving at every turn, together with the conveniences and utilities which govern the work of social nest-building insects in general, would result, if carried out to perfection, in the very same form. Hence the theory of adaptation regards the honey-cells as modifications of similar but rougher structures of the same sort, determined by the further utility of simple saving in working with a costly material; and whatever evidence there is that the bees' instinct is determined toward the ideally perfect type of the honey-cell is directly convertible into proofs that it is so determined by these simple conveniences and utilities.

stem, and not in their outward relations, which require exposure, expansion, and elbow-room. The apparently inconsistent means of these two ends are both realized, however, without interference, in the actual cycles of natural arrangements. Through the simplicity of these cycles leaves, not very remote on the stem, are brought nearer to each other, and into more direct internal connection than they would have but for this simplicity; while in the more prevalent natural forms of the cycle leaves, that are nearest to each other on the stem, are separated as widely as is possible under this condition. That this prevalence is due to selection, through the utility already considered, has been shown to be sufficiently probable. I propose now to connect the prevalence of simplicity in these cycles with another utility. Leaves that are successive, or nearest each other on the stem, may be regarded as rivals, and as rendering each other no service. Those that are more remote may come into relations of dependence, one on the other. Between the leaf and the stem the relations of nutrition are reciprocal. At first, and for the development of the leaf, the stem furnishes nutriment to it. Afterwards the leaf furnishes nutriment for the further lateral expansion of the stem. The development of the stem itself, first in length, while the leaves are expanding, and afterwards in breadth and firmness through the nutrition afforded by the developed leaves, has the effect, and, we may presume, the use or function, of a still more important distribution of the leaves than that we have considered. We have hitherto attended only to the distribution effected by the character of the divergences of leaves around the stem. Their distribution along the stem, or their separation by the internodes of the stem, is a still more direct and effective mode of accomplishing at least one of the uses of the property of distribution, namely, exposure to light and air. The special accomplishment of this important end in the higher plants is secured by two different means; by the firm fibrous structure and the breadth of stems, branches, and trunks in grasses, shrubs, and trees, and by the climbing powers and prehensile apparatus of climbing plants; and in the latter we find the highest degree of specialization or development in the vegetable world. The distribution effected by the separation of leaves along the stem in great measure supersedes the value of their distribution around it, so far as the ultimate functions of leaves are concerned, and independently of their relations in development or in the bud; and this gives freer play to the means of securing whatever advantage there may be in the simpler cyclic arrangements, like the $\frac{1}{2}$ and $\frac{2}{3}$ systems. Accordingly we find, in general, the simpler cycles on the stems of those plants that have the

longest internodes; and, on the other hand, the more complicated cycles are found only in cases of very short internodes, or in great condensations of leaves. There is no evidence, however, that in the condensed form in which undeveloped leaves exist in the bud the cycles are any more complicated than on the stem. Nor ought we to expect such evidence; for it is a false analogy that would lead us to seek for types in the early and rude forms of embryonic life; though, if the simpler cycles were really derived from the more complicated ones, rather than from the utility common to all, we ought, by the analogy of embryology, to find some traces of the process in the bud. No doubt the types exhibited by the mature forms of life exist in the embryo or bud, though not in a visibly embodied form; but rather in a predetermined mode of action in vital forces, embodied in gemmules rather than the visible germ. But while the distribution effected by the internodes of the stem thus allows the simpler cycles to occur, it does not account for their occurrence. This, moreover, must depend on relations in mature, or else in growing leaves, to those below them; and not on their earlier relations in the bud; since, as we have seen, the more complicated cycles are the best fitted for these relations, and in mature stems are only found in great condensations of leaves; such as the bud also presents; yet without any greater complication than the stem has. The simplicity of the cycles in stems with long internodes has the effect that the absolute distance between two leaves standing one over the other is not so great as it otherwise would be. There is, no doubt, a disadvantage in long internodes, or in the separation of growing parts by long intervals from their sources of nutrition; a disadvantage, which only a better exposure to light and air for their subsequent functions could compensate. On the theory of adaptation there would seem to be, then, some advantage to the younger leaf in standing directly over an older one, and not far above it; a greater advantage than in any other position at the same height; and this advantage could apparently be no other than an internal nutritive one, having reference to the sources or movements of sap and the nutrition conveyed by it. But sap circulates with nearly equal facility around and along the stem; and if the lower leaf were really a special source of nutrition to the growing one above it, it could furnish nutrition almost as readily to any other position on the stem at the same height as to the point directly above it, or on the same side. The new leaf is not sensibly nearer the market on account of this feature in the arrangement. But may there not be some advantage to the older leaf in standing directly under the

younger? Next to the advantage of being near a market, or a source of supplies, is the advantage of being in the line of traffic. This, indeed, is in part what it is to be a market, rather than a mine. A leaf is not only a productive or industrial centre, but a commercial one. It effects exchanges, both giving and receiving supplies. When mature, or fully established in this capacity, it draws from the roots its raw material of water and mineral salts, and from the air its more costly material, and in exchange sends forth into the great commerce of the stem its wonderfully intricate fabrics of atoms, woven on the sunbeam, its soluble colloids. Now, although sap may flow with nearly equal facility in all directions in the stem, it probably does flow with greatest rapidity in the direct lines of the forces that impel it, the lines of osmotic force. Sap flows most freely from that side of a perforated tree in the Spring which is immediately below the largest branch. This shows that even in the least active condition of the circulation, when the trunk is surcharged with sap, the forces of circulation are not simply diffusive or hydrostatic; and they must be much less so when definite outlets of this supply become established in the growing buds and leaves of the springtime. The character of the circulation is principally determined by the hydraulic action of osmotic forces. Water *may* flow with equal facility in any part of a river-bed, and across as well as along; but it actually does flow fastest along the middle. The growing leaf has different needs from those of the mature one; hence they are not rivals, or competitors in the market, but buyer and seller, or borrower and lender. The mature leaf needs from the stem water and mineral salts; the growing leaf needs the organic materials of new tissues. The mature leaf helps to prepare the latter by concentrating it, withdrawing the water, and adding its own contribution of organic material in return. But while aiding its younger fellow in this way, it is aided in return, or its efficiency is increased, by the increased circulation produced through the forces of movement above it. In place of a glut in the market we have an active exchange. There is, undoubtedly, a tendency in these physiological causes, however feeble, to that vertical allignment of not very distant leaves, which the cyclic character of the spiral arrangements exhibits, and most markedly in the $\frac{1}{2}$ or alternate system.

We have thus assigned more or less probable utilities to two prominent features in the particular forms of the spiral and verticil arrangements of leaves; their distributive and cyclic characters. We now come to a much more obscure problem, which connects the verticil and spiral arrangements in general

with their probable utilities, and through these with their origin in lower forms of vegetable life. But before entering upon the study of this as an actual physical problem, it is necessary to consider what are the real meanings of the terms "spiral" and "whorl." Are they only conventional modes of representing the phenomena of arrangement, or are they strictly descriptive of the facts in their physical connections? About the whorl there can be no doubt. The actual physical connections and separations of leaves in this type of arrangement are directly indicated by the term; but the ideal geometrical line connecting successive leaves in the so-called spiral arrangements may be a purely formal element in the description of them, and of no material account, — a mode of reducing them to order in our conceptions of them, but implying no physical relationships. There are several ways in which we can so represent the features of these arrangements. Connecting by an ideal line (which may have no physical significance) the leaves nearest to each other on the developed stem, and by the shorter way round, is one way, — the more common way of representing their arrangements. The direction in which this should be drawn, whether to the right or the left, is quite arbitrary in the $\frac{1}{2}$ or alternate system. Connecting, for other cases, the leaves in the same succession, but by the longer way round (as I have chosen to do for convenience), is another way. These are distinctly different spiral paths, but not the only ones by which the parts of these arrangements might be represented geometrically. By connecting them alternately, as 1 with 3, and this with 5, &c., and 2 with 4, and this with 6, &c., we would connect the leaves of the various arrangements by two spiral paths, and these either by the longer or the shorter way round. Or again, by connecting the series 1, 4, 7, &c., and 2, 5, 8, &c., and 3, 6, 9, &c., we would include all the leaves in three spiral paths; and so on. In some cases these lines would not be spiral, but the vertical allignments we have considered. For example, in the last case they would be vertical for the cycle $\frac{2}{3}$; since in this the leaves 1 and 4, or 2 and 5, are the beginnings of distinct successive cycles. If the leaves 1, 2, 3, were in this case of the same age, or at the same height on the stem, and were succeeded at an interval on the stem by 4, 5, 6; also coeval, and so on; we would have the main feature of the verticil arrangement, but not the kind of alternation that belongs to natural whorls. Between 1, 2, and 3 in the natural whorl equal intervals exist, namely, $\frac{1}{3}$; and also between 4, 5, and 6, and so on; but between 3 and 4 the interval in natural three-leaved whorls is either

$\frac{1}{8}$, $\frac{1}{2}$, or $\frac{5}{8}$, according as we choose our spiral paths, or determine which member of the upper whorl shall be counted as the fourth leaf. We see, therefore, that there is no continuity or principle of connection between spiral arrangements and the whorls; and, moreover, that these spiral paths are purely ideal or geometrical lines, so far as we have yet seen. Is there any good reason for supposing that the *simplest* of these, which connects successive leaves on the stem the shorter way round, is any less formal or conventional than the others; or indicates a real connection of the leaves on this path, or any closer original real connection among them? There are two significant facts bearing on this question to which I have already adverted. The first is that the natural fractions of the lower group of our table, or those peculiar to the last two series of the theory of Phyllotaxy, represent the less frequent forms of spiral arrangements, and that if the successive members of these arrangements are connected in the usual mode by this simplest path, or the shorter way round, these members are seen to have less angles of divergence than those of the more common arrangements; or are much nearer each other on this line than the others are. We should thus have the fractions $\frac{2}{7}$, $\frac{3}{11}$, $\frac{1}{4}$, $\frac{2}{9}$, $\frac{1}{5}$; all of which indicate comparatively small divergences, smaller than any among the more common ones. The second fact is the observation that these arrangements are relatively more common among fossil plants than among surviving ones. These facts agree well with the supposition that this simplest spiral path is unlike the others, and is not a merely formal assumption for the representation of leaf-arrangements, but the trace of a former physical connection of the members, or even of a continuity of leafy expansion along this path; a leaf-like expansion resembling a spiral stairway. The leaves, according to this supposition, are the relics of segments made in such a spiral leaf-like expansion around the stem; remnants of it grown smaller and smaller, or more widely separated as they became more advantageously situated through the developments of the stem in length and firmness; and expanding, perhaps, in an opposite direction along the leaf-stems; or, losing their leaf-character and expansion altogether, as they became adapted to other uses in the economy of the higher vegetable life, namely, the use of the leaf-stem itself, as in the tendril, and the uses of leaf-like extensions, as in the reproductive organs of the flower. But are there any surviving instances of such continuous spiral leaf-like expansions on vegetable stems; or, in default of these, could there be any utility in such an arrangement itself to justify the supposition of it as

the basis of the development of more special forms? Before considering this question, however, I will consider what other resources of explanation hypothesis can command. The spiral arrangement might be supposed to be the result of a physiological necessity among the laws of growth, through which single leaves would be produced at regular intervals or steps of development, and placed so as to compass the utilities we have already considered, namely, those of horizontal and longitudinal distribution in successive leaves, and vertical allignment in remoter ones. This would account for the spiral arrangements, and it may be a superior mode of growth, or involve some physiological utility; but that it is not a necessity, is proved by the arrangements of the whorl, in which all the members of a group of leaves are simultaneously produced. The existence of the whorl, then, sets this hypothesis aside. Again, we might suppose on the theory of types that these two great types of arrangement are two fundamental facts in the higher vegetable life, parts of a supernatural plan; two aboriginal and absolute features in this plan. But this, as we have seen, is not to solve the problem, but to surrender it; or rather to demand its surrender, and forbid its solution. Again, the production of adventitious buds in plants, or in separated parts of plants, as in cuttings, dependent only, apparently, on a favorable situation for nutrition, is of common occurrence even in the higher plants. If we could suppose that the definite horizontal distributions of successive leaves were wholly superseded in their utility by the distributions along the stem, or that the leaves could thus be sufficiently exposed to light and air; the power of the adventitious production of buds or leaves in favorable situations might have caused an arrangement without this feature of spiral regularity. But they would still be brought into vertical allignments, if the physiological advantage of the simpler cycles, which has been pointed out, be a real and effective one; for even the so-called adventitious production of buds may reasonably be supposed to be governed by supplies of nutriment. Moreover, these vertical lines would be placed at equal intervals around the stem, on account of the advantage there would be in such a distribution, both for internal and external nutrition. But though leaves would thus be placed at convenient distances along equidistant vertical lines, there would be no consideration of utility to govern their relations to each other on different lines, so as to throw them into whorls, or into definite spiral arrangements. It might, however, be advantageous for leaves on a line between two others to be placed in intermediate positions with respect to the leaves

of these two, and if the latter were placed at the same heights we should have a sector of three whorls; that is, two leaves of the highest and two of the lowest whorl, and one leaf of the intermediate whorl. But such an arrangement disregards or sacrifices in the structure of the whorl itself the advantage, if it be one, of such an alternation. It cannot be reasonable to suppose that a leaf on an intermediate line would seek distance and isolation from those of the lines beside it, and, at the same time, seek close connection horizontally with those of its own whorl. This would be directly opposed to the accommodation of uses in spiral arrangements. The structure of whorls, and the alternation in successive ones, appear, therefore, to be of distinct origins. Whatever advantage there is in the former appears to be sacrificed by this alternation, and by the spiral arrangements; or, if it be a disadvantage, it is avoided by these. It is probably on the whole a disadvantage; since it is ill-fitted for great extensions and branchings in stems, for which the simpler spiral arrangements appear peculiarly fitted. This contrast, however, cannot be regarded as the origin of the contrasted types themselves, and the soundest conclusion appears to be, that, whatever adaptations they may have, these are only incidental, and are not concerned in their origination, either directly through physiological laws of growth, or indirectly by Natural Selection. They are properly genetic characters. This is confirmed by the fact that the particular arrangement for each plant is provided for, or already completed in the bud; that is, it is not a result of laws of development in general, but of the special nature of the plant, or the predisposition of its vital forces. In regard to the causes which I have supposed to control the so-called adventitious production of buds or leaves, it should not be supposed that these exert in actual plants any considerable influence; though the plant's particular laws of growth are probably not in opposition to them. They should only be considered as modifying agencies reacting on the formative forces; but they fail, as we have seen, to account for the spiral and verticil arrangements, and their contrasts through any utility which could modify these forces. But in concluding therefore that these general types of arrangement ought to be regarded as only genetic characters in the higher plants, and as presenting no important advantage or disadvantage, independently of the special forms which they have acquired, or in present forms of life; we are not precluded by such a conclusion from the further inquiry as to what *former* advantage there could have been in less specialized forms, before these genetic characters had lost their

special significance (if any ever existed), and when they could have stood in more immediate and important relations to the conditions of the plant's existence. In this inquiry our principal guide must be hypothesis, but it will be hypothesis under the check and control of the theory of adaptation. It will not be legitimate to assume any unknown form as a past form of life, and as a basis for these arrangements, without showing that such an hypothetical form would have been a useful modification of a still simpler one, which still exists and is known. In this way we may be able to bridge over the chasm that separates the higher and lower forms of vegetable life.

Our problem then becomes, Whether, in the absence of any surviving instances of continuous spiral leaf-like expansions on vegetable stems, we can find any utility in such an arrangement that could act to modify simpler known forms, and convert them into this? If we suppose our hypothetical spiral leaf-blade to be untwisted, it becomes a single-bladed frond, or a frond with one of its blades undeveloped. In considering what advantage there could be in the twist, we should revert to the general objects or functions of leaf-like expansions. They are obviously to expose a large surface to the action of light on its tissues, and to bring it into the most complete contact with the medium in which the plant lives,—with water, or, in more advanced plants, with the air. Secondly, to accomplish this with the least expenditure of material; not by an absolute, but a relative economy, which has reference to the needs of other parts, like the stem or the roots. In many of the higher plants the developments of the stem serve to diminish to the utmost the amount of this material, and the needed expansion, by giving to them advantageous positions. The first of these objects is secured in the simplest and rudest manner in the *algæ*, as represented by the seaweeds. This is a simple expansion of cellular tissue. But even here we do not find perfectly plane surfaces, facing only two ways, and allowing the water to glide smoothly and unobstructed over them. The corrugated surfaces of many of them, and in the large leaves of some land-plants, are doubtless due to unequal growths in the cellular tissues; but such a physiological explanation of this feature does not preclude the supposition of its being a fixed character in a plant, or becoming such in consequence of its utility. It certainly serves the purpose of opposing the leaf-surface to many directions, both with reference to the incidence of light, and to the movement of the surrounding medium,—to water-currents, or to breezes. *Segmentation*, again, such as is seen in the

fronds of brakes or ferns, is another way of bringing the moving medium to impinge on the leaf-surface; but the feasibility of this depends on the fibrous framework which the leaves of land-plants have acquired for the support of their softer tissues. Such a segmentation also appears among the higher plants in compound leaves and in whorls; and, indeed, the whole foliage of trees and shrubs may, from this point of view, be regarded as the reduced segments of the blades of branching fronds, turned in all directions in search of light, and inviting the movements of air through their expanded interstices. Such is the kind of utility that may be claimed for the structure of our hypothetical spiral frond. Another utility in this structure is obvious when we consider the transition of plant-life from aquatic conditions to those of the dry land and the air; as vegetation slowly crept from its watery cradle, or was left stranded by the retiring sea. In default of strength in its material, such as a slowly acquired fibrous structure or framework ultimately gave to it in this transition, the *strongest form* would be the most advantageous in sustaining the weight of the no longer buoyant plant. A spiral arrangement of the blade around a comparatively firm, and, perhaps, already somewhat fibrous stem, would come nearer fulfilling this condition than any other conceivable modification of the frond.

We have, so far, in conformity to the spiral arrangement in leaves, supposed this twisted frond to be a single-bladed one, or with only one blade developed. This would be a first step in that reduction of leaf-expansion which a more advantageous situation of it would allow; and might be required, even at this early stage of atmospheric plant-life, on account of the greatly increased importance of the roots and stem. But this hypothesis is not necessary in general for the ends we have considered. A two-bladed frond might be similarly twisted and give rise to a double spiral surface like a double spiral stairway, or like the blade of an auger; or such a surface as the two handles of the auger describe as they are revolved, and, at the same time, carried forwards in the direction of the boring. The simplest segmentation of such a twisted frond, after the stem had acquired sufficient strength, and such a subsequent reduction of the segments as might be required for the nutrition of the stem, would give rise to parts, which, turned upwards to face the sky, and also separated, perhaps, by the growth of internodes in the lengthening stem, would result in what we may regard as the original form of whorls, namely, a continuous leaf-like expansion around the stem. The origin of the whorl arrangement itself would thus

be distinct, as we have found that it ought to be, from the origin of the relations in the parts of whorls to one another, and to those of adjacent whorls. These would be results of a subsequent segmentation, and would be determined by the utilities which we have considered in this and in the spiral arrangements. And so both this and the spiral arrangements as general types of structure, though originating, as I have supposed, in useful relations to former conditions of existence, may be regarded in relation to later developments as useless, and merely inherited or genetic types; the bases on which subsequent utilities had to erect existing adaptations of structure. The segmentation of the single spiral frond would at first have little or no relation to these more refined utilities of arrangement, but out of all the variable and possible arrangements so produced there would be a gradual selection, and a tendency toward the prevalence of those special forms, which are at present the most common ones. The typical or unique angle of the theory of Phyllotaxy would thus appear to be the goal toward which they tend, rather than the origin of the spiral arrangements. But since a simple cyclic arrangement appears to have also an important value, we cannot concede to the typical angle the exclusive dignity of even this position.

The segmentation I have supposed in this process should not be regarded as an hypothetical element in it; since it is a well-established law of development. Distinct organs are not separately produced from the beginnings of their growth, but make part of their progress in conjunction, or while incorporated in forms, from which they become afterwards separated; and become then more and more special in their characters, or different from other parts. It is this differentiation and separation of parts out of already grown wholes which distinguishes development from mere growth. The analogy of the phases of development in embryonic or germinal life to development in general is liable, however, to be carried too far; and the fact is liable to be overlooked, that these phases of growth are special acquisitions of the higher forms of life, which have features of adaptation peculiar to them. But the more general features of them, and the useless, or merely genetic phases, may safely be regarded as traces of past characters of adaptation, which a change in the mode and order of individual development has not obliterated; while new adaptations have been added, that have no relation to any past or simpler forms of life, but only to the advantages which embryonic or germinal modes of reproduction have secured. If we should follow out the phases of general

development in the progress of the leaf along the line of its highest ascent in development, from the segmentations we have supposed in the twisted frond, we would soon arrive at the steps already familiar in the principles of vegetable morphology. In these we have the same law of segmentation or separation of parts, and the same successive relations of genetic and adaptive characters. What was produced for one purpose becomes serviceable to a new one; and in its capacity as a merely genetic character, or as an inherited feature, becomes the basis for the acquisition of new adaptations. Thus the fibrous structure, at first useful in sustaining the softer tissues of the leaf, becomes the means of a longitudinal development of it, and its more complete exposure to light and air by the growth of the foot-stalk. This stalk acquires next a new utility in climbing-plants to which it becomes exclusively adapted in the tendril. The adaptive characters of the tendril are its later acquisitions. Its genetic characters, such as its position on the stem, and its relations to the leaves, become useless or merely inherited characters. The contrast of genetic and adaptive characters appears thus to have no absolute value in the structure and lives of organisms, but only a relative one. The first are related principally to past and generally unknown adaptations; the second to present and more obvious ones.

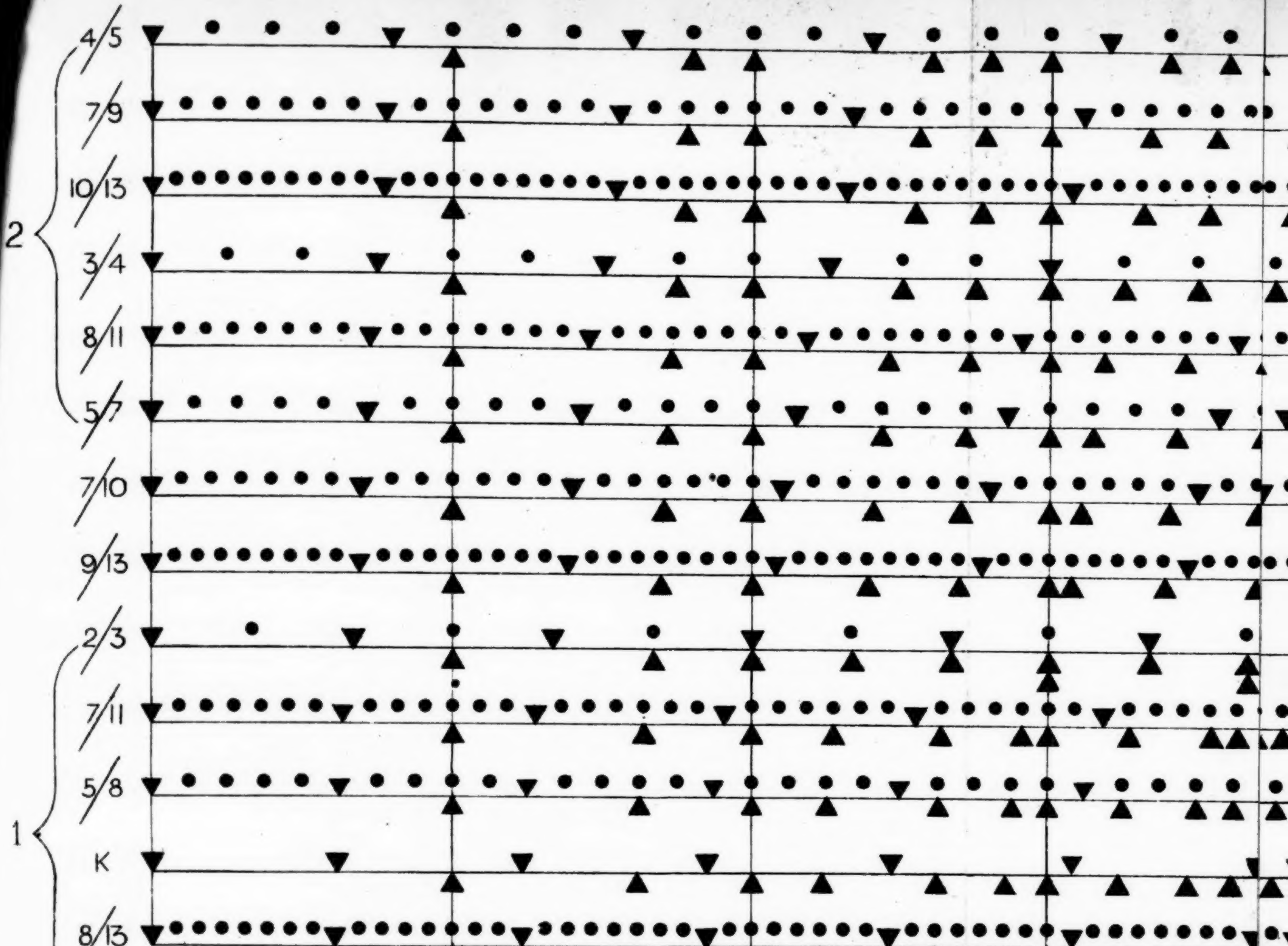
In accordance with this law I have supposed that the general features of the two types of leaf-arrangement, for which no present utilities appear in the lives of the higher plants, were nevertheless useful features in former conditions of vegetable life. The more special features of these arrangements should not, from this point of view, be regarded as derived one from another, much less from the typical or unique form of the theory of Phyllotaxy. In one sense they may, indeed, be said to be derived from this form, at least some of them; yet not from it as an actually past form or progenitor, but rather from the utility which it represents in the abstract. I have, however, pointed out that another utility, shown in the simpler cyclic arrangements, has an equal claim to this spiritual paternity. The actual forms of the spiral arrangements in leaves should, therefore, be regarded as forms independently selected, and as selected on the two principles of utility, which we have considered, out of a very large variety of original forms. We have seen that even those forms which survive include almost all possible ones that could be distinguished; though the more prevalent ones are at present in the minority. We have also seen that the latter fact, and the more frequent occurrence of inferior

forms among fossil plants, are almost the only grounds on which the inductive foundation of the theory of Phyllotaxy could be regarded as well established. On these grounds, and on this foundation, I have sought by hypothesis to reconstruct the continuity of higher and lower forms in vegetable life; and through this to find the *origin* of the principal types of arrangement in leaves. The speculation lies wholly within the limits prescribed for legitimate hypothesis in science. It does not assume utilities in themselves unknown, but assumes only unobserved or unknown applications of them, and raises to the rank of essential properties relations of use, which, at first sight, appear to be only accidental ones. Attention may be claimed at the least for it as an illustration of the method by which the principle of Natural Selection is to be applied as a working hypothesis in the investigations of general physiology or physical biology.

Many features in the structure of leaves, not relating to their arrangements, fall beyond the proper province of this inquiry, but equally illustrate the relative nature of the distinction between genetic and adaptive characters. The general character common to all leaves and leaf-like organs has an obvious utility with reference to the function of nutrition. Some special modifications have the purposes of defence, as in the thorn; of mechanical support, as in the tendril; and of reproduction, as in the parts of the flower. But the vast variety of forms which leaves and the parts of flowers present do not suggest any obvious uses. On the theory of adaptation they would naturally be referred to a combination of adaptive and inherited features. A fixed proportion between the two principal tissues in a plant due to some past utility may, without being changed, become adapted to new external relations, or to new physiological conditions, through various arrangements of them in the structure of the leaf; and this would give rise to a great variety of forms. The forms of notched and sinuated leaves are referable to that process of segmentation and reduction in leaf-expansions, which we have seen to be so important a process in the derivation of the higher plants. But another principle of utility comes into play in the lives of the higher plants, similar to that which appears to be the origin of some of the more conspicuous external characters of animals, namely, what produces distinguishableness and individuation in an animal race. No doubt the laws of inheritance and Natural Selection account for much of the character of individuality in races, or for the fact that variation has a very limited range compared to the differences between species, so

far as it affects any useful quality or character. But variation, not only in animals, but also in many of the higher plants, is much more limited than these causes seem capable of accounting for. It is, apparently, as limited in respect to useless though conspicuous features as in those that are of recognized value to life. Sexual Selection, through which the characters of animals are chosen by themselves, or brought into relation to their perceptive and other psychical powers, is the cause assigned for this fact in the case of animals; that is, forms are chosen for their appearance, or for the pleasure they give to the senses. But plants have no senses, except a sense of touch; and they have no other known psychical powers. Nevertheless they present many conspicuous features of beauty to the eye, and many give forth agreeable and characteristic odors. And such characters are apparently as fixed in many of the higher plants as in animals. The theory of types and the doctrine of Final Causes regard this fixedness and individuality as ends in themselves, or else as existing for the service of some higher form of life, or ultimately even for the uses of human life. But the theory of the adaptation of every feature in a form of life to its own uses is not without resources for the explanation of these characters in plants; for though the plant has no sense to appreciate, or power to select, its own features of individuality and beauty, yet the lives of many of the higher plants are essentially dependent on such powers in insects; so that whatever character renders them attractive to insects, or distinguishable by their sight, may be said to be of use to plants for the ends of reproduction, and tends in this way to become a fixed or only slightly variable character. That this cause may have acted not only to determine definite shapes, colors, and odors in flowers, but also definite features in the foliage of plants, as the marks or signs of these, and that the value of such signs may have determined a greater degree of fixedness or constancy in the arrangements, as well as in the shapes of leaves, is an hypothesis that may be added to those we have already considered, concerning the utilities of these arrangements. This cause would tend to give prominence to those features in arrangement which are most conspicuous to the eye, namely, those of cyclic regularity and simplicity. Such an explanation of this cyclic character, or the simple and definite arrangements of leaves at short intervals in vertical lines on the stem, or the utility of this as a distinguishing character of the plant, is not inconsistent with the physiological utility in these arrangements, which I have pointed out; but the two in co-operating to the production of the same forms would illustrate a

principle in the economy of life which has a wide application, — the principle of indirect utility or correlative acquisition, dependent on ultimate laws in physical and mental natures, — through which independent utilities are realized by the same means, or the same means made serviceable to more than one distinct end. In such ultimate, underived relations of adaptation in nature, we find principles of connection and a unity of plan which cannot be referred to any accidents of history or development.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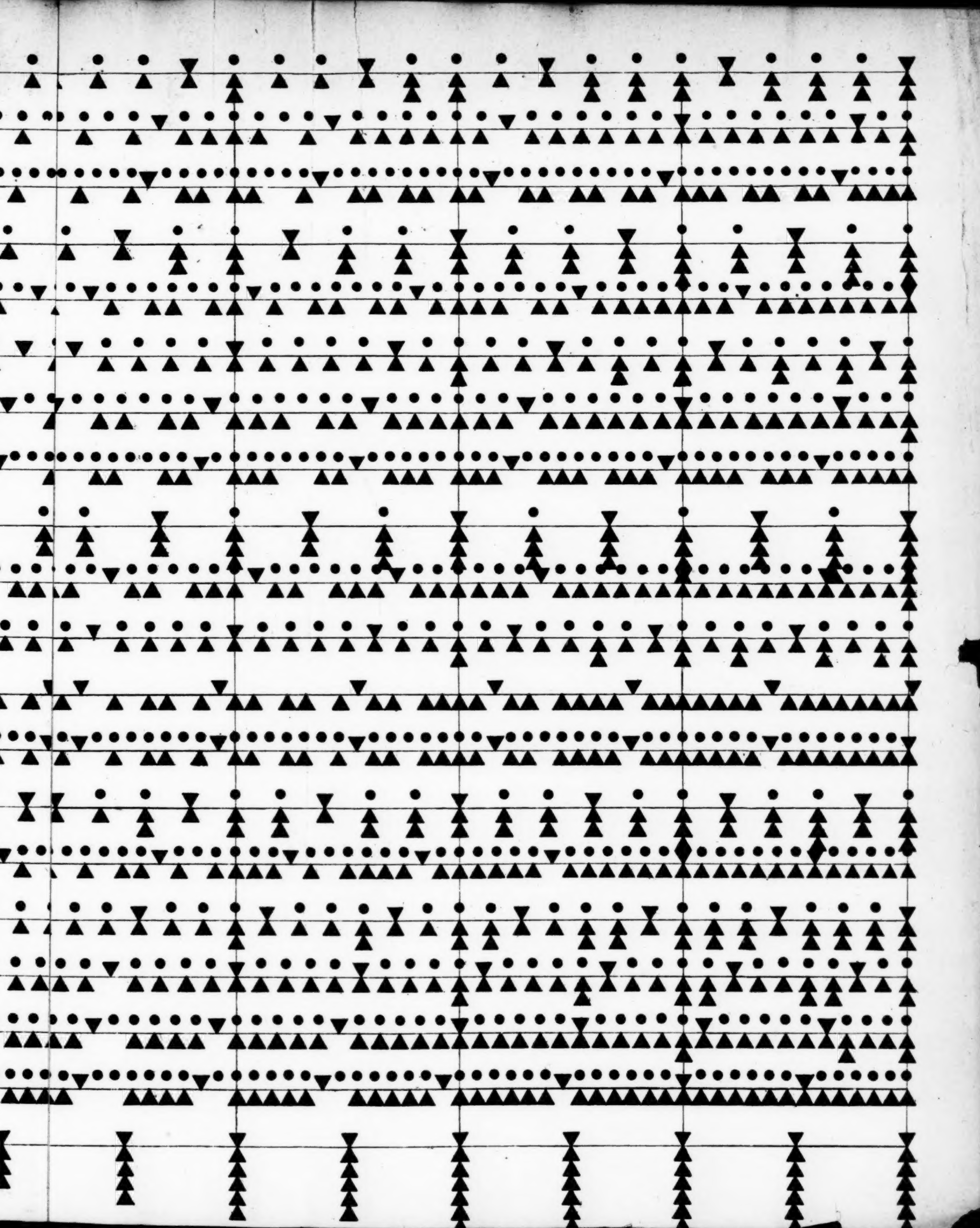

XIV.

On the Derivation of the Mass of Jupiter from the Motion of certain Asteroids.

BY G. W. HILL.

Communicated October 11, 1870.

That the discussion of the observations of certain asteroids, provided they extend over a sufficient period of time, will furnish a far more accurate value of the mass of Jupiter than can be obtained from measurements of the elongation of the satellites, or from the Jupiter perturbations of Saturn, it is the object of the present note to show. And it is to be hoped that the observers will hereafter pay particular attention to those asteroids which are best adapted for the end in question.

The magnitude of the Jupiter perturbations of an asteroid depends at once on the magnitude of the least distance of the two bodies, and the greater or less degree of approach to commensurability of the ratio of their mean motions, and also on the magnitude of the eccentricity of the asteroid's orbit.

Those asteroids which lie on the outer edge of the group, and whose mean motions are nearly double that of Jupiter, will best fulfil the two first conditions named above. For they will have inequalities of long period whose coefficients will be of the order of the first power only of the eccentricities, while all other classes of long-period inequalities are necessarily of higher orders, and hence demand longer periods in order to have their coefficients brought up to an equal magnitude.

In order to exhibit the relative value of these asteroids for the purpose in view, I have computed the terms of the lowest order in the coefficients of these inequalities of long period for all the asteroids, yet discovered, whose daily mean motion lies between the limits 550″ and 650″; and have appended herewith tables, by which the value of these terms can be readily computed for any which may hereafter be discovered between these limits.

The formulæ for computing these terms are found in the *Mécanique Céleste*, Tom. I. pp. 279–281. Here i must be put equal to 2, in the terms which involve the simple power of the eccentricities. We will employ the usual notation for the designation of the elements of orbits, and make some reductions in Laplace's formulæ for the sake of more ready computation.

If we put $\gamma = \frac{2\mu' - \mu}{\mu}$ or in Laplace's notation $\frac{2n' - n}{n}$, and recollect that we need the formulæ only for the case of an inferior perturbed by a superior planet; and moreover make

$$\gamma F^{(2)} = -H, \text{ and } \gamma G^{(2)} = J,$$

$F^{(2)}$ and $G^{(2)}$ being Laplace's symbols; we shall have

$$H = \frac{1}{1-\gamma^2}\left\{\frac{2\gamma(7-\gamma^2)}{1-\gamma}\,\alpha\,b_{\frac{1}{2}}^{(2)} + \frac{3-11\gamma+3\gamma^2-\gamma^3}{2-\gamma}\left[\alpha^2\frac{d\,b_{\frac{1}{2}}^{(2)}}{d\,\alpha} + \frac{4}{1-\gamma}\,\alpha\,b_{\frac{1}{2}}^{(2)}\right] - \gamma\,\alpha^3\frac{d^2\,b_{\frac{1}{2}}^{(2)}}{d\,\alpha^2}\right\},$$

$$J = \frac{\alpha^2}{2(1-\gamma^2)}\left\{(3-8\gamma+\gamma^2)\left[3\,\frac{b_{\frac{1}{2}}^{(1)}}{\alpha} + \frac{d\,b_{\frac{1}{2}}^{(1)}}{d\,\alpha} - 4\right] + 8\gamma\left[3\,\frac{b_{\frac{1}{2}}^{(1)}}{\alpha} - \frac{1}{4}\,\alpha\,\frac{d^2\,b_{\frac{1}{2}}^{(1)}}{d\,\alpha^2} - 3\right]\right\}.$$

If, in the next place, K and β are derived from the equations

$$K\cos(\beta - \pi) = H\sin\varphi - J\sin\varphi'\cos(\pi' - \pi),$$
$$K\sin(\beta - \pi) = \qquad\qquad - J\sin\varphi'\sin(\pi' - \pi),$$

the inequality in longitude we are computing is

$$\frac{m'}{\gamma^2}\,K\sin\left[L - 2L' + \beta\right].$$

H and J may be regarded as functions of α, and are positive between the limits corresponding to $\mu = 550''$ and $\mu = 650''$. The common logarithms of these quantities are here tabulated for every 0.001 of α between the limits above mentioned; the values of $b_{\frac{1}{2}}^{(1)}$ and $b_{\frac{1}{2}}^{(2)}$ and their differentials were obtained from Runkle's *Tables of the Coefficients of the Perturbative Function.*

α	log H.	log J.	α	log H.	log J.
0.595	0.3153369	9.871828	0.605	0.3269054	9.884214
.596	.3165277	.873131	.606	.3280187	.885370
.597	.3177113	.874420	.607	.3291236	.886511
.598	.3188875	.875695	.608	.3302199	.887636
.599	.3200561	.876956	.609	.3313075	.888745
.600	.3212173	.878202	.610	.3323864	.889836
.601	.3223707	.879434	.611	.3334562	.890910
.602	.3235163	.880652	.612	.3345169	.891967
.603	.3246540	.881855	.613	.3355683	.893007
0.604	0.3257838	9.883043	0.614	0.3366103	9.894030

a	log H.	log J.	a	log H.	log J.
0.615	0.3376427	9.895036	0.638	0.3582866	9.912409
.616	.3386652	.896022	.639	.3590269	.912874
.617	.3396777	.896990	.640	.3597519	.913310
.618	.3406801	.897939	.641	.3604614	.913718
.619	.3416723	.898869	.642	.3611550	.914097
.620	.3426539	.899780	.643	.3618323	.914446
.621	.3436248	.900671	.644	.3624928	.914764
.622	.3445848	.901542	.645	.3631366	.915051
.623	.3455337	.902392	.646	.3637632	.915306
.624	.3464714	.903221	.647	.3643722	.915528
.625	.3473975	.904028	.648	.3649632	.915717
.626	.3483119	.904814	.649	.3655358	.915871
.627	.3492144	.905578	.650	.3660897	.915991
.628	.3501047	.906320	.651	.3666246	.916076
.629	.3509827	.907040	.652	.3671400	.916125
.630	.3518480	.907736	.653	.3676354	.916136
.631	.3527005	.908408	.654	.3681103	.916108
.632	.3535399	.909056	.655	.3685644	.916040
.633	.3543659	.909679	.656	.3689972	.915933
.634	.3551782	.910277	.657	.3694082	.915785
.635	.3559767	.910850	.658	.3697969	.915595
.636	.3567612	.911396	.659	.3701628	.915362
0.637	0.3575313	9.911916	0.660	0.3705053	9.915085

The values of the elements of Jupiter's orbit for the epoch 1850.0 which we will use are

$$m' = \tfrac{1}{1050},$$
$$\mu' = 299.1286,$$
$$\log a' = 0.7162372,$$
$$\varphi' = 2^\circ\ 45'\ 54''.55,$$
$$\pi' = 11^\circ\ 55'\ 2''.$$

The values of the corresponding elements of as many of the asteroids as lie between the limits above mentioned are contained in the following table. The longitudes of the perihelia are referred to the mean equinox of 1850.0.

	μ	log a	φ	π
Hygea	634″.3118	0.4984692	5° 44′ 56″.4	234° 58′ 40″.6
Themis	636.7634	0.4973523	6 42 52.9	139 56 11.2
Euphrosyne	633.8508	0.4988680	12 44 10.3	93 27 51.5
Doris	647.1295	0.4926769	4 23 42.9	74 10 11.3
Pales	655.6209	0.4889025	13 43 18.3	32 3 13.1
Europa	650.0877	0.4913564	5 49 14.3	101 45 37.6
Mnemosyne	632.6897	0.4992106	5 58 17.1	52 58 47.8
Erato	640.8591	0.4954961	9 46 4.3	33 55 38.0
Cybele	560.8775	0.5340920	6 54 36.4	258 11 24.3
Freia	569.0505	0.5299038	10 49 12.0	93 2 36.6
Semele	652.9848	0.4900690	11 49 36.5	28 25 39.1
Sylvia	543.5800	0.5431620	4 39 22.6	337 8 6.1
Antiope	632.3591	0.4993618	11 39 2.7	293 49 3.5

The expression of the inequalities, and the length of their periods which result from the substitution of these values of the elements in the formulæ, are

Hygea	$14676''.2 \sin [L - 2L' + 228^\circ\ 58'\ 1''.4]$,	97.96 years.
Themis	$14606.2 \sin [L - 2L' + 146\ 4\ 4.5]$,	91.72 "
Euphrosyne	$28996.5 \sin [L - 2L' + 97\ 58\ 58.4]$,	99.23 "
Doris	$5086.7 \sin [L - 2L' + 85\ 41\ 49.4]$,	72.27 "
Pales	$11639.2 \sin [L - 2L' + 33\ 36\ 12.6]$,	61.57 "
Europa	$6584.4 \sin [L - 2L' + 111\ 29\ 19.2]$,	68.14 "
Mnemosyne	$12956.0 \sin [L - 2L' + 60\ 9\ 1.9]$,	102.58 "
Erato	$13654.9 \sin [L - 2L' + 36\ 21\ 16.9]$,	82.91 "
Cybele	$13145.4 \sin [L - 2L' + 251\ 13\ 31.6]$,	94.49 "
Freia	$32243.5 \sin [L - 2L' + 98\ 15\ 25.5]$,	120.93 "
Semele	$10860.7 \sin [L - 2L' + 29\ 55\ 45.1]$,	64.54 "
Antiope	$28567.8 \sin [L - 2L' + 288\ 44\ 31.6]$,	103.57 "

These expressions can be regarded as rough approximations only to the actual values of these inequalities, since all terms of the third and higher orders with respect to the eccentricities and inclinations, and of the second and higher orders with respect to the disturbing masses, have been neglected. Yet they are sufficiently exact to show the order of magnitude of the Jupiter perturbations of the asteroids in question.

The effect of these inequalities at the time of opposition will be magnified in the proportion roughly of a to $a - 1$. Thus in the case of Freia, the determination of the mass of Jupiter will depend on the observation of an arc of $12^\circ.7$.

XV.

The History of Balanoglossus and Tornaria.

BY ALEXANDER AGASSIZ.

Communicated December 10, 1872.

Among the many pelagic invertebrate larvæ discovered by Müller, of which the development remained long unknown, Tornaria is one of the most interesting. Previous to the preliminary notice given by Metschnikoff[1] in 1869, it had been taken for granted that Tornaria was the larva of some genus of star-fish. Discovered by Müller[2] in 1848, Tornaria was subsequently studied by Krohn[3] in 1853. Müller gave a very full account of the younger stages; Krohn's paper, though not adding materially to our knowledge of this larva, complemented the descriptions of Müller in a few particulars. Nothing further was written on the subject till 1866, when I published a short paper on some points of the development of Tornaria[4] which had not been noticed in the young stages described by Müller. This additional information seemed to settle definitely the star-fish nature of the larva, and I accepted unhesitatingly, as well as my predecessors, Müller's views of the affinities of Tornaria. It was only subsequent to the publication of my paper that Dr. Fritz Müller[5] called my attention to a so-called heart, discovered by him in a species of Tornaria from the shores of Desterro. This heart, situated at the base of the canal leading from the water-system to the dorsal pore, is the only organ of Tornaria throwing some doubt on the then generally accepted systematic position of the larva, and I began seriously to doubt the correctness of the homologies I had carried out between the arms of Brachiolaria edged with vibratile cilia and dotted with pigment-cells and the similar ciliated bands of Tornaria.

Müller, Krohn, and myself attached altogether too little value to the large promi-

[1] Metschnikoff, El. Göttinger Nachrichten, 1869, No. 15, p. 287.

[2] Müller, J. Ueber die Larven u. die Metam. d. Echinod. Abhandl. II., Berlin, 1849; Ueber die Larven u. die Metam. d. Holoth. u. Aster. Abhandl. III., Berlin, 1850.

[3] Krohn, A. Müller's Archiv für Anat. u. Phys., 1853.

[4] Agassiz, Alex. Tornaria, Ann. Lyceum Nat. Hist., VIII., New York, 1866.

[5] Müller, F. In Keferstein's Bericht for 1867.

nent band of vibratile cilia separating the body of Tornaria into two such unequal portions. The presence of a somewhat similar circular band in the larvæ of Holothurians and of Comatulae seemed a powerful argument, in addition to other important structural evidence, in favor of the echinodermoid character of the larva. The analogy between Tornaria and the earlier stages of Brachiolaria is so great that in my paper on the Embryology of the Star-fish I called one of the early stages of Bipinnaria the "Tornaria stage." This striking resemblance is, however, only an analogy, as a memoir published by Metschnikoff in 1870[1] leaves no doubt that Tornaria is the larva of an Annelid and has nothing to do with the Star-fishes.

The exceptionable character of the development of Tornaria made it desirable that the observations of Metschnikoff should be repeated, and the genus of Annelids, of which it is the larva, definitively determined. Metschnikoff threw out the hint that it was, if not Balanoglossus, at least a genus most closely allied to it, — a view which is fully confirmed by the observations I have made of Tornaria, and of a species of Balanoglossus from the coast of New England, connecting the young Annelid raised directly from Tornaria with very small specimens of Balanoglossus found buried in the sand. Metschnikoff certainly showed great sagacity in recognizing as the larva of Balanoglossus the young Annelid he first found pelagic in Naples;[2] but the discrepancy between the two stages then known is so great that his view could hardly be accepted without more tangible proof. The young Annelids which I succeeded in raising directly from Tornaria are considerably older than any observed by Metschnikoff, and they are at the same time but slightly younger than very young specimens of the Balanoglossus found living upon our beaches.

Huxley,[3] in his report upon the researches of Müller, proposed to unite the Echinoderms with the Articulates; but, as he based his opinion entirely upon the figures of Müller, and not upon original investigations, his conclusion, however ingenious and original it may be, and based upon very striking analogical resemblances, is nothing but a hint thrown out for the benefit of investigators. Müller[4] himself regarded the analysis of his paper as most ingenious, but by no means as conclusive; although at that time he had already discovered the peculiar mode of development of some Planarians apparently most closely allied in its general features with the plan of develop-

[1] METSCHNIKOFF, EL. Untersuchungen ueber die Metamorphose einiger Seethiere, Zeitschr. f. Wiss. Zool., 1870, p. 131.

[2] METSCHNIKOFF, EL. Die Larve v. Balanoglossus, Müll. Arch. 1866, p. 592.

[3] HUXLEY, T. H. Report upon the Researches of Müller, Ann. Mag. N. H., VIII., 1851.

[4] MÜLLER, J. Ueber den Allgemeinen Plan in der Entwickelung d. Echinodermen, 1853, p. 19. See also AGASSIZ, ALEX. Embryology of the Starfish, 1864.

ment of Echinoderms. The majority of English naturalists have adopted Huxley's views without further investigation.

Tornaria presents the startling anomaly of an apparently genuine Echinoderm larva, at least till lately so considered by all writers on the subject, developing into an Annelid, and seems at first sight a conclusive proof of the views entertained by Huxley; but, as I think I shall show in the description of the development of Tornaria, the position taken by Huxley is not strengthened, and the gap left between the mode of development of Planarians and other Annelids, as compared with the development of Echinoderms, is as great as ever, in spite of the very striking analogy in the mode of development of some Echinoderms (especially Holothurians and Comatulae) with that of Nemertians, as shown by Müller[1] and Metschnikoff.[2] The history of Tornaria tends, on the contrary, to show a much closer relationship between the Nemertians and the Annelids proper than is generally credited, the development and anatomy of Balanoglossus showing it to be closely allied to Terebellidae, Clymenidae, and allied Annelids, as already suggested by Metschnikoff and Kowalevsky. And, now that we know its ultimate development, the larva presents points of resemblance to well-known Annelid larvæ (Lovén's Larva) which are apparent enough when demonstrated, but so completely hidden by the more prominent pseudo-echinodermal features as readily to have escaped notice.

The presence of the large water-system, riding with its spurs upon the anterior extremity of the alimentary canal, and connecting with the exterior by means of a canal and a dorsal pore, exactly as in the larvæ of the Echinidae, Star-fishes, Ophurians, and other Echinoderms, seems at first sight an overwhelming proof of the position taken by Huxley. But, as I have already shown in my Embryology of the Star-fish,[3] Huxley, misled by the names given by Müller to some of these larvæ ("Wurmförmige Larven"), has revived the old opinion of Oken, and associated the Echinoderms with the Articulates. The hypothetical form to which Huxley reduces these larvæ to make his comparisons and to draw his inferences is one which has never been observed, and as far as we now know does not exist. The larvæ of all the principal types have been described by Müller, Krohn, Thomson, Metschnikoff, and myself. The development of the water-system from the digestive cavity has not been traced in Tornaria; while the lappets of Tornaria, which have nothing to do with the water-

[1] Müller, J. Ueber eine eigenthümliche Wurmlarve ...... Müll. Arch., 1850, p. 485.

[2] Metschnikoff, El. Studien üb. d. Entwickel. der Echinod. u. Nemertinen, Mém. Acad. St. Petersb., 1869, XIV. No. 8.

[3] Agassiz, Alex. Embryology of the Star-fish, 1864, p. 60.

system are developed from the digestive as diverticula. So that the water-system of Tornaria, in spite of its dorsal pore, can in no way be homologized with the water-system of Echinoderms, even if, in addition to the different mode of its development, the presence of a heart and of a muscular band supporting it did not show that we had to deal with an organ which has no homologue in any of the numerous Echinodermoid larvæ thus far observed.

As far as we know the Embryology of the Planarians, of which the development is somewhat analogous to that of Echinoderms, as known from the observations of Müller and Metschnikoff, we still have between the two modes of development radical differences. In all Echinoderms, without exception, the young Echinoderm is developed upon the water-system of the Pluteus as a bud, as it were, and gradually encroaches upon the scaffolding which has supported it, and finally resorbs the whole Pluteus within itself. No such mode of transformation exists in any known Planarian or other Annelid larva. In this group we find without exception that the transformations consist of a very gradual change of one stage into another, and that by an elongation or contraction of the different parts of the larva at its various stages of growth, and by a gradual modification of the topography of the organs, the larva passes little by little into its adult stage, — a condition of things entirely fulfilled by the development of Tornaria into Balanoglossus, which, as far as the last phase of its growth, the Balanoglossus, certainly shows nothing bearing any affinity to the Echinodermoid mode of budding upon the water-system of the Pluteus.

We possess fortunately an admirable anatomy of Balanoglossus by Kowalevsky,[1] who has rediscovered, as it were, the Balanoglossus first figured by Delle Chiaje,[2] and about which nothing of any value, except the short notices of Keferstein[3] and of Quatrefages,[4] had been written since Delle Chiaje's time. I shall therefore, in referring to Kowalevsky's memoir, be able, from the study of the young immediately following the Tornaria stage and of younger specimens of Balanoglossus than those Kowalevsky has investigated, to give an explanation of the nature and function of many of the organs of this interesting animal which could not be explained merely from the study of the adult.

As is well known from Müller's figures and descriptions of the youngest stages of Tornaria, the large circular belt of vibratile cilia is only developed in the older

[1] Kowalevsky. Anatomie du Balanoglossus, Mém. Acad. St. Pet., 1866, X. No. 3.

[2] Delle Chiaje. Mem. sull. Stor. e Not. degl. Anim. s. Verteb., Pl. LVI., f. 36.

[3] Keferstein. Zeitschrift f. Wiss. Zool., 1863, XII. 91.

[4] Quatrefages. Ann. Sc. Nat., 1846, VI. p. 184.

stages; in the youngest specimens observed, the muscular band connecting the extremity of the water-system to the base of the eye-spots is present. These two features are not known in any other Echinoderm larva, and were sufficient to have shown the possibility of Tornaria proving the larva of an Annelid. In the well-known Annelid larva of Lovén[1] (which, according to my observations,[2] develops into a Nemertian, while Schneider[3] thinks it is the larva of Polygordius) we find the same muscular band starting from the base of the eye-specks, and in older stages the circular anal belt of vibratile cilia. Owing, however, to the great number of rings below the anterior part of the larva, and the presence of a prominent anterior ring of vibratile cilia above the mouth, the close resemblance between Lovén's Larva and Tornaria is not so striking at first sight as it really is. In Tornaria the prominent belt of large vibratile cilia appears at a comparatively late period, yet long before any of the rings of the posterior part of Balanoglossus; while in all Annelid larvæ which have a similar anal belt of vibratile cilia this belt is the earliest to make its appearance, long before there is any trace of the formation of the rings of the posterior part of the body. I would refer to the figures of Annelid larvæ given by Milne-Edwards,[4] Sars,[5] Busch,[6] Müller,[7] Claparède,[8] Max Müller,[9] Krohn,[10] Metschnikoff,[11] and myself,[12] for comparison with Tornaria.

The figures of Tornaria given in this paper precede immediately its transformation into young Balanoglossus; for the earlier stages I would refer to the figures given by Müller, Metschnikoff, and myself. The oldest stages of our Tornaria differ materially from those described by Metschnikoff. He speaks of a second smaller anal band of vibratile cilia, between the broad band and the anal opening. I have not found this second band in any specimens of our Tornaria, and in our species this band does not exist. Metschnikoff represents the "Wurstförmige Körper" as appendages of the intestine, and as different from what he has called the lateral plates, or appendages of the

[1] LOVÉN. Jakttagelse öfser Metamorfos hos en Annelid, K. Vet. Akad. Handl. Stockholm, 1840, p. 93.

[2] AGASSIZ, ALEX. On the Young Stages of a few Annelids, Ann. Lyceum Nat. Hist., New York, 1866, p. 303.

[3] SCHNEIDER. Bau u. Entwickelung v. Polygordius, Müll. Arch., 1868, p. 51.

[4] MILNE-EDWARDS. Ann. Sc. Nat., III., 1845; Ann. Sc. Nat., VIII., 1847.

[5] SARS. Archiv f. Naturg., I., 1845.

[6] BUSCH. Bemerk. ueber Anat. u. Entwickel. einig. Wirbellos. Thiere, 1851; Müll. Arch., 1847.

[7] MÜLLER. Sitzungb. Akad. Berlin, 1851, 48.

[8] CLAPARÈDE. Beob. über Anat. u. Entwickel an d. Küste v Normandie, 1863.

[9] MAX MÜLLER. Müller's Archiv, 1850, 1855.

[10] KROHN, A. Müller's Archiv, 1851; KROHN u. SCHNEIDER. Müll. Arch, 1867, Annelid larven mit porös. Hüllen.

[11] CLAPARÈDE u. METSCHNIKOFF. Zeitsch. f. Wiss. Zool, 1868.

[12] AGASSIZ, ALEX. Young Stages of Annelids, g, a.

stomach. In our Tornaria these appendages are both of the nature of lateral plates, and are situated above the intestine. They are formed, as I have shown in a former paper on Tornaria, as diverticula of the stomach; and I have not seen, as Metschnikoff and Keferstein seem to think, the first formation of the water-system from the œsophagus. I merely took it for granted that the odd part of the water-system of Tornaria was analogous to the odd branch of the Brachiolarian water-system, and that the lateral plates corresponded to the independent branches found in Brachiolaria. The function of these lateral plates remains problematical: they do not become connected, as I formerly supposed, with the odd water-system; and their subsequent changes in the young Balanoglossus I have not been able to follow. The lateral plates are hollow at first, but eventually their walls become closely pressed together, forming lappets of considerable size, more or less banded and striated, attached along the upper edge and standing off at an angle, like parts of two inverted funnels, from the walls of the stomach, placed one within the other.

In our Tornaria the water-system is more distant from the dorsal side than in the Mediterranean species, the dorsal pore connecting with it by means of a long tube inserted on one side of the water-system: the dorsal pore also is quite eccentric, and not situated along the median dorsal line, as figured in Metschnikoff's Tornaria. The walls of the water-system are quite stout: the heart, first seen by Fritz Müller, consists of an independent vesicle, situated immediately adjoining the opening of the dorsal canal into the water-system in a sort of depression of the posterior portion of the water-system. In older stages, the heart, which can be distinctly seen to pulsate quite slowly in the earliest stages, is surrounded by an opaque membrane which does not participate in its pulsations. From the rounded posterior extremity of the water-system project two short sharp spurs riding upon the upper part of the stomach: the water-system is regularly arched laterally, slightly conical at the anterior extremity where it is connected with the anterior part of Tornaria (the base of the eye-specks) by a broad, flat, powerful muscular band which by its contraction can draw in the whole of the anterior part of Tornaria towards the mouth. I shall call anterior, dorsal, ventral, posterior, those parts of the Tornaria which correspond ultimately to the anterior, dorsal, ventral, or posterior parts of Balanoglossus.

The natural position of Tornaria in the water, while moving, is usually with the eye-specks uppermost. They revolve quite rapidly upon their longitudinal axis, and at the same time, inclining this axis, advance by a motion of translation, or revolve upon either of the extremities as a fulcrum. Previous to the transformation of Tornaria it is quite transparent: the brilliant carmine, violet, or yellow pigment-

spots are closely crowded along the broad belt of anal vibratile cilia, as well as smaller spots on the longitudinal bands of smaller cilia. The eye-specks are black and extremely prominent. The large and powerful cilia of the broad anal belt move comparatively slowly, more like the cilia of the embryos of Molluscs, as has already been observed by Müller.

Metschnikoff, in the oldest stage of Tornaria observed by him, speaks of two round bag-shaped diverticula of the œsophagus, having an internal structure similar to it, which with advancing age only increase in size. In our Tornaria the mode of formation of the pouches has been traced, and the gills (I will call them at once by their true name, from their function in the adult Balanoglossus) have been traced from their first appearance, a single pair, till there were four such pairs in the Tornaria; this is previous to any other changes. The gills are at first simply slight folds, one on each side of the dorsal part of the œsophagus: the folds gradually change into loops, then we have a series of folds, and subsequently a series of four loops, and finally, when seen from the dorsal side, the loops appear closed, forming a set of four funnels on each side, opening into the œsophagus, which from above look like round rings adjoining one another. These changes are readily followed in the figures illustrating this part of the development.

The passage of Tornaria into young Balanoglossus is very sudden, taking place in a few hours; but, unlike the transition from the Pluteus into the Echinoderm, there is no resorbition of any portion of the larva. The whole transformation consists simply in a lateral contraction of certain parts and an elongation of others, but which is sufficient, with the disappearance of the longitudinal bands of cilia, to alter completely the general aspect of the larva. The first trace of the change is the opaqueness of the larva: it loses its transparency, and somewhat its activity; the whole of the stomach is drawn down towards the intestine; the œsophagus becomes greatly elongated; and the transverse oral vibratile band is now above the junction of the stomach with the œsophagus. The œsophagus is lengthened so much that the water-system no longer rides upon the anterior part of the stomach, but is placed immediately above the opening of the mouth. The intestine has become depressed into a triangular pouch, and the stomach forms a few folds to accommodate itself to its present circumscribed space: these folds are the first trace of convolutions similar to those of the adult. During this process of lengthening of the posterior part of Tornaria, the anterior part is also greatly elongated, assuming somewhat the shape of an elliptical proboscis. In front of the circular band of cilia are plainly seen the four pairs of gills on each side of the œsophagus: this is now

divided into two portions, — one the mouth-opening of the œsophagus, the other the convoluted gill-bearing part. The pigment-cells of the longitudinal undulating bands of vibratile cilia, as well as the bands themselves, have almost disappeared. The eye-specks alone are still extremely prominent; the muscular band attached to the anterior part of the water-system has disappeared, but powerful longitudinal muscular bands as well as less marked transverse bands appear on the proboscis. The walls of the water-system have become contracted; the water-system occupies a comparatively much smaller space in the proboscis of the young Balanoglossus than in the anterior part of Tornaria. The heart can now no longer be distinguished through the opaque walls of the contracted water-system. The opening of the dorsal pore is plainly seen near the base of the proboscis.

In somewhat older stages all trace of the undulating longitudinal bands of vibratile cilia have disappeared; minute cilia, no longer arranged in bands, cover uniformly the walls of the body and proboscis. The young Balanoglossus can now be separated into three well-marked regions, — the proboscis, the collar, and the short, triangular abdominal portion. The elongation of all the parts of the Tornaria behind the proboscis is quite marked, and the distance between the collar and the anal vibratile band has become considerable. The proboscis grows more and more elongated, the body comparatively narrow and slender, while the collar-segment is better distinguished from the segment above the anal vibratile band. The part of the œsophagus from which the gills have developed lengthens more rapidly during the last stages than any other part of the young Balanoglossus. The mouth is already, as in the adult, a broad circular opening immediately under the base of the proboscis, leading into an open cavity becoming strictly an œsophagus only at the point where the gills have commenced to develop. The walls of the little worm become more and more opaque with advancing age: near the collar they generate already in the youngest stages a moderate quantity of transparent mucus, which is so abundantly and rapidly generated by the adult Balanoglossus. The few irregularly scattered pigment-cells still found upon the proboscis are the remnants of the undulating vibratile bands. In the broad anal band the cilia have lost much of their energy, vibrating but feebly and extremely slowly. The little worm no longer swims freely about, as it does in the earliest stage of its Balanoglossus existence (slightly older than the pelagic one caught by Metschnikoff), but creeps rapidly over the bottom by means of its proboscis, which acts as a sort of propeller, taking in water at the minute opening of the anterior extremity of the proboscis, and expelling it through an opening on its ventral side immediately in front of the

mouth. The eyes, during these changes, have gradually lost their prominence, becoming somewhat indistinct, and are at last completely absorbed in the walls of the proboscis in somewhat older stages. The cavity of the body is formed by two extremely thin walls extending from the base of the collar to the intestine, developed probably by the extension of the problematic bodies of the stomach, though I have never succeeded, owing to the opaqueness of the outer walls, in actually tracing this transition.

When seen from below or above, two very distinct vessels — one dorsal, the other ventral — can be traced, extending from the base of the collar to the intestine. The vessels are apparently disconnected, being pointed at the two extremities. A circular canal is formed round the œsophagus by the two forks of the water-system which saddled the upper part of the anterior portion of the stomach in Tornaria. It is connected with the heart, and opens outwards through the dorsal pore, but seems to have no connecting link as yet with the outer vessels, though in older stages this connection apparently exists.

The gills, at first circular openings leading out from the œsophagus, become gradually elliptical, then the walls nearest the middle line send out a loop, and form the first trace of the complicated folds of the gills: they do not open externally, — at least in the earlier stages raised directly from Tornaria I could not trace the opening so clearly seen in the Mediterranean species by Metschnikoff; it was only in much more advanced stages of Balanoglossus that the outward opening was discovered. Metschnikoff figures the gills as two large, funnel-shaped bodies opening on each side of a deep dorsal furrow, which is also wanting in our young Balanoglossus. I only succeeded in finding the skeleton observed by Metschnikoff as a minute granular plate in very small specimens of Balanoglossus soon after their transformation from Tornaria. The only trace of the skeleton at the base of the proboscis of Tornaria consists in an accumulation of granules similar to the granular chord lying between the chitine supports of the gills in adult specimens.

The oldest stage I succeeded in raising directly from Tornaria is a good deal older than any described by Metschnikoff, so that, unless there is an error in his observations, the formation of the gills in the European Balanoglossus must take place very slowly, as his young Balanoglossus only showed one pair of gills in the oldest stage he figures; our species commencing at once with four pairs of rudimentary gills developed almost simultaneously already during the Tornaria stage.

The smallest specimens of Balanoglossus dug up in the sand, although considerably larger than those raised directly from Tornaria, are yet sufficiently different from

the adult to leave no doubt of its identity with the Balanoglossus raised from Tornaria. The differences between them are simply differences of size and quantity. The collar is more distinct, the number of gills greater, and the convolutions of the intestine more numerous; the anal extremity has lost completely its circle of vibratile cilia. The walls of the body are scarcely more opaque than in the young Balanoglossus raised directly from Tornaria, with the exception of the proboscis, in which the muscular bands, both transverse and longitudinal, are more fully developed. The water-system and the heart could only be seen by compression; the latter, closely contracted, appeared like a solid ball at the base of the proboscis within its cavity. The anterior opening of the cavity of the proboscis is very distinct: the posterior opening is situated on the ventral side, immediately in advance of the mouth; it is quite a long, slender slit. The so-called skeleton of the proboscis is very different from that figured by Metschnikoff and Kowalevsky; it is pointed anteriorly, with two branches curving towards the ventral side, pointing towards the posterior base of the proboscis. It consists of two parts, — one apparently chitine, semi-transparent, forming the two bent forks arching towards the mouth and the other the main shaft from which the forks arise. At the head of the shaft there is a flat saucer-shaped expansion, of granular character, quite flexible, a sort of support for the base of the limber proboscis. From the base of the proboscidal skeleton there extends a granular chord as far as the extremity of the gills, to the sides of which the three branched, semi-transparent supports of the gill-folds are attached; this forms an internal skeleton in the anterior part of Balanoglossus, supporting the gill-folds, which is without parallel in the Invertebrates, unless we except some of the Ascidians. The nature and function of this skeleton is not known; the portion at the base of the proboscis may be homologized to the operculum of some of the Annelids, with which I am inclined to associate Balanoglossus. The gills are numerous, in many pairs, along the dorsal line of the anterior part, but as yet forming only slightly complicated folds, not to be compared with the complicated folds of the gills of the adult; and thus the resemblance to the young Balanoglossus raised from Tornaria is still quite complete. At the terminal extremity of the gill-system we find young gills developing from the œsophagus exactly as we have seen them develop in Tornaria.

With this preface I can now describe our common species, which differs in some very important points from the two species of Balanoglossus distinguished by Kowalevsky. I shall follow his nomenclature, and as I have nothing of any great importance to add to his exhaustive anatomy of the adult, will only compare our species with

his two, B. clavigerus, Dell. Ch., and B. minutus, Kow. I shall hereafter speak of our species as Balanoglossus Kowalevskii. The New England Balanoglossus is found at low-water mark, buried in the sand (only in the cleanest sand-beaches) to a depth of about twelve or fourteen inches. It is readily tracked on the surface by the peculiar elliptical coils of sand which are thrown out at the top of the hole. The hole in which they live is lined by a thick mucous layer, forming a sort of sheath of considerable diameter, in which the worm must evidently be able to move up and down with considerable ease. Owing to the extreme tenuity of the walls of the posterior part of the body, and its great length in adult specimens, it is quite difficult to obtain complete full-grown specimens; but of the smaller sizes, where the posterior part of the body behind the gills has not yet taken a great development, it is quite easy to obtain whole specimens.

The proboscis is elongate, slightly pyriform, somewhat flattened, rounded, or pointed at the anterior extremity; it is of a pinkish-yellow color. The proboscis is attached to the upper part (dorsal) of the collar; it is rounded posteriorly, slightly indented on the median dorsal line. Seen in profile, its base of attachment is found to be quite slender; the body of the proboscis is bevelled anteriorly; the space between the lower part of the proboscis and the cavity of the collar is occupied on the base of the proboscis by the avenue leading to the mouth, a large elliptical opening capable of little expansion or contraction. The whole proboscis is occupied by a cavity opening anteriorly by a small orifice; a second opening, subsequently formed, is placed on the lower surface of the proboscis immediately in front of the mouth. The proboscis, as has already been stated by Kowalevsky, is the main organ of locomotion: water and sand are taken in at one extremity and forced out at the other, the animal moving the rest of the body by drawing it after the proboscis, which thus acts as a kind of sucker. The capacity for motion of the body situated behind the collar is limited to simply twisting and gliding, while the moving force is applied by the proboscis and collar. The proboscis is extremely variable in outline; it is capable of great expansion and contraction, the whole proboscis being lined with longitudinal and transverse muscles, which make their appearance in the earliest stages of the young Balanoglossus; the longitudinal muscles are by far the most numerous; concentric muscles are found round the two openings of the proboscis. The cavity of the proboscis has no connection whatever with the cavity of the body into which the mouth opens, as has been correctly maintained by Kowalevsky, in opposition to the statements of Delle Chiaje and Keferstein. On the dorsal side, at the base of the proboscis, is situated the skeleton of which I have spoken above, which

probably serves as a support or fulcrum for the slender base by which the proboscis is attached to the collar. The shape of the proboscis is slightly different from that of B. minutus, being pointed anteriorly and not truncated.

Immediately behind the proboscis comes the collar, a part of the body somewhat similar to the collar of Sabella, Clymene, and allied Annelids; the anterior extremity is deeply hollowed out, the edges projecting so as to conceal the slender connecting stem of the base of the proboscis. The whole collar (as well as the proboscis, but in a less degree) is filled with glands from which an immense quantity of white transparent mucus is constantly and rapidly generated. The color of the collar is somewhat darker than that of the proboscis; it flares out both at the anterior and posterior extremity, and is slightly corrugated along the median dorsal part. The broadly open mouth leads into what corresponds to the œsophagus of Tornaria. That part of the œsophagus which is flanked by the gills is about twice as long as the proboscis, the latter being from five to six times as long as the collar.

Following the œsophagus we come to the convolutions of the former stomach, which have taken an immense development, the intestine proper remaining very much as it is in the younger stages, and occupying only a small portion of the posterior extremity of the body. The alimentary canal is connected dorsally and ventrally with the outer walls of the body along the median line, occupied by folds which are strongly ciliated and send out small branches through the windings of the alimentary canal, on each side of the large dorsal and ventral vessels, as described by Kowalevsky, the branches of the two median vessels connecting them laterally. Beyond the gills the alimentary canal is not as intimately connected with the walls as in the anterior part. The alimentary canal becomes differentiated and forms diverticula, — small, narrow folds which eventually connect with the main alimentary canal only by a narrow slit; these diverticula draw down the adjacent outer wall, forming a large number of small, elongate, narrow folds of a greenish color; these folds, lined with whitish cells, give that part of the body a most peculiar appearance. Kowalevsky calls these diverticula the liver. The limitation of the liver organs is not as well marked in the American Balanoglossus as Kowalevsky describes it in the Mediterranean species, the folds become more and more distant towards the posterior extremity, and extend far towards the anal end in the cylindrical portion of the termination of the body. Behind the liver the alimentary canal is simply formed of more or less closely packed convolutions, and it becomes almost straight near the anal extremity. In Balanoglossus Kowalevskii, immediately behind the collar, along the median dorsal line are situated the gills; they grow gradually smaller towards the pos-

terior extremity, and disappear entirely at a considerable distance behind the collar on the dorsal side. The median dorsal part of the body beyond the gills is somewhat flattened, becoming more so for the greater part of the distance occupied by the liver, the position of which is somewhat different from that which it occupies in B. clavigerus, where it extends only a short distance below the gills towards the collar; the lateral folds do not take a great development, and do not unite dorsally behind the collar, as is the case in B. minutus. The whole surface of body, as has been mentioned by previous writers on Balanoglossus, is thickly covered by minute cilia.

The gills, as I have shown from their mode of formation, consist originally of folds of the œsophagus, forming subsequently elliptical, funnel-shaped diverticula from it; from the dorsal sides of these, new folds are formed, dividing the funnels into two; and so additional folds are formed, increasing greatly the complexity of the gills, but never, in our species, forming the remarkable system of network described by Kowalevsky; nor have I been able to make out any special order in the mode of formation of the folds of the gills. Their mode of opening externally is quite different from that described by Kowalevsky. Near the dorsal median line we find a series of inverted pouches which the slightest compression will throw out like an inverted finger of a glove, forming a flat cylinder opening into a narrow slit next to the dorsal vessel, through which the gills communicate externally. The inner walls of this cylinder are strongly ciliated. The supports of the gill-folds are quite simple; the folds of the gills are supported by three prongs starting from a common curved base and attached to the more or less granular chord extending between them along the dorsal line; there is nothing to be seen of the complicated skeleton support of the gills figured by Kowalevsky for B. minutus. It is immediately on the edge of the folds that the most powerful vibratile cilia are found; owing to the increased lengthening of the central and lateral folds of the gills, they occupy a greater part of the gill-opening, and, becoming laterally crowded, appear like numerous folds placed side by side, while in reality we see only the edges of the folds, and of their skeleton supports in profile.

The genital organs occupy the same position as described by Kowalevsky for the Neapolitan species, between the liver and the anterior part of the body, forming singular bags on either side of the median line. Only a few eggs were found, and all attempts to raise them by artificial fecundation failed completely. Nor did I succeed, while digging over a large extent of ground occupied by Balanoglossus, in finding any trace of strings of eggs, as Kowalevsky suggested they might be found. The posterior part of the body is quite cylindrical, the alimentary canal having but

few convolutions. The anus terminates the short intestine as a broad opening; the edges of the walls are lined by powerful vibratile cilia. On the lower side we find on each side of the median-ventral vessel a series of small folds closely packed, extending a short distance from the median line, forming a flat, corrugated band, gradually becoming narrower towards the anal extremity, extending from the collar to the posterior extremity; this band is of a light dirty-pink color, and, flanked as it is by the dark-green convolutions of the alimentary canal, is a prominent feature of the ventral side. Balanoglossus can easily be kept alive; I have kept them several weeks in confinement in jars, of which the bottom was covered with sand. The proboscis is kept continually expanding and contracting, and the sides of the body, especially of the posterior extremity, are in incessant motion.

Kowalevsky is inclined to associate Balanoglossus with the Annelids proper, and not with the Nemertians, in spite of its proboscis and its want of bristles and other appendages. This remarkable type recalls the Tunicates, from the nature of the gills and their mode of formation. It has, like Echinoderms, a ring canal; its larva is eminently echinodermoid, allied to Star-fish larvæ, which in their turn are more closely allied to the larvæ of Holothurians and Crinoids than to those of Echinoids and Ophiurans. The larva is, however, most closely allied to genuine Annelid larvæ, as Lovén's larva, though the close homology is not at first apparent, owing to the disproportion in the development of the anterior and posterior extremities in these two types. It has, like Lovén's larva, the peculiar thickening of the outer wall immediately below the two large eye-specks, as well as the muscular band leading from them. Neither of the Annelids developing from these two larvæ have any bristles, and if Schneider is correct in assigning Polygordius as the adult of Lovén's larva, we find the explanation of the two cavities lined with cilia, which he figures on each side of the anterior part of the body of Polygordius, as rudimentary gills still in the condition in which they first appear in Balanoglossus in the Tornaria stage. Both are distinctly articulated. The opening of the mouth and the structure of the alimentary canal are strikingly similar. The collar, however, is a feature which we find nowhere among Annelids except among the Sabellidae, Terebellidae, Serpulidae, Maldaniae, and the like. The presence of gills as found in Balanoglossus is a feature totally unlike that of any other group of Annelids, nor can we in any way homologize the gills with the dorsal cirri found in any group of Dorsibranchiates, as in one case they communicate directly with the œsophagus, in the other with the perivisceral cavity. In a species of Tomopteris, which is quite common on our coast, I have observed in the lateral appendages the openings first seen by Claparède, which might

be compared to the gills of Balanoglossus, but like other gill-like organs in Annelids they connect with the perivisceral cavity. Tomopteris has, however, several features which had led Claparède to regard it as intermediate between the Annelids proper and Nemertians: the absence of well-defined articulations and of setæ along the lateral appendages of the body, the setæ being limited to the cephalic appendages. We must, I think, look upon Balanoglossus as the type of a family intermediate between Tubicolous Annelids and Nemertians, to which its mode of development is closely analogous, while its structural features recall more strongly those of several families of Tubicolous Annelids. We have among Sabellidae genera in which we find a most rudimentary proboscis immediately above the opening of the mouth, on the dorsal side, under the collar; as, for instance, the genus Artacama of Malmgren.[1] Then we have such forms as Myriochile, MALM.,[2] where we find the first trace of a collar totally destitute of cephalic appendages of any sort, these taken in connection with such genera as Sabellaria, where the development of the posterior part is great, and independent, as it were, of that of the anterior part of the body, where we have a collar, gills, and dorsal cirri, as well as setæ, with all the intermediate passages afforded by the Maldaniae, Terebellidae, Sabellidae, Hermellidae, show us many features which are dimly recognized in Balanoglossus, and which link together families thus far as disconnected as the Nemertians and Tubicolous Annelids, — an association which the great similarity between the Lovén type of Annelid larva and the larvæ of Nereis and Phyllodoce shows not to be so far-fetched as might at first be imagined; hinting at a more intimate relationship between the different orders of Annelids than had thus far been recognized. It must, however, not be forgotten that the peculiar structure of the proboscis, with its openings and diverticula from the alimentary canal, are features thus far not known except among Nemertians. The lateral cephalic splits of some of the Nemertian genera correspond with the openings formed by the attachment of the proboscis, for its whole length, to the base of the collar, and their articulations are quite as distinct as in some of the Annelids. It may be that future investigations may give a different explanation of the skeleton supports of the base of the proboscis and of the gills, which would homologize them in part with the proboscidal armature of some Nemertians. Keferstein has described something analogous to the ciliated furrows of Balanoglossus in some of the Nemertians, where the lateral part of the intestine forms pouches re-entering towards the dorsal part, this being the rudimentary structure of what is so excessively developed in Balanoglossus. Van Bene-

[1] MALMGREN, A. Nordiska Hafs Annulater, Pl. XXIII. f. 60.

[2] MALMGREN, A. Annul. Polychaeta, 1867, p. 101. Pl. VII. f. 7.

den[1] has also suggested in Nemertians the existence of a rudimentary liver as diverticula from the alimentary canal. Keferstein questions the existence of two openings in the proboscis of Nemertians, as stated by Williams;[2] but from what we have found in Balanoglossus, such a structure is by no means an impossible one, even completely disconnected from the main cavity of the alimentary canal, though the explanation given by Williams of the existence of two independent stomachs is not correct; he has probably observed the water entering and leaving the proboscis, as noticed by Kowalevsky and myself in the proboscis of Balanoglossus. Kölliker[3] considers the proboscis of Nemertians as an organ of locomotion, — an opinion fully sustained by the structure and function of the proboscis of Balanoglossus. The same view is also taken by Claparède,[4] after studying the ramifications of the alimentary canal of Eurylepta.

The history of Balanoglossus as given above, while showing great analogy between the development of Echinoderms and Nemertians, by no means proves the identity of type of the Echinoderms and Annuloids. It is undoubtedly the strongest case known which could be taken to prove their identity; but when we come carefully to analyze the anatomy of true Echinoderm larvæ, and compare it with that of Tornaria, we find that we leave as wide a gulf as ever between the structure of the Echinoderms and that of the Annuloids.

[1] Van Beneden. Bull. Acad. Brux., XXXII. 1861.
[2] Williams. Report Brit. Ann., 1852.
[3] Kölliker. Schweiz. Naturforsch. Ges. 1844.
[4] Claparède, E. Recherches Anat. sur les Annel. Turbell. . . . . d. Hébrides, 1861, p. 76.

XVI.

On the Determination of Transatlantic Longitudes by Means of the Telegraphic Cables.

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After the Telegraphic Cable had been successfully laid between Trinity Bay in Newfoundland and Valencia in Ireland, no time was lost by the Superintendent of the U. S. Coast Survey in making it available for the determination of differences of longitude between the principal meridians of the British Islands and the principal meridians of the United States. The processes and the results of this operation have been fully and ably explained in other publications.[1] In the winter of 1869 – 70, advantage was taken of the French cable, which was then open between Duxbury, Mass., and Brest, France, by the way of the island of St. Pierre, near Newfoundland, to connect, by time-signals, Brest with Duxbury and Duxbury with Cambridge, and hence with Washington. Again, in the summer of 1872, Cambridge exchanged time-signals with St. Pierre by another route, and St. Pierre exchanged signals with Brest by the French cable. Moreover, Mr. J. E. Hilgard, of the U. S. Coast Survey, was despatched to Europe to superintend the exchange of time-signals between Brest and Paris, and between Paris and Greenwich. When all the computations which are required for deducing the final result from the work of the last summer are finished, the Coast Survey Office will possess three independent determinations, by the telegraphic method, of the difference of longitude between Greenwich and Washington. The comparison which has already been made, by anticipation, between the determinations which have been thoroughly computed and those which, as yet, are only known approximately, justifies the expectation that, when the labor of the computers is finished, all the different results will correspond to a surprising degree. The present communication is limited to giving some details in regard to the campaign of 1869 – 70, and the subsequent calculations which grew out of it.

The astronomical station occupied by the U. S. Coast Survey at Brest was under

[1] Smithsonian Contribution to Knowledge, No. 223. Amer. Jour. Sci. N. S. XLIX. p. 228.

the charge of Mr. George W. Dean, of Fall River, Mass., assisted by Mr. F. Blake, experienced officers in the Coast Survey Service. Mr. Dean acknowledges, in his report, the obligations of the Survey to Mr. Thomas Andrews, General Superintendent of the French cable at Brest, for his active and cheerful co-operation in the objects of the expedition, and also to Mr. J. D. H. Dickson for valuable aid in arranging the batteries and other instruments in the cable office.

The U. S. Coast Survey station at Brest is situated near the southeastern part of the grounds attached to the *Établissement des Pupilles de la Marine*, and 126.44 metres west of the flag-staff on the tower of the St. Louis Church, which served as one of the principal points in the primary triangulation of France. Mr. Dean con nected this flag-staff with his station by a triangle, with a measured base of 117.33 metres, extending due north from his own position. The Coast Survey Station was distant about one quarter of a mile from the office of the Cable Telegraph, and was temporarily connected with it by two wires. Its latitude is 48° 23′ 21″.4 N.

For the determination of the local time at Brest, U. S. C. S. Transit No. 4, made by Troughton and Simms of London, in 1848, was used. Its principal focus is forty-six inches in length, and its aperture two inches and three quarters of an inch. The power employed was ninety-five. It was noticed that some of the threads, particularly the middle one (D_3), were disturbed by the hygrometric changes of the atmosphere. The level marked (B) was used for ascertaining the inclination of the mechanical axis of the instrument. The arc-value of one division of the scale was found in 1868 to be 0.99 of a second of arc, or 0.066 of a second of time. It was known, by observations made at Salt Lake City in 1869, that the *lamp-end* pivot was the largest, and required a correction amounting to $0^s.013$. The *thread-intervals* were subjected to a new determination, which will be given in another part of this paper.

A Sidereal Chronometer, made by Charles Frodsham of London, and numbered 3451, was loaned to the Coast Survey Service by Professor Joseph Winlock, Director of the Harvard College Observatory. This instrument is provided with a break-circuit, invented by Mr. Frodsham in 1868, so as to record the *alternate* seconds upon a chronograph-sheet; but at the beginning of each minute *two successive* seconds are registered. Mr. Dean observes that this instrument, though its rate was not so uniform at Brest as at the Observatory, has on the whole performed satisfactorily. The Chronograph employed (which was U. S. C. S. No. 1) worked well.

The astronomical station of the U. S. Coast Survey at Duxbury, Mass., was under the charge of Mr. Edward Goodfellow. It is situated on ground belonging to Mr. William Paulding, at a distance of 881 feet E. N. E. from the office of the Cable Tele-

graph. From it the station of Manomet, in the primary triangulation of the Coast Survey, and the secondary station at West Duxbury are both visible. By a preliminary triangulation, resting on a measured base of 380.17 feet, the centre of the Transit-instrument was found to be 6161.73 feet east, and 4861.19 feet north, of the secondary station at West Duxbury. Its latitude is 42° 2′ 53″.1 N. This position had the advantage of an open meridian line to the south of two thousand feet in length. A granite block, with a copper bolt inserted in it, marks the precise spot over which the centre of the Transit-instrument stood.

The observations for instrumental corrections, thread-intervals, and time were made here, as at Brest, according to the chronographic method, and the instruments used were C. S. Transit No. 6, the Krille Sidereal Clock, and U. S. C. S. Chronograph No. 2. The focal length of the Transit-instrument is forty-six inches, and its aperture two inches and three quarters of an inch. The value of one division of level (B) is equal to 0.86 of a second of arc, or 0.057 of a second of time. The correction to be applied to the lamp-end of the axis for inequality of pivots, as deduced by observations on three different nights, is 0.027 of a second of time, the *lamp-end* being the smallest. The performance of the Krille Clock is reported as tolerably satisfactory, though it appeared to be *under*-compensated. The Chronograph No. 2 had been cleaned and put in good order by the Messrs. Bond before it was taken to Duxbury, but Mr. Goodfellow found that it was difficult to keep it in good adjustment. A fresh determination of the thread-intervals was made at Duxbury. When this instrument was unpacked, it was found that threads (C_4) and (C_5) had become hopelessly twisted together, so that they were useless throughout the campaign.

The observations at Cambridge were under the charge of Professor Joseph Winlock, Director of Harvard College Observatory, those for instrumental corrections and for time being made with the Sidereal Clock and the Chronograph of the Observatory by Mr. E. P. Austin, then an assistant at that place. Coast Survey Transit No. 5 was used. Its position was twenty-eight feet west of the dome of the Observatory, or 0.025 of a second of arc. The value of one division of the level is 0.96 of a second of arc, or 0.064 of a second of time. The *lamp-pivot* is the smallest, and requires a correction of 0.014 of a second of time. No new observations were made for the determination of the thread-intervals. The diaphragm of the Transit-instrument at each of the three stations, Cambridge, Duxbury, and Brest, consists of twenty-five threads, arranged in five tallies.

Whenever the weather permitted, a large set of observations were made *before* and *after* exchanging longitude signals, in order to ascertain the instrumental corrections,

and the error and rate of the clocks or chronometer with the highest degree of accuracy.

In the Coast Survey operations for obtaining differences of longitude between various points in the United States, by the Air Lines of telegraph, it is possible to make the longitude-signal record itself on the chronograph-sheets which register the time, at the station from which the signal goes and also at the station where it is received. But the electrical currents which are practicable with the Cable Lines are too weak to make this record at the latter station. They have barely strength to deflect a delicately suspended magnet, with an attached mirror, from which a beam of light is reflected upon a scale in a dark room. When the signal arrives and this deflection is seen, the observer touches his key and records the moment of its arrival, with as little loss of time as possible, upon the chronograph-sheet.

The longitude signals arranged beforehand between Mr. Dean and Mr. Goodfellow are of four kinds.

I. At the beginning of a minute or any five-seconds mark on a mean-time chronometer or watch, the observer at the first station sends a positive current, of exactly wo seconds' duration, into the cable; at the beginning of the next five-seconds mark he sends a negative current of the same duration, to be followed at the third five-seconds mark by a positive current of equal duration. After an interval of ten seconds succeed four alternating currents in like manner, the first of which is negative. After another interval of ten seconds, three other alternating currents follow, the first of which is negative. These ten signals make what is called a *set*, which may be written thus: —

$$(P^2)\underline{\ ^5\ }(N^2)\underline{\ ^5\ }(P^2)\underline{\quad ^{10}\quad }(N^2)\underline{\ ^5\ }(P^2)\underline{\ ^5\ }(N^2)\underline{\ ^5\ }(P^2)\underline{\quad ^{10}\quad }(N^2)\underline{\ ^5\ }(P^2)\underline{\ ^5\ }(N^2).$$

The object in dividing the set in this way into three groups is for the convenience of identifying the individual signals with facility, when they are recorded upon the chronograph-sheets. After an interval of ten seconds a second set is sent from the same station. In recording these signals at the second station the observer, with his finger upon his chronograph-key, watches closely the bright band, reflected by the mirror on the galvanometer magnet, as it moves towards the *end* of the scale, and at the instant this band darts towards the *centre*, he taps his key, and records upon the chronograph sheet the moment when the cable begins to *discharge* its positive or negative current. After this, two sets of signals are sent in the same way from the second station, and are observed and recorded in a similar manner at the first station. The whole operation

is then repeated, from the first station to the second, and again from the second to the first.

II. The second class of signals differs from the first class in two ways: 1. The positive and negative currents which are sent into the cable are continued for only one half of a second. 2. The observer records the moment when the light begins to move *from* the centre, towards the right or left end of the scale. This class of signals may be written thus:—

$$(P^{\frac{1}{2}})\underline{\;5\;}(N^{\frac{1}{2}})\underline{\;5\;}(P^{\frac{1}{2}})\underline{\quad 10\quad}(N^{\frac{1}{2}})\underline{\;5\;}(P^{\frac{1}{2}})\underline{\;5\;}(N^{\frac{1}{2}})\underline{\;5\;}(P^{\frac{1}{2}})\underline{\quad 10\quad}(N^{\frac{1}{2}})\underline{\;5\;}(P^{\frac{1}{2}})\underline{\;5\;}(N^{\frac{1}{2}}).$$

III. In the third class of signals, alternating positive and negative currents, to the number of six, and each of five seconds' duration, are sent at intervals of ten seconds from the first station, and the moment is recorded at the other station when the light upon the scale darts towards the *centre*. After an interval of fifteen seconds a similar set is sent again. Then the same number of signals are despatched from the second station, to be recorded at the first station. Afterwards, the whole operation is repeated from the first station to the second, and also from the second to the first. This series of twelve signals in two groups may be written thus:—

$$(P^{5})\underline{\;10\;}(N^{5})\underline{\;10\;}(P^{5})\underline{\;10\;}(N^{5})\underline{\;10\;}(P^{5})\underline{\;10\;}(N^{5})\underline{\quad 15\quad}(P^{5})\underline{\;10\;}(N^{5})\underline{\;10\;}(P^{5})\underline{\;10\;}(N^{5})\underline{\;10\;}(P^{5})\underline{\;10\;}(N^{5}).$$

IV. The fourth class of signals differs from the third as follows. Four alternating currents make a group, and two groups a set. The duration of each current is ten seconds. The interval between the end of one current and the beginning of the next is five seconds, and the interval between groups is ten seconds, thus:—

$$(P^{10})\underline{\;5\;}(N^{10})\underline{\;5\;}(P^{10})\underline{\;5\;}(N^{10})\underline{\;5\;}(P^{10})\underline{\;5\;}(N^{10})\underline{\quad 10\quad}(P^{10})\underline{\;5\;}(N^{10})\underline{\;5\;}(P^{10})\underline{\;5\;}(N^{10})\underline{\;5\;}(P^{10})\underline{\;5\;}(N^{10}).$$

It will be noticed that the signals which are recognized when Classes I., III., and IV. are used, are *discharge* signals; that is, that the moment is recorded when the cessation of the battery current reaches the remote station, and the needle of the galvanometer suddenly returns to the centre. On the other hand, when signals of Class II. are employed, the moment is recorded when the needle begins to show the effect of the *charge*. The velocity with which the signal travels is diminished to a slight extent by the resistance of the battery, and when the sending batteries at the two stations are different, the transmission time derived from currents sent alternately from the two stations will be vitiated to the extent of half this difference in the resistance of

the batteries. With this qualification, this class of signals is useful for the determination of longitude and of the velocity with which the *charge* travels. On the other hand, signals which belong to Classes I., III., and IV. are employed for an exact measurement of the velocity with which the discharge of the battery is propagated through the cable. A preliminary trial, made by Mr. Varley on December 19, revealed the fact that two seconds was insufficient to charge the cable so as to give unequivocal signals, and that the signals of five seconds' duration were better. Accordingly, signals of Class I. were not used at all, and those of Class III. were employed more frequently than those of Class II. The signals of Class IV. were seldom used. Moreover, the original plan laid out beforehand by Mr. Dean was modified so as to make Class II. to comprise six sets of signals, of ten each, and Class III. to comprise six sets of signals, of six each, to be sent in each direction. A good night's work would, therefore, include one hundred and ninety-two signals.

The following data in regard to the Atlantic cable which connects Duxbury with Brest are extracted from the day-book of Mr. Goodfellow. They were communicated to him by R. T. Brown, Esq., Superintendent of the Duxbury Station of the French Transatlantic Cable Company: —

Length of cable from Duxbury to St. Pierre, ...	749	nautical miles.
Length of cable from St. Pierre to Brest, ...	2580	" "
Mean cable resistance per knot of 6087 feet, at 75° Fahr., between Duxbury and St. Pierre, ...	11.99	ohms.
between St. Pierre and Brest, .	3.16	"
Mean gutta-percha resistance per knot, at 75° Fahr., between St. Pierre and Brest, .	2405	megohms.
between Duxbury and St. Pierre,	2300	"
(when laid and without regard to temperature and pressure.)		
Electrostatic capacity per knot between Duxbury and St. Pierre, ...	0.358	farads.
between St. Pierre and Brest, ...	0.404	"
Weight of the conductor per knot between Duxbury and St. Pierre, ...	107	lbs.
between St. Pierre and Brest, ...	400	"
Weight of the gutta-percha per knot between Duxbury and St. Pierre, . .	150	"
between St. Pierre and Brest, ...	400	"
Diameter of the cores between Duxbury and St. Pierre, ...	0.282	inch.
between St. Pierre and Brest, ...	0.468	"
Specific conducting power compared with pure copper, between Duxbury and St. Pierre,	92.9	per cent.
between St. Pierre and Brest, .	94.3	" "
Resistance of the galvanometer used in the observations for longitude (temp. of 56° Fahr.), ...	1188	ohms.
Average distance of the scale from the mirror of the galvanometer, . . .	39	inches.

Value of ten divisions on the scale, ...	0.25 inch.
Number of cells of Minotto's battery used at Duxbury in the longitude campaign, .	40
Number of cells of Minotto's battery used at Brest, ...	30
Electromotive force of the Duxbury battery expressed in Daniell's cells, . .	33.7
Electromotive force of the Brest battery expressed in Daniell's cells, . . .	25.3

On the nights of January 5 and 8, the mirror of the Thomson galvanometer at Duxbury was suspended by a single silk fibre at the top and bottom of the tube. On all other occasions, the mirror was suspended by a single fibre from the top of the tube, the bottom fibre being cut away in order to render the needle more sensitive.

As the Transit room of the Coast Survey at Brest was distant about one quarter of a mile from the cable office, it was necessary to use a Daniell's battery of five cells in the local circuit which connected the key in the cable office with the chronometer and chronograph. At Duxbury, the Transit room and cable office were only one sixth of a mile apart, and a Daniell's battery of three cells was sufficient to work the clock and chronograph circuit.

In order to be able to exchange signals with the Harvard College Observatory, the Coast Survey station at Duxbury was joined to the Boston air-line of telegraph by a loop made of the "American compound telegraph-wire," and with the office of the cable line by a second loop of the same wire, which were kindly loaned by Mr. Moses G. Farmer. This wire is of the diameter known as "steel-core No. 15," its external diameter being No. 13. It weighs one hundred and thirty-four pounds to the mile. Its conductivity is about equal to No. 7 of galvanized iron. The distance from Duxbury to Cambridge by the wire is between forty-three and forty-four miles. The battery used was about fifteen bi-chromate of potash cells. Sometimes, however, thirty-five cells were required, and, at other times, ten cells were sufficient.

The signals used between Cambridge and Duxbury were of two kinds. I. *Clock-signals.* In this case each second break of the clock at one station not only recorded itself upon the chronograph at that station, but, operating upon the main line and the relay magnet at the second station, was also registered by means of the local battery upon the chronograph at the latter station. In sending the signals from Duxbury to Cambridge, the Krille Sidereal Clock was always used. On January 14, Professor Winlock reported that the signals from Duxbury were not well received when his sidereal clock was in the main circuit, so that on and after that date the Mean Time clock, which broke the circuit only every alternate second, was substituted in sending clock-signals to Duxbury. But the times of sending and receiving all kinds of signals were recorded on the chronograph by the sidereal clock. The clock-signals were sent for

one or two minutes at a time. II. The other kind of signals used may be called *hand-signals* or *key-signals*.[1] On December 15, 23, and 31, and on January 3, six sets, of ten breaks each, were sent to Cambridge and received from Cambridge. A few failed of being properly recorded on the chronograph-sheets. On January 14, 22, 24, 26, and 28, and on February 9 and 10, the Cambridge and Duxbury line was connected, by means of a relay-magnet, with the cable key and clock and chronograph circuit at Duxbury, and when the cable signals belonging to Classes II., III., or IV. were received from Brest or sent to Brest, they were recorded on the Duxbury and Cambridge chronograph-sheets. These are called *cable-key* signals, to distinguish them from hand-signals made with a telegraph key at arbitrary times.

After the programme already described had been successfully carried out, with no essential variation, by the officers of the Coast Survey in charge respectively of the three stations, the materials were all placed in my hands by the Superintendent, Professor Benjamin Peirce, in order that I might deduce from them the differences of longitude between Cambridge, Duxbury, and Brest. The computations have been made under my direction, and have been carefully re-examined by me. Those designed to ascertain the clock and instrumental corrections at Cambridge were made by Mr. Henry Gannett. Those necessary to obtain the clock and instrumental corrections at Duxbury and Brest, and also those intended to give a precise determination of the differences of longitude, corrected for the Personal Equation in observing transits and noting cable signals, were made by Mr. Lucius Brown. I shall now describe the methods of computation which have been followed.

The plan which has been adopted for the precise determination of the local time at each of the three stations is essentially the same as that upon which F. G. W. Struve proceeded in working up the results of his two chronometric expeditions to ascertain the differences of longitude between Pulkowa and Altona, and between Altona and Greenwich, and which has become familiar in the Coast Survey Service by the labors of the late Mr. Sears C. Walker and Dr. B. A. Gould. I have not thought that observations made with small portable Transit-instruments, such as those which are used at the Coast Survey stations, would be accurate enough to justify the added labor of assigning different weights to the stars observed for time, according to their declination, after the manner indicated by Struve and J. A. C. Oudemans.[2]

[1] These are sometimes called *arbitrary* signals, to distinguish them from *clock* signals.

[2] Dissertatio Astronomica Inauguralis.

Formulæ used in the Computation of Instrumental and Clock Corrections.

a	$= T + \Delta T + \tau$ (τ being the correction *in time* for errors in azimuth, level, inequality of pivots, and collimation.)
τ	$= a\dfrac{\sin(\phi-\delta)}{\cos\delta} + b\dfrac{\cos(\phi-\delta)}{\cos\delta} \pm \dfrac{c}{\cos\delta} = Aa + Bb^0 \pm Cc$,* ($a$) being the azimuth constant, (b) being the level constant, and (c) being the collimation constant.
ϕ	= the geographical latitude of the station.
a	= the right ascension of the star.
δ	= the declination of the star.
M	= the mean of the observed tallies.
f	= the mean of the equatorial thread-intervals.
ρ	= the correction for rate, and its log is 0.000005 for a gain of 1 sec. daily: 0.00119 for a mean time clock.
σ	= the sine correction when star is near the pole: log σ being additive to log F; when F sec δ is less than $2^{m.}$ it may be neglected.
R	$= F \sec\delta . \sigma . \rho$; and is to be added to M to obtain the time of transit over the mean of all the threads.
b_0	= the level constant in time, corrected for inequality of pivots: it is positive for W. end high.
Star constant A	$= \dfrac{\sin(\phi-\delta)}{\cos\delta}$.
Star constant B	$= \dfrac{\cos(\phi-\delta)}{\cos\delta}$.
Star constant C	$= \sec\delta$.
	$180° - \delta$ is used instead of δ when the star is below the pole.
	A is positive, except for stars between the Zenith and North Pole.
	B is positive, except for stars at lower culmination.
	C is positive, except for stars at lower culmination.
k	= the diurnal aberration $= 0^s.021 \cos\phi \sec\delta$. It is (—) in upper, (+) in lower culminations.
Bb_0	= correction for level and inequality of pivots.
T	$= M + R =$ time of transit over the mean of all the threads.
t	$= T + Bb_0 + \kappa$.
ΔT	$= \Delta t =$ correction of the clock.
ω	$= a - t = \Delta t + Aa \pm Cc$.
Cc	= correction for collimation.
Aa	= correction for azimuth.
ω_0	$= \omega \mp Cc$, upper sign for lamp west.

The collimation constant (c) is determined from reversals on circumpolar stars, and is to be obtained from the equation

$$t_e - t_w = \omega_w - \omega_e = 2\,Cc.$$

If the collimation is known, and the corresponding corrections are applied, reduce the value of ω_0 for the several stars to an arbitrary time T_0, by applying the correction for daily rate. Calling this reduced value (ω_0),

$$(\omega_0) = \omega_0 \frac{t - T_0}{24 \text{ hrs.}} \times \text{daily rate.}$$

* Mayer's formula, corrected for inequality of pivots and aberration.

The local time and azimuth are obtained thus: Assume an approximate value of the clock correction $= \theta$ for the arbitrary time T_0, and make $(\omega_0) - \theta = \omega_0'$.

For each star, $\Delta t + Aa = (\omega_0) = \omega_0' + \theta$.

If $\Delta t - \theta = \Delta\theta$ we have $Aa + \Delta\theta = \omega_0'$, ω_0' being a small residual.

Following the method of least squares, multiply this equation for each star by the coefficients of the unknown quantities, and we obtain the normal equations: —

(1.) $$\Sigma\, A\, a + \Sigma\, \Delta\theta = \Sigma\, \omega_0'.$$

(2.) $$\Sigma\, A^2 a + \Sigma\, A\, \Delta\theta = \Sigma\, A\, \omega_0'.$$

From these two equations, we can compute the values of a, $\Delta\theta$, and hence derive the value of Δt for the time T_0.

When the collimation constant has not been determined by reversals, but enters into the equations as one of the unknown quantities, we have for each star

$$Aa \pm Cc + \Delta\theta = \omega',$$

(ω' being equal to $(\omega) - \theta$). In this case the normal equations are: —

(1.) $$\Sigma\, Aa \pm \Sigma\, Cc + \Sigma\, \Delta\theta = \Sigma\, \omega.$$

(2.) $$\Sigma\, A^2 a \pm \Sigma\, A\, Cc + \Sigma\, A\, \Delta\theta = \Sigma\, A\, \omega'.$$

(3.) $$\Sigma\, A\, Ca \pm \Sigma\, C^2 c + \Sigma\, C\, \Delta\theta = \Sigma\, C\omega'.$$

In the computations for Brest and Duxbury, only two normal equations were used, the value of (c) being assumed from the result of the reversals. In the computations for Cambridge, the value of (c) was assumed from reversals, for December 17 and 31, for January 3, and for February 1. On January 4 the value of (c) was taken from January 3. On January 12, the values calculated for January 11 and 16 were adopted. On January 19, the values calculated for January 18 and 26 were adopted. On February 11, the value calculated for February 10 was adopted. On the other nights (c) was calculated by means of the three normal equations.

Adopted Values of the Equatorial Thread Intervals.

	C. S. Transit Instrument, No. 4.		C. S. Transit Instrument, No. 5.		C. S. Transit Instrument. No. 6.
	From 31 Observations.		From 24 Observations.		From 38 Observations.
B_1	+34.174	B_1	+36.450	B_1	+35.590
B_2	+31.817	B_2	+33.924	B_2	+33.094
B_3	+29.269	B_3	+31.320	B_3	+30.607
B_4	+26.860	B_4	+28.800	B_4	+28.038
B_5	+24.308	B_5	+26.202	B_5	+25.403
C_1	+19.433	C_1	+20.950	C_1	
C_2	+17.173	C_2	+18.307	C_2	
C_3	+14.533	C_3	+15.691	C_3	+15.399
C_4	+12.019	C_4	+13.051	C_4	+12.693
C_5	+ 9.826	C_5	+10.498	C_5	+10.239
D_1	+ 5.035	D_1	+ 5.232	D_1	+ 5.107
D_2	+ 2.484	D_2	+ 2.551	D_2	+ 2.589
D_3	— 0.043	D_3	— 0.082	D_3	+ 0.099
D_4	— 2.364	D_4	— 2.630	D_4	— 2.450
D_5	— 4.755	D_5	— 5.271	D_5	— 5.059
E_1	— 9.691	E_1	—10.413	E_1	—10.071
E_2	—12.205	E_2	—12.972	E_2	—12.821
E_3	—14.630	E_3	—15.565	E_3	—15.331
E_4	—17.153	E_4	—18.201	E_4	—17.950
E_5	—19.470	E_5	—20.787	E_5	—20.440
F_1	—24.451	F_1	—26.102	F_1	—25.504
F_2	—26.818	F_2	—28.671	F_2	—28.073
F_3	—29.399	F_3	—31.522	F_3	—30.660
F_4	—31.799	F_4	—34.057	F_4	—33.107
F_5	—34.153	F_5	—36.703	F_5	—35.782

The equatorial thread-intervals used in the computations were obtained for the C. S. transit-instruments No. 4 and No. 6 from the observations made by Mr. Dean and Mr. Goodfellow in the course of the campaign. The equatorial thread-intervals for C. S. transit-instrument No. 5 are the same as those found by Mr. Isaac Bradford, from the observations made at Cambridge, in 1869, by Mr. A. F. Mosman, and Mr. F. Blake, Jr. No observations were made for this purpose in the campaign of 1869–70.

The computations to find the clock and instrumental corrections, for a *single* night, are given in detail, as an illustration of the method adopted.

Computation of Transits observed at Cambridge on January 3, 1870.

Star	η Piscium.	o Piscium.	β Arietis.	50 Cassiop.	ι Cassiop.	ι Cassiop.
Lamp	W.	W.	W.	W.	W.	E
Threads	C D E	C D E	C D E	B* C D	B C	B C
α	$1^h\ 24^m\ 31^s.11$	$1^h\ 38^m\ 31^s.35$	$1^h\ 47^m\ 27^s.27$	$1^h\ 52^m\ 22^s.63$	$2^h\ 18^m\ 23^s.72$	$2^h\ 18^m\ 23^s.72$
δ	14° 40′ 26″	8° 30′ 3″	20° 10′ 16″	71° 47′ 35″	66° 49′ 5″	66° 49′ 5″
M	$26^m\ 28^s.887$	$40^m\ 28^s.853$	$49^m\ 24^s.946$	$53^m\ 36^s.956$	$19^m\ 23^s.015$	$21^m\ 23^s.541$
f	+ .024	+ .024	+ .024	+ 14.362	+ 23.519	— 23.519
log. f	8.38021	8.38021	8.38021	1.15721	1.37142	1.37142*n*
secant δ	.01441	.00480	.02749	.50522	.40488	
log. ρ						
log. σ						
log. R	8.39462	8.38501	8.40770	1.66243	1.77630	1.77630*n*
R	.025	.024	.026	45.965	59.744	— 59.744
b_0 or b_1	— .136	— .150	— .159	— .162	— .186	— .116
A	.481	.564	.403	— 1.572	— 1.051	
B	.916	.839	.986	2.788	2.313	
C	1.034	1.011	1.065	3.200	2.540	
k	— .016	— .016	— .016	— .050	— .039	— .039
Bb_0 or Bb_1	— .125	— .126	— .157	— .452	— .430	— .268
T	$26^m\ 28^s.912$	$40^m\ 28^s.877$	$49^m\ 24^s.972$	$54^m\ 22^s.921$	$20^m\ 22^s.759$	$20^m\ 23^s.797$
t	26 28,771	40 28.735	49 24.799	54 22.419	20 22.290	20 23.490
ω	1 57.661	1 57.385	1 57.529	1 58.789		
Cc	.242	.237	.249	.749		
ω_0	1 57.903	1 57.622	1 57.778	1 59.538		
(ω_0)	1 57.888	1 57.609	1 57.766	1 59.527		

* The observation on thread B_2 was lost.

Computation of Transits observed at Cambridge on January 3, 1870.

Star	5 Urs. Min.	γ Ceti.	β Urs. Min.	ζ Arietis.	α Persei.	γ^2 Urs. Min.	δ Persei.
Lamp	E. (L. C.)	E.	E. (L. C.)	E.	E.	E. (L. C.)	E.
Threads	C D E	C D E	D_5 E F	C D E	C D E	C D E	C D E
α	$2^h\ 27^m\ 46^s.43$	$2^h\ 36^m\ 33^s.73$	$2^h\ 51^m\ 3^s.23$	$3^h\ 7^m\ 25^s.89$	$3^h\ 15^m\ 3^s.42$	$3^h\ 20^m\ 53^s.87$	$3^h\ 33^m\ 40^s.91$
δ	76° 16′ 4″	2° 41′ 3″	74° 41′ 1″	20° 33′ 37″	49° 23′ 49″	72° 17′ 40″	47° 22′ 11″
M	$29^m\ 39^s.371$	$38^m\ 31^s.695$	$54^m\ 19^s.845$	$9^m\ 24^s.080$	$17^m\ 2^s.091$	$22^m\ 47^s.800$	$35^m\ 39^s.640$
f	+ .024	— .024	— 21.842	— .024	— .024	+ .024	— .024
log. f	8.38021	8.38021n	1.33929n	8.38021n	8.38021n	8.38021n	8.38021n
secant δ	.62462	.00049	.57814	.02858	.18654	.51693	.16925
log. ρ							
log. σ							
log. R	9.00483	8.38069n	1.91743n	8.40879n	8.56675n	8.89714	8.54946n
R	.101	— .024	—1 22.686	— .026	— .037	+ .079	— .035
b_0 or b_1	— .126	— .149	— .191	— .221	— .244	— .257	— .214
A	3.697	.639	3.371	.397	— .188	2.988	— .128
B	— 2.020	.770	— 1.722	.992	1.525	— 1.373	1.470
C	— 4.213	1.001	— 3.786	1.068	1.537	— 3.288	1.476
k	+ .066	— .016	+ .058	— .017	— .024	+ .051	— .023
Bb_0 or Bb_1	.255	— .115	+ .329	— .219	— .372	+ .352	— .315
T	$29^m.39^s.472$	$38^m\ 31^s.671$	$52^m\ 57^s.159$	$9^m\ 24^s.054$	$17^m\ 2^s.054$	$22^m\ 47^s.879$	$35^m\ 39^s.505$
t	29 39.793	38 31.540	52 57.546	9 23.818	17 1.658	22 48.282	35 39.167
ω	1 53.363	1 57.810	1 54.316	1 57.928	1 58.248	1 54.412	1 58.267
Cc	+ .986	— .234	+ .886	— .250	— .359	+ .769	— .345
ω_0	1 54.349	1 57.576	1 55.202	1 57.678	1 57.889	1 55.181	1 57.922
(ω_0)	1 54.342	1 57.570	1 55.197	1 57.676	1 57.887	1 55.180	1 57.922

Computations of Transits observed at Cambridge on January 3, 1870.

Star	η TAURI.	ζ PERSEI.	α CAMELOP.	i AURIGÆ.	11 ORIONIS.	α AURIGÆ.
Lamp	E.	E.	E.	E.	W.	W.
Threads	C D E	D E F	C D E	C D E	C D E	B C D
α	$3^h\ 39^m\ 45^s.67$	$3^h\ 45^m\ 58^s.02$	$4^h\ 41^m\ 9^s.86$	$4^h\ 48^m\ 32^s.18$	$4^h\ 57^m\ 8^s.87$	$5^h\ 7^m\ 5^s.92$
δ	23° 42′ 1″	31° 29′ 41″	66° 7′ 6″	32° 57′ 25″	15° 13′ 9″	45° 51′ 44″
M	$41^m\ 43^s.941$	$47^m\ 38^s.027$	$43^m.\ 9^s.715$	$50^m\ 30^s.547$	$59^m\ 6^s.502$	$8^m\ 41^s.248$
f	− .024	+ 15.680	− .024	− .024	+ .024	13.666
log. f	8.38021n	1.19535	8.38021n	8.38021n	8.38021	1.19496
secant δ	.03826	.06921	.39272	.07619	.01551	.15715
log. ρ						
log. σ						
log. R	8.41847n	1.26456	8.77293n	8.45640n	8.39572	1.35211
R	− .026	+ 18.389	− .059	− .029	+ .025	+ 22.491
b_0 or b_1	− .188	− .173	− .249	− .268	− .152	− .141
A	.350	.223	− .994	− .195	.473	− .087
B	1.035	1.152	2.261	1.176	.922	1.433
C	1.092	1.173	2.470	1.192	1.036	1.436
k	− .017	− .018	− .038	− .018	− .016	− .022
Bb_0 or Bb_1	− .195	− .199	− .563	− .315	− .140	− .202
T	$41^m\ 43^s.915$	$47^m\ 56^s.416$	$43^m\ 9^s.656$	$50^m\ 30^s.518$	$56^m\ 6^s.527$	$9^m\ 3^s.739$
t	41 43.703	47 56.199	43 9.055	50 30.185	56 6.371	9 3.515
ω	1 58.033	1 58.179	1 59.195	1 58.005	1 57.501	1 57.595
Cc	− .253	− .274	− .578	− .279	+ .242	+ .336
ω_0	1 57.780	1 57.905	1 58.617	1 57.726	1 57.743	1 57.931
(ω_0)	1 57.780	1 57.907	1 58.629	1 57.735	1 57.753	1 57.943

Computation of Transits observed at Cambridge on January 3, 1870.

Star	β Orionis.	β Tauri.	Groomb. 966	δ Orionis.	α Leporis.	ε Orionis.	ω Draconis.
Lamp	W.	W.	W.	W.	W.	W.	W. (L. C.)
Threads	E* F	C D E	C D E	C D E	C D E	C D E	D E F
α	$5^h\ 8^m\ 17^s.87$	$5^h\ 18^m\ 4^s.90$	$5^h\ 22^m\ 24^s.48$	$5^h\ 25^m\ 22^s.38$	$5^h\ 27^m\ 0^s.45$	$5^h\ 29^m\ 37^s.47$	$5^h\ 37^m\ 39^s.35$
δ	S 8° 21′ 23″	28° 29′ 38″	74° 57′ 6″	S 0° 23′ 59″	S 17° 55′ 11″	S 1° 17′ 21″	68° 49′ 1″
M	$10^m\ 40^s.516$	$20^m\ 2^s.735$	$24^m\ 23^s.559$	$27^m\ 19^s.803$	$28^m\ 57^s.563$	$31^m\ 34^s.885$	$38^m\ 52^s.055$
f	— 24.953	+ .024	+ .024	+ .024	+ .024	+ .024	— 15.680
log. f	1.39712n	8.38021	8.38021	8.38021	8.38021	8.38021	1.19535n
secant δ	.00464	.05607	.58565	.00001	.02160	.00011	.44208
log. ρ							
log. σ							
log. R	1.40176n	8.43628	8.96586	8.38022	8.40181	8.38032	1.63743
R	— 25.220	+ .027	+ .092	+ .024	+ .025	+ .024	+ 43.394
b_0 or b_1	— .139	— .128	— .124	— .121	— .120	— .117	— .110
A	.783	.273	— 2.074	.679	.913	.691	2.580
B	.640	1.105	3.246	.734	.521	.724	— 1.001
C	1.011	1.138	3.852	1.000	1.051	1.000	— 2.767
k	— .016	— .018	— .060	— .016	— .016	— .016	+ .043
Bb_0 or Bb_1	— .089	— .141	— .403	— .089	— .063	— .085	+ .110
T	$10^m\ 15^s.296$	$20^m\ 2^s.762$	$24^m\ 23^s.651$	$27^m\ 19^s.827$	$28^m\ 57^s.588$	$31^m\ 34^s.909$	$39^m\ 35^s.449$
t	10 15.191	24 2.603	24 23.188	28 19.722	28 57.509	31 34.808	39 35.602
ω	1 57.321	1 57.703	1 58.708	1 57.342	1 57.059	1 57.338	1 56.252
Cc	+ .237	+ .266	+ .901	+ .234	+ .246	+ .234	— .647
ω_0	1 57.558	1 57.969	1 59.609	1 57.576	1 57.305	1 57.572	1 55.605
(ω_0)	1 57.570	1 57.982	1 59.722	1 57.590	1 57.319	1 57.586	1 55.620

* The observation on E_1 was lost.

Computations for the Error of the Clock and for Instrumental Corrections, January 3, 1873.

Star.	Lamp.	(ω_0)	A	(ω_0')	A^2	$(A\omega_0')$	$A\alpha$	$\varDelta T_0$	Residuals.	(Residuals)²
		m s						m s		
η Piscium	W.	1 57.888	.481	— .012	.231	— .006	.436	1 58.324	.275	.076
o Piscium	W.	1 57.609	.564	— .291	.318	— .164	.511	1 58.120	.071	.005
β Arietis	W.	1 57.766	.403	— .134	.162	— .058	.366	1 58.132	.083	.007
50 Cassiop.	W.	1 59.527	— 1.572	1.627	2.470	— 2.558	—1.425	1 58.102	.053	.003
5 Urs. M. (L. C.)	E.	1 54.342	3.697	—3.558	13.669	—13.134	3.352	1 57.695	—.354	.125
γ Ceti	E.	1 57.570	.639	— .330	.409	— .211	.580	1 58.150	.101	.010
β Urs. M. (L. C.)	E.	1 55.197	3.371	—2.703	11.364	— 9.112	3.057	1 58.254	.185	.034
ζ Arietis	E.	1 57.676	.397	— .224	.157	— .149	.261	1 57.937	—.112	.013
α Persei	E.	1 57.887	— .188	— .013	.035	+ .002	— .171	1 57.716	—.333	.111
γ³ Urs. M. (L. C.)	E.	1 55.180	2.988	—2.720	8.927	— 8.127	2.710	1 57.890	—.159	.025
δ Persei	E.	1 57.922	— .128	.022	.016	— .003	— .115	1 57.807	—.242	.059
η Tauri	E.	1 57.781	.350	— .119	.122	— .042	.317	1 58.098	.049	.002
ζ Persei	E.	1 57.907	.223	.007	.049	+ .002	.201	1 58.108	.059	.003
α Camelop.	E.	1 58.629	— .994	.929	.989	— .923	— .900	1 57.929	—.120	.001
i Aurigæ	E.	1 57.735	.195	— .165	.038	— .032	.176	1 57.911	—.138	.019
11 Orionis	W.	1 57.753	.473	— .147	.224	— .070	.429	1 58.182	.133	.018
α Aurigæ	W.	1 57.943	— .087	.043	.006	— .004	— .078	1 57.865	—.184	.034
β Orionis	W.	1 57.570	.783	— .330	.612	— .026	.709	1 58.279	.230	.053
β Tauri	W.	1 57.982	.273	.082	.075	+ .022	.248	1 58.230	.181	.033
Groomb. 966	W.	1 59.722	— 2.074	1.822	4.300	— 3.779	—1.821	1 57.901	—.148	.022
δ Orionis	W.	1 57.590	.679	— .310	.461	— .210	.615	1 58.205	.156	.024
α Leporis	W.	1 57.319	.913	— .581	.833	— .530	.828	1 58.147	.098	.010
ε Orionis	W.	1 57.586	.691	— .314	.477	— .217	.626	1 58.212	.163	.027
ω Draco. (L. C.)	W.	1 55.620	2.580	—2.280	6.656	— 5.884	2.340	1 57.969	—.080	.006
			+14.757	—9.699	+52.610	—45.445				

Collimation Constant.

Star.	t	Lamp.	$2\,Cc$	c
ι Cassiopeæ	20ᵐ 22ˢ.290	West	+1.200	+.236
	20 23.490	East		

$$t_e - t_w = w_w - w_e = 2\,Cc$$

$$c = \frac{t_e - t_w}{2\,C}$$

Hourly rate of clock $= -\,0^{s}.007$

Normal Equations for the weight of $\varDelta\theta$.

$$\varDelta\theta + .616\,a - 1 = 0$$

$$14.757\,\varDelta\theta + 52.610\,a = 0$$

$$p \text{ (or weight of } \varDelta\theta) = 6.41$$

$$\epsilon = \frac{(\text{Residuals})^2}{22} = {}^{s}.181$$

$$r = .6745\,\epsilon = {}^{s}.122$$

$$r_0 = \frac{r}{\sqrt{p}} = {}^{s}.043$$

Normal Equations.

$$\Sigma\varDelta\theta + \Sigma Aa + \Sigma\omega_0 = 0$$

$$24\,\varDelta\theta + 14.757\,a - 9.699 = 0$$

$$\varDelta\theta + .616\,a - .404 = 0$$

$$\Sigma A\varDelta\theta + \Sigma A^2 a + \Sigma A\,\omega_0' = 0$$

$$14.757\,\varDelta\theta + 52.610\,a - 45.445 = 0$$

$$14.757\,\varDelta\theta + 9.076\,a - 5.965 = 0$$

$$43.534\,a - 39.480 = 0$$

$$a = +.907$$

$$a = +.907$$

$$\varDelta\theta = -.150$$

$$\varDelta T_0 = -\,1^{m}\,58^{s}.050$$

$$\theta = -\,1^{m}\,57^{s}.900$$

$$T_0 = 3^{h}\,30^{m}$$

The probable error of $\varDelta\theta$ and of $\varDelta T_0$ is $\pm\,{}^{s}.043$.

In the following tables, the first column contains the dates, the second (a) the correction for the azimuth of the transit instrument, the third (c) its correction for collimation, the fourth (ΔT_0) the error of the clock or chronometer, the fifth the hourly rate of the clock or chronometer, and the sixth the number of stars of which the transits were recorded on the corresponding date.

CAMBRIDGE.

Date.	a	c	ΔT_0	Hourly Rate.	Number of Stars.
1869.	s	s	m s	s	
Dec. 14	+1.797	+0.220	2 0.695	+0.029	7
" 15	+1.863	+0.154	2 0.212	+0.019	12
" 17	+0.794	0.198	1 59.513	+0.007	14
" 23	+0.674	0.241	1 58.962	+0.004	18
" 31	+0.840	0.258	1 58.402	+0.005	18
1870.					
Jan. 3	+0.907	+0.236	1 58.054	—0.007	24
" 4	+0.223	+0.236	1 57.867		2
" 7	+1.047	+0.312	1 57.392	+0.013	8
" 11	+0.846	+0.234	1 55.618	+0.020	11
" 12	+0.603	+0.276	1 55.096	+0.019	4
" 16	+0.978	+0.257	1 53.636	+0.016	21
" 18	+0.878	+0.259	1 52.844	+0.021	13
" 19	+0.839	+0.258	1 52.243	+0.022	8
" 26	+0.915	+0.254	1 49.744	+0.012	15
" 28	+0.890	+0.216	1 49.316	+0.013	16
Feb. 1	+0.853	+0.311	1 47.806	—0.021	4
" 10	+0.827	+0.235	1 42.430	+0.024	15
" 11	+0.796	0.235	1 41.850	+0.025	8

BREST.

Date.	a	c	ΔT_0	Hourly Rate.	Number of Stars.
1870.	s	s	s s	s	
Jan. 5	+0.299	—0.004	— 3.703 ± 0.039	+0.067	12
" 5	+0.286	—0.004	— 3.810 ± 0.033	+0.067	11
" 8	+0.266	+0.0005	— 6.899 ± 0.034	+0.033	9
" 9	+0.145	+0.0005	— 7.474 ± 0.015	+0.015	15
" 9	+0.223	+0.0005	— 7.627 ± 0.029	+0.015	6
" 10	—0.507	—0.005	— 7.533 ± 0.016	+0.004	15
" 14	+0.426	—0.048	— 9.009 ± 0.030	+0.030	18
" 17	+0.057	—0.003	—11.447 ± 0.018	—0.004	19
" 17	+0.027	—0.003	—11.408 ± 0.019	—0.004	6
" 22	+0.229	+0.010	— 6.685 ± 0.022	—0.030	17
" 24	+0.244	—0.032	— 5.180 ± 0.016	—0.048	14
" 26	+0.433	—0.020	— 2.125 ± 0.016	—0.070	14
" 26	+0.426	—0.020	— 1.824 ± 0.020	—0.070	6
" 28	+0.504	+0.073	+ 1.218 ± 0.022	—0.060	13
" 28	+0.488	+0.023	+ 1.454 ± 0.040	—0.060	6
" 29	+0.473	—0.015	+ 2.453 ± 0.017	—0.030	14
Feb. 10	+0.785	+0.002	— 0.020 ± 0.009	—0.012	9
" 10	+0.888	+0.002	+ 0.026 ± 0.022	—0.012	12

DUXBURY.

Date.	a	c	ΔT_0	Hourly Rate.	Number of Stars.
1869.	s	s	s s	s	
Dec. 14	—0.367	+0.472	+13.548 ± 0.042	—0.160	5
" 14	—0.262	+0.472	+13.868 ± 0.026	—0.160	8
" 15	+0.278	—0.261	+ 2.482 ± 0.021	—0.180	10
" 15	+0.168		+ 3.132 ± 0.031		7
" 23	—0.241	—0.015	— 2.343 ± 0.013	—0.040	19
" 23	—0.320	—0.015	— 2.143 ± 0.016	—0.040	11
" 31	+0.087	—0.015	+ 4.328 ± 0.015	—0.030	13
" 31	+0.081	—0.015	+ 4.361 ± 0.024	—0.030	7
1870.					
Jan. 3	+0.232	—0.010	+ 6.627 ± 0.019	—0.030	16
" 3	+0.122	—0.010	+ 6.806 ± 0.021	—0.030	8
" 4	—0.098	—0.032	+ 7.433 ± 0.015	—0.025	9
" 5	+0.038	—0.023	+ 7.786 ± 0.032	0.000	4
" 5	—0.184		+ 7.951 ± 0.030	—0.045	10
" 7	+0.117	—0.034	+ 7.532 ± 0.027	+0.010	7
" 8	+0.089	—0.051	+ 7.343 ± 0.015	+0.045	11
" 9	+0.478	—0.076	+ 6.584 ± 0.020	+0.012	10
" 11	+0.180	—0.046	+ 6.607 ± 0.025	+0.000	18
" 16	—0.121	—0.037	+ 6.163 ± 0.026	+0.000	9
" 18	+0.013	—0.151	+ 9.462 ± 0.034	—0.015	7
" 19	+4.234	—0.198	+10.026 ± 0.023	—0.018	17
" 21		—0.035		—0.030	
" 22	—0.340	—0.065	+11.638 ± 0.023	—0.030	6
" 23	—0.139	—0.074	+12.420 ± 0.027	—0.040	3
" 26	+0.309	—0.100	+15.579 ± 0.022	—0.045	12
" 26	+0.082		+16.092 ± 0.016		10
" 28	+0.102	—0.085	+18.019 ± 0.018	—0.040	9
" 28	—0.192		+18.453 ± 0.016		8
Feb. 10	—0.196	—0.001	— 2.256 ± 0.021	0.000	6
" 10	—0.373	—0.001	— 2.117 ± 0.024	0.000	7
" 11	—0.015	—0.000	— 2.298 ± 0.026	0.000	4

After the instrumental and the clock corrections have been found, the difference of longitude is computed in the following manner: —

T_1 is the *mean* of the recorded Brest clock times of sending a *set* of westward signals.

T_1' is the *mean* of the recorded Duxbury clock times of receiving these signals.

T_2 is the *mean* of the recorded Brest clock times of receiving a *set* of eastward signals.

T_2' is the *mean* of the recorded Duxbury clock times of sending these signals.

$T_1 - T_1'$ is the mean difference for the westward set.

$T_2 - T_2'$ is the mean difference for the eastward set.

$\Delta T_1 - \Delta T_1'$ is the difference of the clock corrections at Brest and Duxbury, when the westward signals were sent.

$\Delta T_2 - \Delta T_2'$ is the difference of the clock corrections at Brest and Duxbury, when the eastward signals were sent.

x is the mean transmission-time of signals.

λ is the difference of longitude.

λ and x are computed by these two formulas.

(1.) $$2\lambda = (T_1 - T_1') + (T_2 - T_2') + (\Delta T_1 - \Delta T_1') + (\Delta T_2 - \Delta T_2')$$

(2.) $$2x = (T_2 - T_2') - (T_1 - T_1') + (\Delta T_2 - \Delta T_2') - (\Delta T_1 - \Delta T_1')$$

When signals are sent in only one direction,

(3.) $$\lambda = (T_1 - T_1') + (\Delta T_1 - \Delta T_1') + x, \qquad \text{or}$$

(4.) $$\lambda = (T_2 - T_2') + (\Delta T_2 - \Delta T_2') - x$$

In using the formulas (3) and (4), we must adopt an assumed value of x, and this assumed value may be the one computed from the perfect sets of signals of the same class exchanged on the *same* night, or the more general mean derived from the perfect sets exchanged on *all* the nights.

The following table contains the differences of longitude and the transmission time between Duxbury and Brest, *not* corrected for personal equation.

		Class II.		Class III.	
Date.	Sets.	λ	x	λ	x
Jan. 5	1			h m s 4 24 42.731	s 1.135
" "	2			42.762	1.088
" "	Mean			42.746	1.111
Jan. 8	1	h m s 4 24 42.910	s 1.223	s 42.852	s 1.089
" "	2	42.888	1.227	42.781	1.123
" "	3	42.874	1.187	42.968	1.143
" "	4	42.875	1.167	42.950	1.140
" "	5	42.896	1.103	42.838	1.151
" "	6	42.916	1.140	42.888	1.116
" "	Mean	42.893	1.174	42.879	1.127
Jan. 9	1	s 42.891	s 1.052	s 42.854	s 1.055
" "	2	42.889	1.090	42.825	1.053
" "	3	42.850	1.098	42.844	1.063
" "	4	42.886	1.105	42.840	1.062
" "	5	42.923	1.133	42.895	1.001
" "	6	42.934	1.119	42.889	1.031
" "	Mean	42.895	1.100	42.858	1.044
Jan. 10	1	s 42.512	s 1.132	s 42.503	s 1.052
" "	2	42.494	1.126	42.479	1.079
" "	3	42.520	1.147	42.509	1.045
" "	4	42.496	1.114	42.479	1.057
" "	5	42.523	1.116	42.502	1.054
" "	6	42.546	1.139	42.515	1.050
" "	Mean	42.515	1.129	42.498	1.056
Jan. 14	1	s 42.002	s 1.083	s 42.115	s 1.168
" "	2	42.024	1.107	42.070	1.132
" "	3	42.002	1.039	42.067	1.087
" "	4	42.991	1.114	42.063	1.073
" "	5	42.018	1.079	42.071	1.128
" "	6	42.979	1.087	42.011	1.098
" "	Mean	42.003	1.085	42.066	1.114
Jan. 17	1	s 42.870	s 1.064	s 42.998	s 1.131
" "	2	42.873	1.064	42.998	1.251
" "	3	42.880	1.060	42.994	1.195
" "	4	42.922	1.057	42.903	1.107
" "	5	42.904	1.065	42.989	1.146
" "	6	42.900	1.070	42.982	1.174
" "	Mean	42.892	1.063	42.977	1.167

		Class II.		Class III.	
Date.	Sets.	λ	x	λ	x
Jan. 22	1	42.811*		42.937	1.105
" "	2	42.859*		43.016	1.132
" "	3	42.930*		43.022	1.145
" "	4	42.893*		43.040	1.168
" "	5	42.825	1.170	43.005	1.188
" "	6	42.856	1.139	43.078	1.244
" "	Mean	42.853	1.154	43.016	1.164
Jan. 24	1	43.018	1.152	43.130*	
" "	2	43.009	1.096	43.135	1.383
" "	3	43.067	1.090	43.115	1.314
" "	4	43.073	1.092	43.170	1.310
" "	5	43.037	1.110	43.162	1.285
" "	6	43.023	1.156	43.163	1.249
" "	Mean	43.038	1.116	43.148	1.308
Jan. 26	1	43.080*		43.246	1.176
" "	2	43.077	1.065	43.196	1.144
" "	3	43.122	1.106	43.186	1.170
" "	4	43.102	1.098	43.181	1.131
" "	5	43.081*		43.154	1.187
" "	6	43.145*		43.166	1.134
" "	Mean	43.101	1.090	43.188	1.157
Jan. 28	1	43.151	1.091	43.310*	
" "	2	43.160	1.085	43.296	1.200
" "	3	43.143	1.098	43.325	1.183
" "	4	43.168	1.072	43.263	1.182
" "	5	43.151	1.070	43.226	1.135
" "	6	43.141	1.069	43.259	1.170
" "	Mean	43.152	1.081	43.276	1.174
Feb. 9	1	43.098		43.076*	
" "	2	43.084		43.070*	
" "	3	43.085		43.130	1.228
" "	4	43.096		43.048	1.206
" "	5	43.121		43.048*	
" "	6	43.146		43.028	1.216
" "	Mean	43.105		43.068	1.217
		Class II.		Class IV.	
Date.	Sets.	λ	x	λ	x
Feb. 10	1	42.683			
" "	2	42.629		42.743	
" "	3	42.625		42.652	
" "	4	42.645		42.715	
" "	5			42.610	
" "	6			42.671	
" "	7			42.630	
" "	8			42.617	
" "	Mean	42.645		42.663	

A weight of $\frac{1}{2}$ is given to each of the values marked with an asterisk in obtaining the means of the groups. For these values were obtained from signals in one direction only, with assumed values of the transmission time, derived from other sets of the same group. On February 10, the Brest signals were not received at Duxbury. An assumed value of (x) was applied to the signals of Class II, sent from Duxbury, viz. the mean value of x derived from all the other signals of Class II. An assumed value of (x) was applied to the signals of Class IV., viz. the mean value of x obtained from all the signals of Classes II. and III.

DIFFERENCE OF LONGITUDE BETWEEN BREST AND DUXBURY.

Date.	λ	λ	λ	λ
1870.	Class II.	Class III.	Class IV.	Means.
	h m s	h m s	h m s	h m s
Jan. 5		4 24 42.746		4 24 42.746
" 8	4 24 42.893	42.879		42.886
" 9	42.895	42.858		42.876
" 22	42.853	43.016		42.935
" 26	43.101	43.188		43.144
" 28	42.152	43.276		43.214
Feb. 10	42.645		4 24 42.663	42.654
Mean of Division A				4 24 42.922 ±.051
	h m s	h m s		h m s
Jan. 10	4 24 42.515	4 24 42.498		4 24 42.507
" 14	42.003	42.066		42.034
" 17	42.892	42.977		42.935
" 24	43.038	43.148		42.093
Feb. 9	43.105	43.068		42.086
Mean of Division B (Rejecting January 14)				4 24 42.905 ±.093

Division A includes all the nights on which the local time was obtained at *both* stations by transits observed on the night of the signals. Division B contains all the nights on which the *Duxbury* time was computed from transits observed on the preceding or following nights. The value of the longitude for January 14 is rejected because on that night the time was computed from observations made on January 11 and 16.

The work of the best seven nights makes $\lambda = 4^h\ 24^m\ 42^s.922 \pm .051$

The work of four inferior nights makes $\lambda = 4^h\ 24^m\ 42^s.905 \pm .093$

If weights are assigned to these two results inversely proportional to the squares of their probable errors, the final result is $\lambda = 4^h\ 24^m\ 42^s.918 \pm .045$

The correction for the difference of the Personal Equation in taking transits between Mr. Dean and Mr. Goodfellow is $-\ 0^s.033 \pm .012$

The correction for the difference of the Personal Equation in recording cable-signals between these observers is $-\ 0^s.018 \pm .006$

Applying these corrections, we have $\lambda = 4^h\ 24^m\ 42^s.867 \pm .047$

This result is computed from 1324 signals of Class II., 2245 signals of Class III., and 52 signals of Class IV., in all 3621 signals; of which about half were negative and half positive, and about half sent from Brest to Duxbury and half from Duxbury to Brest.

MEAN RESULTS OF SIGNALS SENT FROM DUXBURY TO CAMBRIDGE.

Date.	Kind of Signal.	Number.	T_1	$T_1 - T_1''$	$\Delta T_1 - \Delta T_1''$	$\lambda - x$
1869			h m	s	s	s
Dec. 14	Sidereal Clock,	58	1 55	—24.156	134.443	110.287
" 14	Sidereal Clock,	59	2 17	—24.185	134.486	110.301
" 15	Sidereal Clock,	59	3 18	—12.776	123.083	110.307
" 15	Key or Hand,	57	3 27	—12.746	123.108	110.362
" 23	Sidereal Clock,	59	3 21	— 6.290	116.727	110.437
" 23	Key or Hand,	60	3 30	— 6.262	116.737	110.475
" 31	Sidereal Clock,	59	3 55	—12.345	122.741	110.396
" 31	Key or Hand,	84	4 6	—12.323	122.742	110.419
1870						
Jan. 3	Sidereal Clock,	59	3 58	—14.508	124.766	110.258
" 3	Key or Hand,	57	4 6	—14.505	124.774	110.269
" 4	Sidereal Clock,	59	4 31	—15.218	125.330	110.112
" 14	Cable Key, Class III.,	63	3 59	— 9.522	120.695	110.173
" 14	Cable Key, Class II.,	48	4 18	— 9.508	120.689	110.181
" 22	Cable Key, Class III.,	70	4 43	—12.396	122.893	110.497
" 22	Cable Key, Class II.,	74	5 3	—12.373	122.899	110.526
" 24	Cable Key, Class III.,	61	4 54	—13.403	124.027	110.624
" 24	Cable Key, Class II.,	110	5 15	—13.421	124.038	110.617
" 26	Cable Key, Class III.,	68	5 8	—15.382	125.654	110.272
" 26	Cable Key, Class II.,	95	5 28	—15 416	125.691	110.275
" 28	Cable Key, Class III.,	62	5 8	—17.307	127.608	110.301
" 28	Cable Key, Class II.,	112	5 28	—17.297	127.636	110.339
Feb. 9	Cable Key, Class III.,	59	6 14	+ 9.796	100.974	110.770
" 9	Cable Key, Class II.,	61	6 37	+ 9.792	100.961	110.753
" 10	Cable Key, Class IV.,	54	6 32	+10.241	100.244	110.485
" 10	Cable Key, Class II.,	20	6 46	+10.256	100.249	110.505

MEAN RESULTS OF SIGNALS SENT FROM CAMBRIDGE TO DUXBURY.

Date.	Kind of Signal.	Number.	T'	$T' - T''$	$\Delta T' - \Delta T''$	$\lambda + x$
1869			h m	s	s	s
Dec. 14	Sidereal Clock,	59	2 5	—24.081	134.462	110.381
" 15	Sidereal Clock,	59	3 33	—12.734	123.126	110.392
" 15	Key or Hand,	59	3 39	—12.765	123.143	110.378
" 23	Sidereal Clock,	59	3 37	— 6.203	116.744	110.541
" 23	Key or Hand,	57	3 43	— 6.241	116.750	110.509
" 31	Sidereal Clock,	59	4 13	—12.295	122.744	110.449
" 31	Key or Hand,	43	4 18	—12.314	122.744	110 430
1870						
Jan. 3	Sidereal Clock,	59	4 12	—14.478	124.781	110.303
" 3	Key or Hand,	46	4 15	—14.496	124.784	110.288
" 22	M. T. Clock,	50	4 12	—12.355	122.885	110.530
" 24	M. T. Clock,	50	4 26	—13.361	124.014	110.653
" 26	M. T. Clock,	50	4 29	—15.299	125.583	110.284
" 28	M. T. Clock,	50	4 48	—17.240	127.578	110.338
Feb. 9	M. T. Clock,	50	5 33	— 9.790	100.995	110.785
" 10	M. T. Clock,	50	5 25	—10.277	100.225	110.502

DIFFERENCE OF LONGITUDE BETWEEN CAMBRIDGE AND DUXBURY, FROM SIGNALS SENT IN BOTH DIRECTIONS.

Kind of Signal.	Date.	$\lambda - x$	$\lambda + x$	x	λ
Sidereal Clocks.	Dec. 14	110.294	110.381	0.044	110.337
	" 15	110.307	110.392	0.042	110.350
	" 23	110.437	110.541	0.052	110.489
	" 31	110.396	110.449	0.027	110.422
	Jan. 3	110.258	110.303	0.022	110.281
	" 4	110.112		0.037	110.149
Key or Hand-signals.	Dec. 15	110.362	110.378	0.008	110.370
	" 23	110.475	110.509	0.017	110.492
	" 31	110.419	110.430	0.006	110.425
	Jan. 3	110.269	110.288	0.009	110.278
Cable-Key at Duxbury and M. T. clock at Cambridge.	Jan. 14	111.176		0.009	111.185
	" 22	110.512	110.530	0.009	110.521
	" 24	110.619	110.653	0.017	110.636
	" 26	110.274	110.284	0.005	110.279
	" 28	110.325	110.338	0.007	110.331
	Feb. 9	110.761	110.785	0.012	110.773
	" 10	110.490	110.502	0.006	110.496

On January 14, 22, and 24, and on February 9, the time at one or both of the stations was computed from the error and rate of the clocks obtained on other nights, in most cases two or three days intervening. The clock-rates were not constant, and the results, therefore, are not trustworthy. On January 4, the clock error at Cambridge was found from poor observations on only two stars. In the following table the observations on all these nights are rejected.

Date	$\lambda - x$	$\lambda + x$	x	λ
Dec. 14	110.294	110.381	0.044	110.337
" 15	110.334	110.385	0.025	110.360
" 23	110.456	110.525	0.034	110.490
" 31	110.407	110.440	0.016	110.423
Jan. 3	110.263	110.296	0.016	110.280
" 26	110.274	110.284	0.005	110.279
" 28	110.325	110.338	0.007	110.331
" 10	110.490	110.502	0.006	110.496
	Mean of 1672 signals	$\lambda = 110^s.375$ $\pm .021$		

The correction for the difference of the Personal Equation in taking transits between Mr. Goodfellow and Mr. Austin is $-0^s.145 \quad \pm .007$

Applying this correction we have $\lambda = 0^h\ 1^m\ 50^s.230 \quad \pm .022$

for the difference of Longitude between Duxbury and Cambridge.

Difference of Longitude between Brest and Duxbury $4^h\ 24^m\ 42^s.867 \quad \pm .047$

Difference of Longitude between Brest and Cambridge $4^h\ 26^m\ 33^s.097 \quad \pm .052$

The Coast Survey Transit-Instrument at the Cambridge Observatory occupies a position which is 28 feet (or .025 of a second) west of the Dome. The Coast Survey Transit-Instrument at Brest stood 413.5 feet (or .41 of a second) west of the Geodetic station on the St. Louis Tower. Reducing the longitudes above given to the Dome of the Observatory and the St. Louis Tower, we have:—

Difference of longitude between Duxbury and Cambridge	$1^m\ 50^s.205$	$\pm$ .022
Difference of longitude between Brest and Duxbury	$4^h\ 24^m\ 43^s.277$	$\pm$ .047
Difference of longitude between Brest and Cambridge	$4^h\ 26^m\ 33^s.482$	$\pm$ .052

In the determination of differences of longitude by telegraphic signals, a correction must be applied on account of the Personal Equation of the observers in noting transits of stars; and when the signals are sent by the cable-lines, another correction is required on account of the Personal Equation in observing and recording signals.

1. What is called the Personal Equation in noting transits is made up of many elements, some of which are personal and others instrumental. When a star has reached a wire of the transit-instrument, the phenomenon does not impress itself upon the eye of the observer instantaneously; then the effect upon the eye must be communicated to the brain; then the will must be aroused so as to send its order to the finger; and this order when sent consumes a measurable fraction of time in travelling through the nerve; there is also another delay in the execution of this order by the muscles and the finger. After the finger touches the key, that must move, contact must be made or broken, the disturbance must go through the local circuit to the electro-magnet which works the pen of the chronograph, and the magnet must act upon its armature. All these various processes require a fraction of time, however small, and their aggregate is the Personal Equation. The Personal Equation, therefore, depends partly on the mental and physical constitution of the observer, and partly upon the instruments through which he operates. If the delay in noting transits was the same at both stations, no error would be introduced into the difference of their local times or into their difference of longitude. Even when the instruments at the stations are strictly the same, the observers are different, and, therefore, the Personal Equations are different, and a correction must be applied on account of the difference of the Personal Equations of the two observers. If it is practicable for the observers to exchange stations with each other for half of the campaign, the difference of their Personal Equations will be eliminated when the separate results of the two parts of the campaign are united into a final mean. Where this cannot be conveniently done, as it obviously cannot be done in operating for transatlantic longitudes, the difference between the Personal Equations of the two observers may be ascertained, at least so far as it is constant, by processes especially devised for the purpose.

The difference of the Personal Equation in time determinations between Mr. Dean and Mr. Goodfellow has been computed from observations made by them at the Cambridge Observatory on three nights, viz. May 13, 17, and 18. On May 13, alternate tallies were taken by them, sometimes one and sometimes the other leading. On May 17 and 18, the first and last tallies were taken by one observer, and the three middle tallies by the other. Twenty-six stars were observed on each of these nights.

The difference of the Personal Equation in time determination between Mr. Goodfellow and Mr. Austin was computed from observations made by the first method on May 15, and by the second method on May 17 and 18.

The following results were obtained: —

DEAN — GOODFELLOW.

1870.	May 13		$-0^s.066 \pm .004$
	" 17		$-0.023 \pm .008$
	" 18		$-0.009 \pm .006$
		Mean	$-0.033 \pm .012$

Mr. Dean records a transit *earlier* than Mr. Goodfellow by .033 of a second.

AUSTIN — GOODFELLOW.

1870.	May 15		$+0^s.129 \pm .011$
	" 17		$+0.163 \pm .015$
	" 18		$+0.144 \pm .008$
		Mean	$+0.145 \pm .007$

Mr. Austin records a transit later than Mr. Goodfellow by .145 of a second.

These results were calculated by the formula

$$(M_1 - M_2) + (F_1 - F_2) \sec \delta = \pm \epsilon$$

in which M_1 represents the mean of the observed threads by the observer who led, and M_2 the mean of the observed threads by the observer who followed; and $F_1 \sec \delta$ and $F_2 \sec \delta$ the corresponding reductions to the mean of all the threads. On May 13 and 15, $F_1 - F_2$ was equal to $-0^s.093$; and on May 17 and 18, $F_1 - F_2$ was equal to $-0^s.060$; Coast Survey Transit No. 5 being used on all the nights.

2. If the electrical current sent through the main line were able to operate a relay-magnet, and through it to control the local circuit and battery, it might record itself upon the chronograph at the place where it is received, as it does at the place from which it is sent. But, in fact, the signals are not thus automatically recorded. The way in which they are recognized and registered is described on page 440. A

fraction of a second intervenes between the real and the recorded arrival of the signal. Various elements enter into this loss of time, partly physiological, and partly instrumental, as in the other case; their aggregate effect is the Personal Equation in noting signals. This is not exactly the same for the two observers at the opposite stations, otherwise it would be eliminated by employing signals sent in both directions alternately. Therefore, it must be determined separately for the two observers, and the proper allowance be made for the difference.

A series of observations was made by Mr. Dean on January 24 and 25, in order to determine his Personal Equation in recording Cable-Signals at Brest. The chronograph was adjusted so that it would make two and one quarter revolutions in a minute of time; one second of time being represented on the chronograph sheet by a distance of three quarters of an inch. A galvanometer, similar to the Thompson galvanometer used in the Cable Office, was adjusted in the *testing-room* of the office. The cable-key remained in its usual place in the *instrument-room*, which is at some distance from the *testing-room*. When this galvanometer and the cable-key were united with a single cell of Minotto's battery, and ten thousand units of resistance coils, in addition to the circuit which passed between the cable office and the Coast Survey Station, and the clock and chronograph, were introduced, the beam of light from the mirror of the galvanometer moved in the same way as when the Second Class of cable-signals came from Duxbury. On January 30, the experiments were modified by placing the cable-key in the *testing-room*, and substituting for the first galvanometer that very one in the instrument-room by which all the longitude signals had been received.

A series of observations was made by Mr. Goodfellow on February 8, 9, and 12, in order to determine his Personal Equation in recording Cable-Signals at Duxbury. The current from a Daniell's battery of three elements was sent through the clock and chronograph of the transit-room of the Coast Survey Observatory, and through the cable-key and the break-circuit keys in the Cable Office. The current from a single cell of Minotto's battery was sent through the same circuit, and the galvanometer used in receiving cable-signals from Brest was shunted so as to make it produce deflections similar to those observed with the cable-signals. The signals proceeded from the battery-room, and the observer who noted and recorded them was placed in another part of the building, out of sight and hearing of anything which occurred in the battery-room. In all these observations made at Brest and Duxbury, to ascertain the retardation in noting cable-signals, these signals were sent through the local circuit which connects the Cable Office with the Coast Survey Station. In order to prevent the interference of the clock-breaks with the passage of these signals, the clock is

switched out of the circuit while the signals are passing, and then introduced for one or two minutes between the different sets of signals, so as to graduate the chronograph sheet into seconds. In this way, the exact time of *sending* and of *noting* each signal would be ascertained. As the transmission time in these experiments is insensible, the difference between the recorded times of sending the signal and of noting it gives the retardation in noting cable-signals.

January 24.			January 25.			January 30.		
Number.	Sums.	Means.	Number.	Sums.	Means.	Number.	Sums.	Means.
10	3.54	$0^s.354$	10	3.39	$0^s.339$	10	3.11	$0^s.311$
10	3.21	0.321	10	3.24	0.324	10	3.14	0.314
9	3.06	0.340	10	3.52	0.352	10	3.38	0.338
8	2.97	0.371	10	3.37	0.337	10	3.16	0.316
10	3.55	0.355	10	3.17	0.317	10	3.29	0.329
7	2.41	0.344	10	3.33	0.333	10	2.95	0.295
6	1.95	0.325	10	3.30	0.330	10	2.75	0.275
6	1.93	0.322	10	3.28	0.328	10	2.99	0.299
9	3.80	0.422	10	3.51	0.351	10	3.19	0.319
10	3.54	0.354	10	3.22	0.322	8	2.33	0.291
10	3.69	0.369	10	3.54	0.354	8	2.25	0.281
10	3.67	0.367	10	3.59	0.359			
			10	3.59	0.359			
			10	3.72	0.372			
			10	3.18	0.318			
			10	3.65	0.365			
			9	2.96	0.329			
105	37.32	$0^s.355$ $\pm.005$	169	57.56	$0^s.341$ $\pm.003$	106	32.54	$0^s.307$ $\pm.004$

February 8.			February 9.			February 12.		
Number.	Sums.	Means.	Number.	Sums.	Means.	Number.	Sums.	Means.
9	2.81	$0^s.312$	5	1.70	$0^s.340$	12	3.71	$0^s.309$
10	3.35	0.335	9	2.62	0.291	12	3.30	0.275
12	3.71	0.309	7	2.19	0.313	12	3.44	0.287
12	3.49	0.291	12	3.58	0.298	12	3.38	0.282
11	3.72	0.338	12	3.69	0.307	12	3.22	0.268
11	3.54	0.322	12	4.11	0.343	12	3.38	0.282
12	3.93	0.327	12	3.68	0.307	12	3.18	0.265
12	3.72	0.310	12	3.51	0.292	12	3.15	0.262
12	3.66	0.305	12	3.66	0.305	12	3.13	0.261
12	3.66	0.305	12	3.74	0.312			
12	3.56	0.297						
12	3.93	0.328						
12	3.47	0.289						
12	3.53	0.294						
12	3.41	0.284						
11	3.17	0.288						
184	56.66	$0^s.308$ $\pm.003$	105	32.48	$0^s.309$ $\pm.004$	108	29.89	$0^s.277$ $\pm.003$

The mean delay of Mr. Dean in recording cable-signals, as computed from three hundred and eighty signals, is $0^s.334 \pm .010$.

The mean delay of Mr. Goodfellow in recording cable-signals, as computed from three hundred and ninety-seven signals, is $0^s.298 \pm .007$.

TRANSMISSION TIME BETWEEN DUXBURY AND BREST.

	January										Feb.	Mean.
	5	8	9	10	14	17	22	24	26	28	9	
CLASS II.		s 1.174	s 1.100	s 1.129	s 1.085	s 1.063	s 1.154	s 1.116	s 1.090	s 1.081	s	s 1.110 ±.008
CLASS III.	s 1.111	1.127	1.044	1.056	1.114	1.167	1.164	1.308	1.157	1.174	1.217	1.149 ±.015
MEAN.	1.111	1.150	1.072	1.093	1.200	1.115	1.159	1.212	1.123	1.178	1.217	1.132 ±.009

The time lost by Mr. Dean in noting signals at Brest was $0^s.334$, and by Mr. Goodfellow in noting signals at Duxbury was $0^s.298$. If we correct the transmission time for the personal and instrumental delay in recording cable-signals, we have

$$x = 1^s.132 \pm .009$$

$$\text{Correction} \quad = \tfrac{1}{2}(0.334 + 0.298) = .316 \pm .006$$

$$\text{Corrected value of } x = 0^s.816 \pm .011.$$

The mean transmission time between Duxbury and Cambridge, as obtained from the Arbitrary Hand-signals, is $0^s.010$; from the Cable Key-signals at Duxbury and the Mean Time Clock-signals at Cambridge, is $0^s.009$; and from the Sidereal Clock-signals at both places, is $0^s.037$.

When arbitrary hand-signals were sent from Duxbury to Cambridge, arbitrary hand-signals were returned from Cambridge to Duxbury. When such signals are sent, the key breaks only the main line at both stations, and the current acts through a relay-magnet at both stations. Therefore, by combining the results of these signals when sent in both directions, the transmission time is correctly eliminated. When clock-signals from the Mean Time Clock were sent from Cambridge, it only broke the main line as an arbitrary hand-signal would have broken it, and therefore such signals are properly combined with the signals sent from Duxbury to Cambridge by the cable-key. When sidereal clock-signals are sent in both directions, the results may be compared, so as to eliminate the transmission time and deduce a correct longitude. Inasmuch as, however, in this case, the sidereal clock breaks directly the main line, without

the intervention of a relay-magnet at the sending station, and acts through a relay-magnet at the receiving station, it is evident that the difference of time registered by the two chronograph-sheets will express not only the difference of longitude and the transmission time, but also the uncompensated resistance of the relay-magnet. If the difference of local times which obtains when such signals are sent from Cambridge to Duxbury is subtracted from the difference of local times when such signals are sent from Duxbury to Cambridge, the remainder will represent not merely twice the transmission time, but twice the transmission time increased by twice the resistance of the relay-magnet. This accounts for the large value of the *apparent* transmission time derived from the sidereal clock-signals when compared with the transmission time indicated by hand-signals or by the Mean Time Clock-signals.

The St. Pierre and Brest cables, when combined, make a total length of thirty-three hundred and twenty-nine nautical miles. The total resistance of the cable between Duxbury and St. Pierre amounts to 8980.51 ohms; the total resistance of the cable between St. Pierre and Brest (the greater diameter of the wire overbalancing the increased length) is only 8152.80 ohms. When the resistance of the galvanometer is added to the sum of the above-named resistances, the whole resistance between Duxbury and Brest is found to be 18321.31 ohms. The degree of insulation is expressed by saying that the gutta-percha resistance between Duxbury and St. Pierre is 1,722,700 megohms, and between St. Pierre and Brest 6,204,900 megohms; in all, 7,927,600 megohms. It appears that the transmission time of the electrical signals through this cable is .816 of a second. It would be a hasty conclusion, however, to suppose that the velocity of electricity is 4,080 miles a second, or that it has any velocity in the sense in which we speak of the velocity of a cannon-ball or of the velocities of sound or light. There is an advantage in substituting for the word *velocity* the phrase *transmission-time*, or in defining the velocity of electricity as the distance passed over in *any particular case* divided by the time. The peculiarity of the motion is this: if the distance to be traversed by the electricity had been greater, not only would an additional time have been required for the additional distance, but more time would be required for the first distance. Ohm, in his admirable but too long neglected treatise, published in 1827, under the title *Die galvanische Kette mathematisch bearbeitet*, after treating of the permanent state of the galvanic circuit, takes up the subject of the distribution of tensions in the variable state of the circuit, and arrives at this formula: —

$$u = \frac{a}{2l}\, x + a\, \Sigma \left(\frac{1}{i\pi} \sin \frac{i\pi(l+x)}{l} \cdot e^{\frac{-k'\pi^2 i^2 t}{l^2}} \right).$$

In this equation (u) is the tension at any point, the distance of which from the middle of the circuit is (x); (a) is the tension at the point of excitation; ($2l$) is the length of the circuit; (k') is the conducting power, divided by a coefficient which expresses the specific electrical capacity of the substance; (t) is the time; (π) is the ratio between the circumference and diameter of a circle; (e) is the Napierian logarithmic base; and (i) is any positive number. M. Gangain, in his commentary upon the treatise of Ohm, which he translated[1] into French, has pointed out the conclusions to be drawn from this formula. He says that it has been established, on the assumption that there was but a single electromotive force brought into play in the circuit, that this force was constant, and that the circuit was homogeneous. In a voltaic pile there are many electromotive forces developed at different points of the circuit; if the resistance of the pile is only a small fraction of the total resistance of the circuit, there will be no sensible error in the supposition that the pile is concentrated in one point, that its resistance is nothing, and that the sum of the electromotive forces is represented by the letter (a) of the formula. When the permanent state of the conductor is established, the value of (u) becomes equal to $\left(\frac{a}{2l}x\right)$; the term comprehended under the symbol (Σ) (which is obviously smaller as the time (t) increases) having become too insignificant to be regarded. This will happen as soon as ($k'\pi^2 i^2 t$) is large, compared with (l^2). Whatever value of (t) may be sufficient for this purpose with any given value of (l), it is evident that, in general, the required value of (t) must be proportional to (l^2). This value of (t) represents the *duration of the propagation* before the permanent state is acquired. It is evident that this value of (t) may be less as the value of (k') is greater. If the velocity of electricity is defined as $\left(\frac{l}{t}\right)$, it will have no determinate value, but may be exceedingly great or exceedingly small according to the distance to be travelled. The passage of electricity is not analogous to the transmission of sound or light, but resembles rather the conduction of heat. This will appear from comparing Ohm's formula with that obtained by Poisson for the conduction of heat along a metallic rod when the two extremities are maintained at a constant temperature.[2]

After Kirchhoff had succeeded in deducing the familiar formulas of Ohm, which express the constant voltaic current, from the principles of statical electricity, he gave his attention to the variable state of the current, and he has obtained expressions[3] for the quantity and intensity of the free electricity at any point of the conductor which

[1] Théorie Mathématique des Courants Electriques, p. 177.

[2] Journal de l'Ecole Polytechnique, XIX. p. 53.

[3] Pogg. Ann. der Physik und Chemie, C. 193 und CII. 529.

admit of two interpretations. One of the factors depends on the length, diameter, and total resistance of the conductor. When this factor is very great, the quantity of electricity tends towards uniformity of distribution as the time progresses, whereas the current approximates towards zero. Moreover, the formulæ, in this case, are analogous to those which represent the propagation of sound in a narrow tube. From this Kirchhoff concludes that two electrical waves are transmitted through the conductor in opposite directions, and with a velocity of 310,765 kilometres (about 193,000 miles) per second. If, on the other hand, the critical factor is very small, a result is reached which shows that the propagation of electricity is analogous to that of heat. In these circumstances there is no more any question about the velocity (in the common sense of the word) of electricity, except that it may be pronounced greater when the conductivity and diameter of the wire are increased.

Almost contemporaneous with these researches of Kirchhoff was an investigation by Sir William Thompson on the "Theory of the Electric Telegraph." He obtained a complex formula for the *potential* at any point of the conducting wire (one end being connected with the battery and the other with the ground), one term of which varies with the time. This term is similar to the variable term in Ohm's formula. Thompson concludes that, though an infinitesimal effect may reach the remote end instantaneously, the time required for the current to reach a *stated fraction of its maximum strength* sufficient to show itself (which may be less as the galvanometer is more delicate) at the remote station will be proportional, in different lines, to the product of the square of the length by the resistance, (electrostatically measured,) and by the electrostatical capacity of the unit of length. Therefore, there is no regular velocity of transmission.

"We may infer that the retardations of signals are proportional to the squares of the distances, and not to the distances simply; and hence different observers, believing they have found a 'velocity of electric propagation' may well have obtained widely discrepant results; and the apparent velocity would, *cæteris paribus*, be the less the greater the length of wire used in the observations."[1] For example, as Professor Stokes has said, if the retardation on the submarine line between Greenwich and Brussels (200 miles in length) is one tenth of a second, the average velocity of the signal is 2,000 miles per second. If a similar cable were extended over a semicircumference of the earth (about 14,000 miles), the retardation would amount to four hundred and ninety seconds! In this case, the average velocity would be seventy times less, or only $28\frac{1}{2}$ miles per second.

[1] Royal Society Proceedings (London), VII. pp. 382 – 99.

These theoretical considerations will go far towards explaining the apparent contradictions between the results obtained by Wheatstone, Walker, Mitchel, Gould, Fizeau, Faraday, Gaugain, Guillemin, Jenkin, Clark, and many others, who have experimented upon the velocity of electricity, — results which range from 288,000 miles per second to 800 miles per second. No two experiments are properly compared with each other unless a variety of conditions is taken into the account. The enormous velocity obtained by Wheatstone favors the supposition that electricity of high tension (as that which exists in a charged Leyden jar) is endowed with a superior power of transmission. But the experiments of Mr. Latimer Clark[1] (quoted by Faraday) have proved that a voltaic battery might be increased from 31 cells to 500 cells, without sensibly changing the velocity, when the current was sent through 768 miles of copper wire covered with gutta-percha. Moreover, Wheatstone's experiment only proved that the electricity went through less than a mile of wire (in addition to the air spaces where the sparks occurred) *at the rate* of 288,000 miles a second. If many persons have hastily come to the conclusion that electricity would actually move through 288,000 miles of the same kind of wire in a single second, they have made the assumption, which neither theory nor observation warrant, that the velocity is independent of the total resistance of the wire and the length to be traversed. Melloni appears to have adopted this view, as an inference from Clark's experiments, already quoted. He says:[2] "The equal velocity of currents of various tension offers, on the contrary, a fine argument in favor of the opinion of those who suppose the electric current to be analogous to the vibrations of air under the action of sonorous bodies. As sounds, higher or lower in pitch, traverse in air the same space in the same time, whatever be the length or the intensity of the aerial wave formed by the vibration of the sonorous body, so the vibrations, more or less rapid or more or less vigorous, of the electric fluid, excited by the action of batteries of a greater or smaller number of plates, are propagated in conductors with the same velocity." The theoretical examinations which have been made of the problem by Ohm, Gaugain, Kirchhoff, and Thompson all point the other way. They all imply that the whole space travelled by electricity is proportional to the square root of the time. A simple computation conducts Gaugain to the conclusion that Wheatstone's experiment, when interpreted in the light of the best theoretical knowledge, only proved that electricity would take one second to run over 268 miles of similar wire. But Gaugain has evidently gone to

[1] London Philosophical Magazine, 1855.

[2] As quoted by Faraday. Experimental Researches in Electricity, III. 577.

the other extreme, and underrated the velocity. He has reasoned as if, in the experiment, the whole distance traversed was only one quarter of a mile of wire. In fact, it was more than this, and the air-spaces besides. What the equivalent in wire would be for these air-spaces it might not be easy to state with precision. Moreover, the tension of the Leyden jar, as Gaugain is ready to see, was not constant during the discharge, and did not, therefore, conform to the conditions of Ohm's calculations. If, however, the theory must not be *rigidly* applied in opposition to Wheatstone's conclusion, it is sufficiently clear that the enormous discrepancy between the velocity to be deduced from his remarkable experiment and that afterwards revealed by direct observation upon long lines of telegraphic wire no longer exists. Indeed, the corrected velocity from Wheatstone's experiment would be unaccountably *small*, when compared with these later determinations, unless they also received the proper qualification demanded by the same theory.

If an objection is made to the inferences drawn from the mathematical theory of electricity, because this theory is perplexed by difficult questions of analysis, a stronger objection holds against the assumption that the velocity is the same for long as for short distances, as this is sustained neither by theory nor experiment. Undoubtedly, the safest way for determining the velocity of electricity — or, more properly stated, the transmission time — over very long wires is by experiments on these same wires. An experiment upon a very large scale was made by Professor Joseph Winlock, at the Observatory of Harvard College, on the nights of February 28 and March 7, 1869. Signals were sent from the Observatory to San Francisco by one line of wire, and returned to the Observatory by the way of Canada, and the times of leaving and returning to the Observatory were recorded upon the same chronograph-sheet. The time required to pass through this enormous loop of 7,200 miles of No. 9 iron wire, and thirteen telegraph-repeaters, was about two thirds of one second. This result agreed closely with the time obtained from signals sent directly between Cambridge and San Francisco with earth connections, and the transmission time deduced by Mr. Bradford in his computation of the Coast Survey operations for longitude between these two cities. The transmission time in this last case was about two fifths of a second; thus showing that the velocity was nearly the same, whether the electricity discharged into the earth or returned by a loop.

If two wires, without any other difference between them but that of length, are compared, the transmission times are as the squares of the lengths, so that the average velocity is inversely as the length. If the diameters and specific conductivities are unlike, we can substitute equivalent lengths of a standard size and quality, and then

compare these reduced lengths. But other things besides the dimensions and quality of the wire influence the velocity of electricity. Ohm's formula, already discussed, contains a letter which represents the electrostatic capacity of the conductor. But he did not live to see its full significance as revealed in subterranean lines of telegraph. In the very year (1854) when Ohm was seized with a mortal illness, Faraday communicated to the Proceedings of the Royal Institution[1] of Great Britain, some remarkable experiments, made originally by Mr. Latimer Clark, and then repeated, under his inspection, at the Works of the Electric Telegraph Company. One hundred miles of copper wire, covered with gutta-percha, were immersed in water, and when a voltaic current was sent through this circuit, phenomena resulted which were never observed when the wire was surrounded by air. The immersed wire was charged like a Leyden jar, the gutta-percha serving as the dielectric, and the water as the outer coating. In this particular case, the copper wire was one sixteenth of an inch in diameter, and the gutta-percha about one tenth of an inch in thickness. Regarded as a Leyden arrangement, the inner coating, or surface of the copper, contained 8,300 square feet, and the outer coating, or the surface of water in contact with the gutta-percha, contained 33,000 square feet. Experiments were made upon wires covered with gutta-percha, and then enclosed in tubes of lead or iron, or buried in the earth, and with similar results. By connecting the various subterranean wires between London and Manchester, a total length was obtained of 1,500 miles. When galvanometers were introduced at intervals of about 400 miles, they were all under the eye of the same observer in London. If a current was sent into this wire the galvanometers were *successively* affected; the last one only after the long interval of two seconds. Faraday explained this slow transmission through such wires, when compared with the velocity of electricity through air-lines (where no near conductor is present to supply the place of the outer metallic surface in the Leyden arrangement), to the increased electrostatic capacity of the wire under the inductive action, and to the time required for the battery to furnish the additional amount of electricity. Sir William Thompson[2] has proved that the electrostatic capacity of such a wire is equal to $\frac{VkS}{4\pi r \log \frac{r'}{r}}$, in which ($V$) is the potential, ($k$) the specific inductive capacity of gutta-percha, (S) the surface of the wire, (r) the radius of the copper or the inner surface of the gutta-percha, and (r') the radius of the outside surface of the gutta-percha. Every hundred miles of wire, similar to that used in Faraday's first experiments, had an electrical capacity at least equal to a Leyden battery with 8,300 square feet of coated sur-

[1] Proceedings of the Royal Institution, January 20, 1854.

[2] Papers on Electrostatics and Magnetism, p. 41.

faces separated by a thickness of only $\frac{1}{32}$ of an inch of glass. When a current is fully established and the wire is charged, an immersed cable-line will conduct as well as an air-line. But when the connection is first made with the battery, or first broken, the charge in one case, and the discharge in the other, will travel more slowly in the cable-line than in the air-line; in other words, the time of the variable state of the conductor will be prolonged.[1]

In the cable between St. Pierre and Brest, the inner surface of the gutta-percha (which represents approximately the surface of copper to be charged) amounts to about 700,000 square feet, and the outer surface to about 2,000,000 square feet. In the cable between Duxbury and St. Pierre, the inner surface contains about 100,000 square feet, and the outer surface about 300,000 square feet. Both cables united correspond to a Leyden arrangement, in which one surface has about 800,000 square feet and the other about 2,300,000 square feet. If the dielectric were glass instead of gutta-percha, the equivalent thickness of the glass would be, according to Thompson's formula already given, $\frac{k'}{k}$ multiplied by .043 of an inch for the St. Pierre and Brest cable, and $\frac{k'}{k}$ multiplied by .084 of an inch for the Duxbury and St. Pierre cable; k' and k representing respectively the inductive capacities of glass and gutta-percha. Experiment[2] shows that the ratio $\frac{k'}{k}$ is about $\frac{1.9}{4.2}$. The electrostatical capacity of the St. Pierre and Brest cable has been already given as equal to .404 of a microfarad for each nautical mile. Its total capacity is about 1,042 microfarads. The electrostatical capacity of the Duxbury and St. Pierre cable is .358 of a microfarad for each nautical mile, and the total electrostatical capacity is about 268 microfarads. The electrostatical capacity of both cables is about 1310 microfarads.

This total electrostatical capacity of both cables expresses the whole amount of electricity they contain when one end is united to a battery having an electromotive force of one volt, and the other end is put *to air;* that is, is disconnected with the ground. As the forty cells of Minotto's battery used at Duxbury were equivalent to about 33.7 of Daniell's cells, its electromotive force would be equal to about 36 volts, that of a simple Daniell's cell being taken as 1.079 volts. If one end of the united cables is connected to this battery, and the other end is insulated in the air, so that the difference of potential between the inner and outer surfaces of the gutta-percha envelope is equal to 36 volts, the whole amount of electricity required to charge the cable is 47,160 microfarads. The insulation of each knot of cable between Duxbury

[1] See also experiments of W. Siemens, Ann. de Ch. et de Ph., XXIX. 394.

[2] Clark and Sabine's Electrical Tables and Formulæ, p. 67.

and St. Pierre being 2,300 megohms, the total insulation is 3,880,000 ohms. The insulation of each knot of cable between St. Pierre and Brest being 2,405 megohms, the total insulation is 2,000,000 ohms. The total insulation of the gutta-percha is expressed by 1,300,000 ohms; and is, therefore, about seventy-two times greater than that of the conductor. If we suppose Duxbury and Brest to be united by a homogeneous cable throughout the entire length, and suppose also that the remote end of the cable is put to *ground*, if the insulation is perfect, the tension will diminish regularly from its maximum at the battery end down to zero at the ground end. Under these circumstances the charge of electricity it would hold, with the battery of 36 volts, would be one half of what the same cable would contain when it was disconnected from the ground; that is, 23,580 microfarads. In the actual case, the two branches of the cable were very different; moreover, the two cables were not immediately joined, but a Leyden condenser was used at St. Pierre, one surface of which was connected with the branch which extended to Brest, and the other surface with the branch which proceeded to Duxbury.

The current sent into the cable would be, according to Ohm's formula, $\frac{E}{R+r}$. If we substitute for (E) the electromotive force of the battery used, viz. 36 volts, and for (R) the resistance of the battery (which was about 68 ohms), and for (r) the total resistance of the conductor, which was about 8,153 ohms for the St. Pierre and Brest branch, and about 8,980 ohms for the St. Pierre and Duxbury branch, adding also the resistance of the galvanometer, we should have $\frac{36}{9409}$ (or .003826 of one veber) for the strength of current between Brest and St. Pierre, and $\frac{36}{10236}$ (or .003517 of one veber) for the strength of current between St. Pierre and Duxbury. When signals were sent from Duxbury to St. Pierre, and then repeated by the condenser between St. Pierre and Brest, 3,517 microvebers flowed into the circuit every second between Duxbury and St. Pierre, and 3,826 microvebers between St. Pierre and Brest. When signals were sent from Brest to St. Pierre and then forwarded by the condenser to Duxbury, the number of microvebers circulating every second in the two branches was less than the numbers just mentioned in the ratio of about 253 to 337, since the battery used at Brest was smaller than that which operated the Duxbury end in this ratio. This maximum strength of the current would not be attained, throughout the circuit, until after connection with the battery had been maintained for an indefinitely long period of time. But ninety per cent of the maximum would be reached in the time allotted to signals of Class IV., and a large fraction of the maximum even in the times assigned to Classes II. and III. For the battery would supply the 18,756 microfarads required to charge fully the longer branch of the cable in five or six seconds, and the 4,824

microfarads for the shorter branch in less than two seconds. As the whole transmission time over both cables amounted to less than one second, it is evident that the end remotest from the battery must have begun to discharge when the cable was imperfectly charged. This might be expected when it is considered that a part of the electricity which constitutes the charge is free to move. As the transmission times for different cables are proportional to the products of the total electrostatical capacities multiplied by the total resistance, the transmission time between Duxbury and St. Pierre would be about one fourth (more exactly $\frac{10}{38}$) of the transmission time between St. Pierre and Brest. Of the whole transmission time between Duxbury and Brest, four fifths (or .639 of a second) would belong to the larger branch, and one fifth (or .177 of a second) to the shorter branch. The former was traversed at an average rate of about 4,000 nautical miles per second, the latter at an average rate of 4,230 nautical miles per second, the whole distance having been passed over at the average rate of 4,080 nautical miles per second.

The transmission time obtained by Dr. B. A. Gould in 1866, for the passage of signals between Valencia and Newfoundland, was about three tenths of one second. If we apply Thompson's formula, viz. that the time which elapses before the current reaches a stated fraction of its maximum strength is proportional to kcl^2 (or the total electrostatical capacity (cl), multiplied by the total resistance (kl),), we have the ratio of the times for signals to pass between Valencia and Newfoundland, and between Brest and St. Pierre, expressed by the fraction $\frac{(1852)^2 \times 3.89 \times .353}{(2580)^2 \times 2.93 \times .429} = \frac{47}{83}$ nearly. When the transmission time observed between Brest and St. Pierre (viz. .639 of one second) is multiplied by this ratio ($\frac{47}{83}$), the product is .36 of a second. Hence there appears to be a satisfactory agreement between the velocities of transmission, as deduced from the longitude campaign of 1866 upon the Anglo-American cable, and the longitude campaign of 1869 – 70 upon the French cable, when the two are reduced to the same standard of length, conductivity, and electrostatical capacity.

Mr. Varley[1] made experiments with a battery, varying from twelve to thirty-six of Daniell's cells, upon a cable coiled in a mass, and upon lengths of 150, 300, and 450 miles, and found the transmission times independent of the force of the battery, but proportional to the squares of the lengths of cable introduced into the circuit. He also experimented upon 270 and 540 miles of submerged cable between Dunwich and Zandvoort, and found the time on the shorter length to be only one quarter of what it was with the double length. Jenkins[2] has described some observations which he

[1] Proceedings of the Royal Society, London, XII. 211.

[2] Proceedings of the Royal Society, London, XII. 198.

made upon the Red Sea cable while it was coiled in iron tanks, and he found that, although the electromotive force had no effect upon the velocity, the rate of transmission was inversely as the square of the length. When Quetelet and Airy despatched signals between Greenwich and Brussels, in order to ascertain the difference of longitude, the observers exchanged stations with each other, during a part of the operation, for the purpose of eliminating the Personal Equation. The length of the line was 270 miles, of which about 180 miles consisted of a subterranean coated wire. The transmission time (one tenth of a second [1]) was comparatively large for so short a line. This was due, doubtless, to the great resistance and electrostatical capacity of the core, but I do not possess sufficiently accurate knowledge of its character to calculate their precise values. The same difficulty applies to Faraday's experiment [2] in which he obtained two seconds as the transmission time of electricity in passing over 1,500 miles of coated wire. Moreover, the galvanometer which he used was not the same as those employed on the transatlantic cables. Varley experimented upon 1,600 miles of the same wire, in 1854, and obtained a transmission time of three seconds, the arrival of the electrical current being recognized by its *chemical* effect.[3] Mr. Latimer Clark found the transmission time over 768 miles of coated wire to be two thirds of one second. In this last case the dimensions of the copper wire and the core are given. The formula, by which the times are compared, when the lengths, diameters, and electrostatical capacities of two lines are known, would lead us to expect in Mr. Clark's experiment a time only one half as great as the transmission time between Brest and St. Pierre. In reality it was quite as large; but no allowance could be made for the difference of galvanometers, or the specific conductivities of the wire. It should also be kept in mind, that in some of the various experiments made upon the transmission time of the current, the conducting wire was simply covered with gutta-percha, whereas, in other cases, it had, outside of the gutta-percha, a protecting armor of iron. Now it is easily conceivable, as was suggested by Mr. Varley, that magnetic induction in the iron would not be without its influence on the rate of transmission. Moreover, the theory of Ohm was framed before the discovery of the *extra* current induced in the conductor, and could not, therefore, have taken into its account any effect which that may exert upon the propagation of the primitive current from the battery. Furthermore, it should be remembered that, thus far, I have supposed that the wire, if a naked one, was uninfluenced by the surrounding

[1] London Athenæum, 1854, p. 54.

[2] Experimental Researches in Electricity, III. 512.

[3] Proceedings of the Royal Society, London, XII. 211.

air; or, if it was a cable wire, coated with gutta-percha, that it suffered no loss of electricity from the surrounding water; in other words, that, in both cases, the insulation was perfect, so that there was no leakage to the electrical current. The influence of this leakage, at least so far as the naked wires are concerned, has been carefully studied by Gaugain,[1] and the deductions from Ohm's theory have been tested by nice experiments, and with one curious result. Gaugain designates the time required by the current to reach its *highest limit* of tension at any point as the *absolute duration of the propagation*, and he calls the time which elapses before the current attains a definite percentage of this maximum the *relative duration of the propagation.* Now, when the disturbing effect of the air is taken into account, in consequence of its imperfect insulation, Ohm's theory indicates that the absolute duration of the propagation would increase *faster* than in proportion to the square of the length of the conducting wire, and that the relative velocity of propagation would increase more *slowly* than the square of the length. The absolute duration of the propagation is of no practical importance as it is indefinitely long in any case; and there can be no question as to its comparative values. But the law in regard to the relative duration of propagation was tested experimentally as follows. Two threads of cotton were taken, each $1^m.65$ in length. When tried separately, the transmission time was on the average eleven seconds. If they were placed end to end, so as to double the length, the transmission time was four times larger, or forty-four seconds. In these experiments the loss by the air was comparatively insignificant. Gaugain tried, next, two threads of silk, each four metres in length, so that the loss by the air might be sensible, compared with their own imperfect conducting power. The transmission time with the united lengths (eight metres) was only three times as great as when either was tried alone. He obtained the same result with two threads of cotton, each only one metre in length, when the current was diverted laterally by means of three pieces of silk edging symmetrically placed. The battery used in all the experiments consisted of 630 elements, of which 140 were those known as the *couronne de tasses,* and 490 were those of Pulvermacher.[2] These results accord with Ohm's more general expression for the tension, in which the influence of the air is not neglected, viz.: —

$$u = \frac{a}{2}\left(\frac{e^{\beta x} - e^{-\beta x}}{e^{\beta l} - e^{-\beta l}}\right) + a \,.\, e^{-k'\beta^2 t} \; \Sigma \frac{i\pi \sin \frac{i\pi(l+x)}{l}}{i^2\pi^2 + \beta^2 l^2} \,.\, e^{\frac{-k'\pi^2 i^2 t}{l^2}}.$$

The formula, complicated as it is, holds good only on the presumption that the disturbing influence of the air is uniform throughout the whole extent of the circuit.

[1] Annales de Chimie et de Physique, 3 S., LX. 326. [2] Annales de Chimie et de Physique, 3 S., LXIII. 201.

Gaugain despairs of realizing this essential condition upon any long telegraphic line, and he therefore concludes that it is difficult, if not impossible, to verify these laws on such a scale of magnitude. Guillemin has made many experiments upon the telegraphic lines in France. In Gaugain's experiments upon short, imperfect conductors, it appeared that the rate of propagation was no greater with strong batteries than with weak ones; and this has been the general verdict in experiments upon air and cable lines of telegraph. But Guillemin found that when the number of elements in his battery was doubled, the transmission time was diminished about ten per cent. To ascertain the rate of transmission, he employed lines varying in length from 280 kilometres (174 miles) to 1,004 kilometres (624 miles). The rate of transmission was much greater than in the simple proportion of the length, but decidedly less than in proportion to the square of the length. With 60 elements of Bunsen the transmission time over 354 miles of iron wire $4^{\text{mm.}}$ (or .157 of an inch) in diameter was only .020 of one second. Therefore the electrical disturbance travelled over this particular distance at the rate of 17,710 miles in one second.[1] It should be understood that generally the transmission time has been ascertained by the arrival of the current in sufficient force to affect a galvanometer; but in Gaugain's experiments the time required for the conductor to acquire a stated tension was determined by means of a delicate gold-leaf electroscope. The resistance *per knot* of the Anglo-Atlantic cable is less than that of the cable between Duxbury and St. Pierre, and very much less than that of the cable between St. Pierre and Brest, partly on account of difference of dimensions and partly because of differences of pressure and temperature where they are laid. Nevertheless, the total insulation of the Anglo-Atlantic cable is about 1,316,000 ohms, and that of the whole length between Brest and Duxbury 1,300,000. I infer, therefore, that little or no allowance need be made, in consequence of any large difference of leakage, in the comparison which has already been presented of the transmission times over these two long cable-lines.

We may hope that the longitude campaigns of the United States Coast Survey may have been useful, not only in determining differences of longitude, but also in throwing light on the delicate problem of the transmission of electrical disturbances. As we have pursued this discussion we have felt our obligation to Professor Winlock of the Harvard College Observatory, and to Mr. Dean and Mr. Goodfellow, and their assistants in the United States Coast Survey service, for the intelligence and energy with which the observations were planned and executed. But the best observations, however numerous, will not bring out a satisfactory result, unless they are skilfully

[1] Annales de Chimie et de Physique, 3 S., LX. 385.

handled by the computer. That this part of the work has been done laboriously and wisely by Mr. Lucius Brown, the result itself is sufficient to demonstrate.[1]

Electrical Tests of Atlantic Cables as given by Clark and Sabine.

Cable	Anglo-Atlantic.	Anglo-Atlantic.	French-Atlantic.	St. Pierre-Duxbury.
Date	1865	1866	1869	1869
Length (in knots)	1896	1852	2584	749
Diameter of Copper	.147	.147	.168	.087
Diameter of Core	.467	.467	.470	.282
Resistance of Conductor, in Ohms, per knot	4.27	4.20	3.16	12.03
Its specific Conductivity [2] (pure copper being 100)	93.09	94.63	94.33	92.63
Resistance of Dielectric in Megohms per knot [2]	349	342	235	266
Electrostatic Capacity per knot in Microfarads	.3535	.3535	.4295	.3740
Resistance of Conductor per knot (when laid) in Ohms[3]	4.01	3.89	2.93	11.12
Resistance of Dielectric per knot (when laid) in Megohms [3]	2945	2437	5200	2910

[1] In the Preface to the *Electrical Tables and Formulæ*, compiled by Latimer Clark and Robert Sabine, it is said: "It may be mentioned that, by common consent, the value at first assigned to the *farad*, as expressing the unit of capacity, has now been assigned to the *microfarad;* this was done to preserve the unity and simplicity of the system." What is called a *farad*, on page 442 of this memoir, is of the same value as the *microfarad*, introduced on page 471. It will be observed that many of the numbers given on page 442 differ from those attached to the French cable on pages 252 - 3 of the work just quoted, and generally for obvious reasons. In the computations I have adopted the more appropriate numbers which belong to the cable, *after it was laid.*

[2] At 24° Centigrade.

[3] Irrespective of Temperature and Pressure.

EXPLANATION OF THE LETTERING.

a Anus.
b Branch of water-system, leading to dorsal pore.
c Collar.
d Dorsal pore.
d′ Central dorsal vessel.
d″ Flat space between *d′* and gills.
v d″ Ventral folds between *d‴* and *s*.
d‴ Central ventral vessel.
d^iv. Flat space between *d′* and genital organs, or alimentary canal.
e Eye-specks.
f, *f′*, *f″* Folds of body-wall enclosing accumulations of mucus-secreting glands.
g Gills.
g′ Openings of gills leading outwardly.
g″ Skeleton supports of folds of gills.
g‴ Gill-folds.
g^iv. Openings of gills leading to œsophagus.
h Heart.
i Intestine.
l Lateral flat part of the sides of the body, so-called lappets of Kowalevsky's Balanoglossus.
lv Folds of liver.
m Mouth.
m b Muscular band running from eye-speck to anterior part of the water-system.
o Œsophagus.
p Skeleton at base of proboscis in Tornaria and young Balanoglossus.
p′ Proboscis.
p″ Anterior opening of proboscis of Balanoglossus.
p‴ Posterior ventral opening of proboscis of Balanoglossus.
q Genital organs.
s Stomach or alimentary canal.
u Upper lappet of posterior extremity of stomach in Tornaria.
u′ Lower lappet of posterior extremity of stomach in Tornaria.
v Circular anal vibratile band of cilia.
v′ Longitudinal undulating bands of cilia.
w Water-system.
w′, *w′* Right and left spurs of water-system.

PLATE I.

1. Tornaria seen in profile; n. s. about 2mm.
2. Tornaria seen from ventral side; n. s. 2mm.
3. Tornaria, seen from dorsal side; n. s. about 2mm.
4. Part of the œsophagus in profile, to show the commencement of the gills as a thickening of the walls on the dorsal side, from a Tornaria somewhat younger than Fig. 1.
5. Œsophagus of Tornaria, seen in profile from mouth, *m*, to its connection with stomach, about in stage of Fig. 1.

5a. Œsophagus somewhat more advanced, seen from the dorsal side; commencement of two gills on each side.

6. Somewhat more advanced than Fig. 5a.
7. Œsophagus seen from the mouth side, about in same stages as Figs. 5 and 5a.
8. Œsophagus considerably more advanced, with four gills on each side, as folds of the walls of the œsophagus.
9. Œsophagus, with gills, somewhat more advanced than in Fig. 8; loops nearly closed.
10. Œsophagus seen from the mouth side, about in the same stage as in Fig. 9.
11. Œsophagus seen in profile, same state of development as preceding Figures.
12. Part of Tornaria, seen from the dorsal side, to show the relative position of the œsophagus, water-system, heart, and gills, to the digestive cavity. This is taken at about the time when Tornaria changes into Balanoglossus.
13. The water-system in profile, in a young Tornaria.
14. The same, seen from the ventral side.
15. Water-system and heart of an older Tornaria, seen in profile.
16. Water-system and heart of an old Tornaria, seen from the dorsal side.
17. Posterior extremity of the stomach of Tornaria, to show the position of the upper and lower lappets, first formed as diverticula of the stomach, seen in profile.
18. The same seen from the ventral side.

PLATE II.

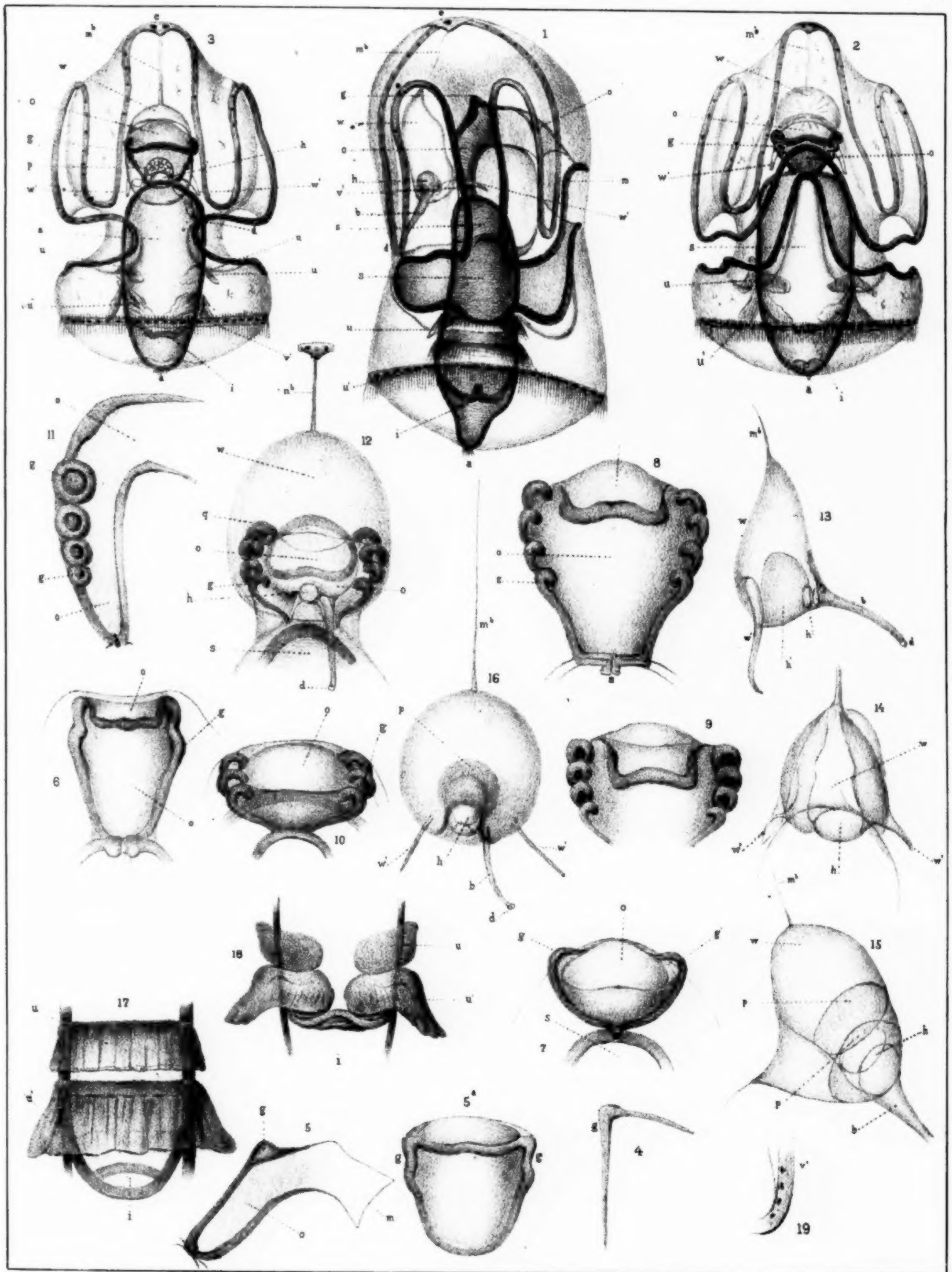
1. Tornaria, seen from the dorsal side, showing first trace of change into Balanoglossus: the posterior part of the Tornaria greatly lengthened; the longitudinal vibratile bands are reduced to indistinct lines of pigment-cells.
2. Balanoglossus in a somewhat more advanced stage, seen from the ventral side.
3. A different individual, nearly in state of Fig. 2, seen in profile.
4. Somewhat older than Fig. 3, seen from the ventral side.
5. Profile of a Balanoglossus, intermediate between Figs. 1 and 4.
6. Balanoglossus considerably older than Fig. 4, seen from the dorsal side.
7. Oldest Balanoglossus raised directly from Tornaria; body has become greatly elongated; the collar, proboscis, and posterior part of the body, are distinctly separated; the anal band of vibratile cilia is scarcely ever in movement.
8. Youngest specimen of Balanoglossus dug up from the sand, magnified two diameters, in profile.
9. The same magnified; the proboscis, collar, and gills are like those of the oldest specimens found; the genital organs and liver have as yet not been formed.
10. Anterior portion of the same somewhat more magnified.
11. Magnified portions of the convolutions of the alimentary canal, same specimen.
12. Anal portion of the same specimen, showing short intestine leading into the alimentary canal.
13. Portion of gills, to show the folds and their mode of communicating externally, from a somewhat older specimen.
14. Portion of the gills, to show mode of formation of the folds.
15. Single gill-opening, leading to œsophagus.
16. Skeleton at base of proboscis, seen from above.

16ª. Same, seen in profile: the forks are curved ventrally.

17. Skeleton supporting the folds of one of the gill-openings.

PLATE III.

1. General view of a full-grown specimen of Balanoglossus Kowalevskii, as seen lying in a dish, from the dorsal side, $\frac{1}{1}$.
2. Magnified portion of the anterior part of another specimen of the same age, seen from the dorsal side.
3. Specimen very much younger, but older than the one figured on Pl. II., f. 8, seen in profile previous to the formation of the liver and of the genital organs, $35^{mm.}$ in length.
4. Magnified portion of anterior part of same (Fig. 3), seen from the ventral side.
5. Anal extremity of specimen of Fig. 1.
6. Anterior extremity of the proboscis, to show terminal opening.
7. Anterior extremity of another specimen, about the age of Fig. 3, still showing traces of the eye-specks.
8. Posterior extremity of the proboscis, to show the ventral opening contracted.
9. Posterior extremity of the proboscis of another specimen, showing the long ventral opening.
10. Portion of anal extremity of the body, seen from the ventral side.
11. Portion of ventral part of the body immediately behind the termination of the gills, covered by glands secreting mucus.
12. Portion of the dorsal part of the body a short distance behind the gills, covered by glands secreting mucus.
13. Part of the anterior portion of the sides of the body in which eggs are developed.
14. Part of the anal extremity of the body.
15. Portion of body showing the long narrow folds forming the so-called liver.



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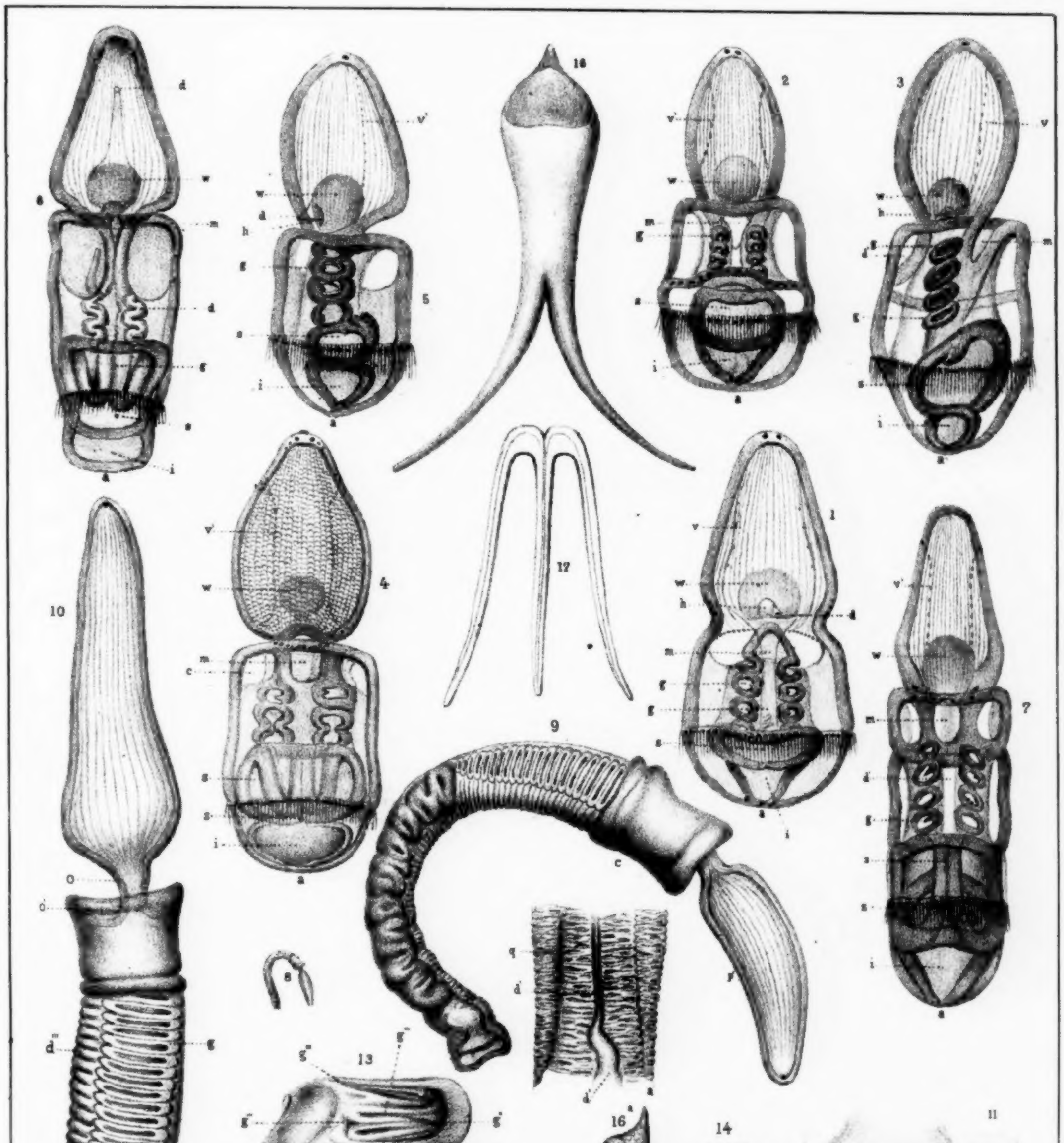

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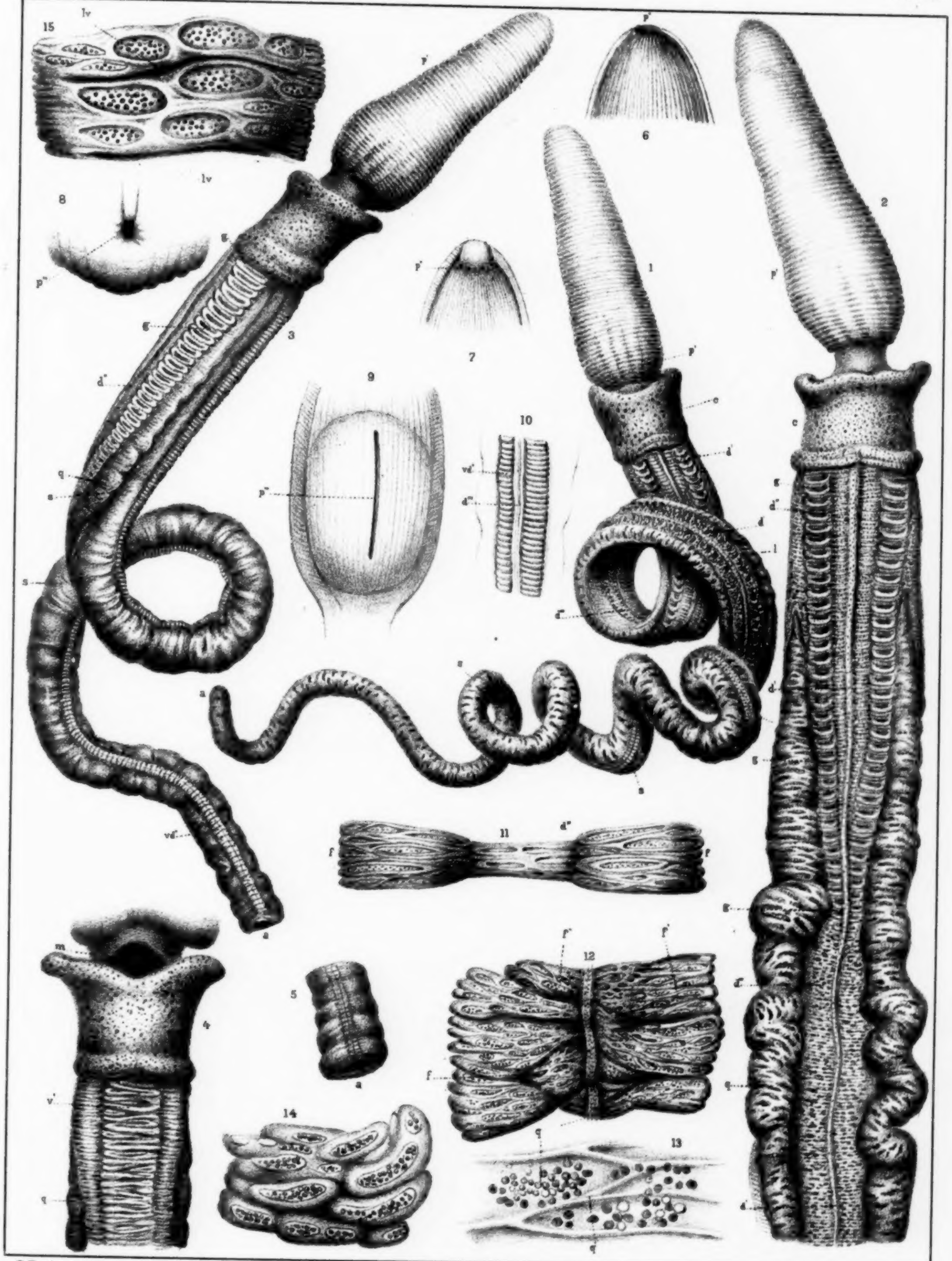
Balanoglossus. Pl. III.



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